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BY

EDWARD OLNEY,

PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF MICHIGAN, AND AUTHOR OF A SERIES OF MATHEMATICAL TEXT-BOOKS.

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PREFACE.

Multiplicity of wants and diversity of tastes are characteristics of advanced civilization. The wigwams of a savage tribe are as uniform in their structure as the nests of a species of birds; and in dress the savages are well-nigh as homogeneous as the birds. On the other hand, scarcely two dwellings in an enlightened community are precisely alike; while endless diversity is seen in the costumes of the inhabitants of the same city. So with intellectual demands. The time has gone by when a single Spelling-Book, a single Reader, and a single Arithmetic will be accepted throughout the country. The necessities of schools are varied, and the views and methods of good teachers are not the same. In reference to Arithmetic, for example, we have every variety of view, — from that which would make the mere *modus operandi* the sole aim, to that which confines attention almost exclusively to the study of principles and to ratiocination; assuming that the pupil well grounded in these cannot be greatly deficient in practical operations.

It is this diversity of demand that has led to the preparation of this text-book on Arithmetic now offered to the

public. While the Author's "ELEMENTS OF ARITHMETIC" has, in a very flattering way, met the wants of teachers who desire to make the study of principles especially prominent; there are still many teachers whose views of the philosophy of teaching, or whose circumstances, demand a less theoretical and more characteristically practical treatise. Such teachers will find their wants met by this book. All statements of principles, definitions, and rules are reduced to the most brief and simple language consistent with clearness and accuracy; and all demonstrations, illustrations, and methods of solution are made as explicit, direct, and practical as possible.

While the examples are taken, in large part, from the "ELEMENTS," some of the more complicated of the latter have been omitted, and a great number of ordinary practical exercises have been added. *As a book of work, it is believed to be richer than any hitherto offered to the public.*

The *Metric System* is presented somewhat earlier and somewhat fuller than in the "Elements;" and a special chapter on *Mensuration* has been added. In fact, the Author has availed himself of the suggestions of many friends, who have examined the "Elements," and have desired a book containing the same freshness of, and practical adaptation of problems, the same thoroughness in the exposition of principles; but have at the same time desired a book which should be somewhat less a development of a method of teaching, and more characteristically a *book of work* for the pupil. With a view to meeting the practical wants of the schoolroom in our great graded schools, this

book, at every step of its progress, has been closely scrutinized, and vigorously criticised by one of our most intelligent, experienced, and successful teachers.

As in the former volume, so in this: the methods known as "Mental" have been carefully incorporated with those known as "Written," in order to present the subject of Arithmetic in one volume as fully and completely as the needs of our public schools require. Thus, for the great body of our schools, but one book in Arithmetic will be required after the primary course.

The Author feels himself specially fortunate in securing publishers who bring out his books in the best style of the printer's art, and on the best material. The good taste of the stereotypers, the elegance of the engraved chapter-headings, the perfection of the press-work, the quality of the paper, the simple elegance and firmness of the binding, are mechanical features of no small moment in a text-book, and which in this cannot fail to challenge admiration.

EDWARD OLNEY.

UNIVERSITY OF MICHIGAN,
ANN ARBOR, July, 1879.

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OLNEY'S Practical ARITHMETIC

CHAPTER I.

FUNDAMENTAL PRINCIPLES AND RULES.

SECTION I.

READING NUMBERS.

What Numbers and Figures are.

1. A **Unit** is one.

How many *units* are there in three? How many in eight? In one? In six?

2. **Number** is an answer to the question, How many? A unit, or any collection of units, is a number.

1. Is six a number? Why?

ANS.—Six is a number because it is an answer to the question, How many?

2. Is fifteen a number? Why? Is one a number? Why?

3. Arithmetic is the elementary branch of the science of number.

---

4. Figures are characters used to represent numbers. There are ten figures; viz., —

0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

5. The last nine of these figures are called **Digits**; and the first, 0, is called **Zero**, or *Cipher*.

6. The name of any digit, and the number of units which it represents, are the same. Hence, to read a number represented by a single figure, simply pronounce the name of the figure.

What is the *name* of this digit, 6? How many units does it represent? What is the name of this, 7? How many units does it represent? Is 0 a digit?

7. The Value of a figure is the number which it represents.

What is the value of 8? Of 6? Of 1? Of 7?

8. Zero, 0, is an auxiliary[1] character, and in itself has no value. When it stands alone it signifies *naught*, or *nothing;* that is, no number.

The digits are often called *Significant Figures*, because they signify something — that is, some number — when standing alone.

How Numbers are Grouped.

9. Numbers are grouped into tens, and this way of grouping them is called the *Decimal System.*

Decimal means by tens; so that *Decimal System* means a system by tens.

Ten *units* make one TEN.

Ten *tens* make one HUNDRED.

Ten *hundreds* make one THOUSAND.

[1] Auxiliary means helping. The character 0 is called an auxiliary character because it *helps* the digits in representing certain other numbers; as 10, 100, 30, 205, etc.

10. The numbers from *ten* to *one hundred* are written and read as follows: —

Written.	Read.	Written.	Read.	Written.	Read.
10,	ten.	24,	twenty-four.	50,	fifty.
11,	eleven.	25,	twenty-five.		Etc., etc.
12,	twelve.	26,	twenty-six.		——
13,	thirteen.	27,	twenty-seven.	60,	sixty.
14,	fourteen.	28,	twenty-eight.		Etc., etc.
15,	fifteen.	29,	twenty-nine.		——
16,	sixteen.		——	70,	seventy.
17,	seventeen.	30,	thirty.		Etc., etc.
18,	eighteen.	31,	thirty-one.		——
19,	nineteen.		Etc., etc.	80,	eighty.
	——		——		Etc., etc.
20,	twenty.	40,	forty.		——
21,	twenty-one.	41,	forty-one.	90,	ninety.
22,	twenty-two.		Etc., etc.		Etc., etc.
23,	twenty-three.		——		——
				100,	one hundred.

The "*teen*" in the numbers from 13 to 19 means *ten:* so that "thirteen" means three and ten; "fourteen," four and ten, etc.

The "*ty*" in the following numbers also means *ten:* but in these cases "twenty" means two tens; "thirty," three tens, etc. Hence "twenty-one" means twenty and one; "twenty-two" means twenty and two, etc.

Exs. — Read the following, giving the meaning of each expression: 14, 13, 19, 40, 70, 30, 54, 73, 82, 48, 71, 32, 23, 56, 65, 33, 55, 91, 98, 36, 43, 34, 41, 21, 26, 62, 78, 64, 31, 71, 17, 50, 13, 80, 61, 16, 81, 18, 88, 99, 47, 74, 96, 69, 79, 89, 77, 51, 15, 28 68, 86.

To Read a Number represented by Three Figures.

PRINCIPLE.

11. *When figures are written side by side, like the letters of a word, and without any other marks, the right-hand figure represents* UNITS, *the next one* TENS, *and the next* HUNDREDS.

1. In 485 which figure represents units?

Which figure represents tens?

Which figure represents hundreds?

2. In 576 what does the 5 represent?

What do the other two figures taken together represent? Then how do we read 576?

ANS. — Five hundred seventy-six.

3. In 700 how many hundreds are represented? How many tens? How many units?

If there are no tens nor units represented in 700, and there are 7 hundreds, how will you read it?

4. In 503 how many hundreds are represented? How many tens? How many units? Then how is 503 read?

ANS. — Five hundred three.

12. Rule.—TO READ A NUMBER REPRESENTED BY THREE FIGURES. *Pronounce the number of hundreds first, and then read the other two figures as directed in the preceding rules.*

5. Read the following: 834, 235, 482, 576, 821, 763, 222, 343, 988, 889, 898, 888, 111, 644, 446, 279, 927, 972.

6. Read 708, 604, 303, 101, 105, 107, 109, 408, 404, 802, 801, 703, 602, 808, 609, 708, 906, 507, 801, 108, 406.

7. Read 530, 640, 760, 990, 810, 960, 870, 440, 110, 250, 340, 820, 910, 750, 450, 340, 880, 730, 690.

---

13. The groups of ten each into which numbers are collected are called ORDERS.

We have now become acquainted with *Units Order*, *Tens*

Order, and *Hundreds Order*, and have seen that ten hundreds make a thousand, which is the next higher order.

Ex. — In 6827 what order is 7 in? What 2? 8? 6?

While each digit always represents the same number of units, the *order* of these units is determined by the place the figure occupies. Thus 5 always represents *five* units; but in the number 2586 the units are *units of hundreds:* so, also, the 2 represents 2 tens-of-hundreds. The number of units represented by a digit is sometimes called its **Simple Value**, while its *place* or *order* value is called its **Local Value**.

Reading Numbers represented by more than Three Figures.

14. When we wish to read a number represented by a large number of figures, as 764312895, we first point the figures off into sets by putting a comma between the third and fourth, sixth and seventh, etc.; that is, after every third figure from the right.

15. Each set of figures thus pointed off is called a PERIOD. Each period, except the one at the left, must have three figures in it. The left-hand period may have one, two, or three figures in it.

16. The second period from the right is called *Thousands Period;* the next, or third from the right, is called *Millions Period;* the next, or fourth from the right, is called *Billions Period.*[1]

1. What is the thousands period in 467312895?

ANS., 312.

What is the millions period? ANS., 467.

2. What is the billions period in 24876403287?

ANS., 24.

[1] The successive periods beyond billions are trillions, quadrillions, quintillions, sextillions, septillions, and octillions, etc.

3. What is the thousands period in the last number? What the millions?

4. Point off 53212671. What is the number which stands in millions period? What in thousands period?

5. Point off 3412756, and then name in order the number standing in each period, beginning at the left.

When pointed off, this is 3,412,756.

Hence we read 3 million, 412 thousand, 756.

6. Point off and read 245642.

When pointed off, this is 245,642.

Hence it is read 245 thousand, 642.

7. Point off and read 50300806.

This is read 50 million, 300 thousand, 806.

8. Point off and read the following numbers: —

348256	12350246710	452506481835
2957643	881100604	2648300500
54203	18203700	30825605100

9. Read 2070582. What are the figures in thousands period? Ans., 070. How much does 070 represent? How many units? How many tens? How many hundreds? When these three figures, 070, stand alone and in this order, what more do they represent than 70?

If 070 is only 70, might we not omit the 0 before the 7 in the number 2070582 without altering the value? Why not? Read 270582. Read 2070582. Are they the same?

17. Rule. — To Read a Number represented by more than Three Figures. *First separate the number into periods by placing a comma before every third figure from the right. Then, beginning at the left, read the number in each period in succession according to the rule for reading a number represented by three figures, and, after the number represented in any period, pronounce the name of that period. Pass in silence over periods filled by zeros.*

Examples for Practice.

Point off and read the following:—

21050707	505050	700000	3003000001
283004005	5050505	1500000	5000011006
2000200	5010210	2000004	6000000112
50034	53457	11111111	50000000
4785	8888888	223344	4260000
124080	30608	4375682	4008000
340077	400000	370000	70060080
3000010	7200000	20009	600000008

The last is pointed off thus: 600,000,008; and read six hundred million, eight.

18. Numeration is naming the orders of the figures in a number for the purpose of reading it.

The following table will show the names of the orders and of the periods at a glance:—

---

19. NUMERATION TABLE.

Names of the PERIODS.	Billions.			Millions.			Thousands.			Units.		
NUMBER.	8	5	3,	4	2	6,	5	7	4,	3	1	9
Names of the ORDERS.	Hundreds of Billions	Tens of Billions	Billions	Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units

1. Numerate, that is, name the orders in, 72156437851, beginning at the right.

2. Numerate the following: —

421561304	142345685087	426005832
31020567	32564875190	31110050

PRINCIPLE.

20. *In the Decimal System ten of any order always makes one of the next higher order, and a thousand of any period always makes one of the next higher period.*

1. How many tens make a hundred?
2. How many hundreds make a thousand?
3. How many thousands make a tens-of-thousands?
4. How many tens-of-thousands make a hundreds-of-thousands?
5. How many thousands make a million?
6. How many millions make a billion?
7. How many billions make a tens-of-billions?
8. How many tens-of-billions make a hundreds-of-billions?

---

21. Two very important practical observations are to be made in closing this section. They lie at the foundation of most of the operations in the fundamental rules, and are of essential importance in understanding decimal fractions.

1. *Any number of figures at the right may be read as so many units.*

2. *Any number of figures at the left may be read as so many of the lowest order of those figures.*

Thus in 2536: 1. We may consider the 3 and 6 as representing 36 units; for 3 tens and 6 units are 36 units. In like manner the 5, 3, and 6 may be considered as 536 units; since 5 hundreds, 3 tens, and 6 units make 536 units. 2. The 25 may be considered as 25 hundreds; for 2 thousands

and 5 hundreds make 25 hundreds. So also the 253 may be regarded as so many *tens;* since 2 thousands, 5 hundreds, and 3 tens make 253 tens. In like manner 47287 may be read 472 hundreds and 87; or 4728 tens and 7; or 4 ten-thousands, 72 hundreds, and 87; or 47 thousand, 28 tens, and 7.

Ex. — Explain in this manner the various ways in which the following may be read: —

348	4285	785401	600820
1501	31058	13482	82654
2036	40000	5000	80140

SECTION II.

WRITING NUMBERS.

22. ***To Write in Figures any Number represented by Three Figures or less.***[1]

1. Write in figures seven; nine; six; four; eight; and two.

2. Write in figures the following numbers: Forty-eight; sixty-three; ninety-seven; eighty-two; thirty-one; fifty-five; seventy-four; seventy-seven; forty-four.

3. Write in figures the following numbers: Four hundred thirty-five; seven hundred twenty-eight; three hundred sixty-two; one hundred thirty-four; two hundred twenty-six; nine hundred forty-one; seven hundred seventy-seven; one hundred eleven; five hundred fourteen; eight hundred thirteen; two hundred twelve; three hundred thirty; seven hundred seventeen.

4. Write six hundred; five hundred; eight hundred; three

---

[1] This article affords a little drill on what it is presumed the pupil has already learned. If he has not, the teacher will need to give a little instruction before assigning the work.

hundred. Which orders will be filled with zeros in writing these numbers? Why?

5. Write four hundred twenty; seven hundred fifty; six hundred seventy; two hundred eighty. Which order will be filled with a zero in writing these numbers? Why?

6. Write four hundred seven; six hundred twenty; eight hundred one; three hundred thirty; one hundred seven; one hundred ten; five hundred fifty-five.

In writing these numbers, what order will be filled by a zero? Why?

23. This method of writing numbers in figures is often called the ARABIC NOTATION, and the ten figures used are called *Arabic Characters.*

[This is because these figures were introduced into Europe by the Moors or Arabs, and were then thought to have been invented by the Arabs. It is now known that they came from farther East, — perhaps from Thibet.]

Examples for Practice.

1. Write all the numbers from one to two hundred.

2. Write the following: Three hundred fifty-six; two hundred twenty-two; seven hundred ninety; seven hundred nine; seven hundred ninety-nine; six hundred; six hundred five; six hundred fifty; six hundred fifty-four; eight hundred eighteen; nine hundred sixteen; seven hundred; seven hundred five; seven hundred sixty-five; three hundred eighty; three hundred eighty-eight; six hundred sixty-six.

3. Write all the numbers from four hundred to six hundred.

4. Write all the numbers from three hundred twenty-six to five hundred forty-two.

5. Write as many different numbers as you can with the three digits, 5, 8, and 3, and read them. With 6, 4, and 2. 1, 3, and 9. 7, 0, and 2. 6, 0, and 0.

[The teacher can readily multiply exercises of this kind at pleasure.*]*

To Write in Figures any Whole Number whatever.

24. Rule. — *Beginning with the highest period, write in succession from left to right the number named in each period as so many hundreds, tens, and units, filling all vacant orders and periods with ciphers.*

Examples for Practice.

1. Write twenty-three million, four hundred fifty-six thousand, five hundred thirty-nine.
2. Write forty-five thousand, seven hundred sixty.
3. Write seventeen billion, one hundred eighty-one million, five hundred sixty-two thousand, two hundred seventy-eight.
4. Write four thousand, one hundred four.
5. Write seventy-five thousand, seventy-five.
6. Write six hundred five thousand, one hundred twenty-three.
7. Write eight hundred seventy-two thousand, five hundred twelve.
8. Write nine million, seven hundred sixty-five thousand, four hundred thirty-two.
9. Write three hundred forty-million, forty-three thousand, five hundred sixty-seven.
10. Write three hundred seventy-four billion, four hundred thirty-eight million, eight hundred sixty-two thousand, eight hundred forty-seven.
11. Write seventy-two million, eighty-three thousand, twenty-seven.
12. Write seventeen thousand, seventeen hundred seventeen.
13. Write 2 hundred 89 million, 1 thousand.
14. Write one million, one thousand, one.
15. Write ten million, ten thousand, ten.
16. Write one hundred sixty-seven thousand, nine hundred thirty-eight.

17. Write ten billion.

18. Write 4 hundred 69 billion, 9 hundred 31 million, 7 hundred seventy-seven.

19. Write nine hundred ninety-nine million, nine hundred ninety-nine thousand, nine hundred ninety-nine.

20. Write one hundred two billion, two hundred thousand, seven.

21. Write 3 hundred forty-seven million, 5 hundred twenty-one thousand, 8 hundred ninety-six.

22. Write 10 billion, 2 hundred 47 million, 327.

23. Write 3 hundred 4 thousand, 26.

24. Write 504 billion, 627 million, 17 thousand, 2.

25. Write 12 million, 8 thousand, six.

26. Write two hundred sixty-one billion, five hundred seventy-eight million, nine hundred thirteen thousand, one hundred twelve.

27. Write ten million, ten.

28. Write 1 million, 1 thousand, 1 hundred 1.

29. Write twenty-two million, two hundred twenty-two thousand, two hundred twenty-two.

30. Write 1 hundred 1 thousand, 2 hundred 2.

SECTION III.

THE ROMAN NOTATION.[1]

25. Dates, numbers of sections and chapters, and of the pages of an introduction to a book, are often represented by the seven letters

I, V, X, L, C, D, M.

---

[1] This section is placed here in deference to custom. It should be omitted by pupils unacquainted with addition and subtraction until they have studied those *subjects.*

26. When used to represent numbers, the values of these letters are as follows: —

I, *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* and M, *one thousand.*

27. This method of representing numbers is called the ROMAN NOTATION.

To Read a Number Represented by the Roman Notation.

28. Rule. — *Add the values of the letters, observing that, when a letter is followed by one of greater value than itself, the difference between these two is to be taken in making up the sum.*

Examples for Practice.

1. Read XXVIII.

SUGGESTION.—X is 10; V, 5; and I, 1. Then, adding the values of *all* the letters, we have 28. Hence XXVIII is 28.

2. Read XIX.

SUGGESTION. — Here the I before X diminishes the value of the latter, making IX *nine.* X is 10. Hence XIX is 10 and 9, or 19.

3. Read MDCCXLVIII.

SUGGESTION. — Here we have 1000, 500, 100, 100, 40 (the XL is 40), 5, 1, 1, and 1. Adding these, there results 1748.

29. By observing what *orders* are represented, we can read such expressions at sight.

4. Read MMDCCCLXXVII.

SUGGESTION. — Here the two M's represent *thousands;* D and the three C's, *hundreds;* L and the two X's, *tens;* V and the two I's, *units.* Hence we read 2 thousand, 8 hundred seventy-seven.

5. Read the following: —

I.	MDCCCLXXV.	XIV.
IV.	MMCCXXII.	XXIX.
LX.	CXL.	XXXIV.
VIII.	MCCVIII.	XLVIII.
XX.	XLV.	CCLXXI.
XL.	MDCXXVII.	MMCLVIII.
XXX.	MCDXCII.	MDCCCXL.
LXXX.	IV.	MDCXX.
CII.	V.	III.
VII.	XIX.	L.
VIII.	XXXVIII.	LII.
IX.	XI.	CCCXLIX.
DVI.	MMMDCCLXXXI.	XXXIX.

To Write any Number less than Four Thousand in the Roman Notation.

30. Rule.—I. *Write the letter of highest value which does not exceed the number to be written. Repeat this letter as many times as possible without exceeding the number.*

II. *Observe how much remains to be represented, and treat it in the same manner, annexing these letters to the former. Continue this process till the entire number is represented.*

Observing that

IV	is written	instead of	IIII for four.
IX	"	"	VIIII for nine.
XL	"	"	XXXX for forty.
XC	"	"	LXXXX for ninety.
CD	"	"	CCCC for four hundred.
CM	"	"	DCCCC for nine hundred.

That is, no letter is written four times in succession.

Examples for Practice.

1. Write 327 in the Roman Notation.

SUGGESTION.—C represents the highest value lower than 327, and can be written three times; thus CCC, which is 300. There now remains 27 to be represented. X is the letter next lower in value, and can be written twice; thus XX, which is 20. 7 now remains; and V is the letter next lower in value, but cannot be repeated without exceeding the number: hence we write V. There now remains 2, which is written II. Collecting the letters, we have CCCXXVII for 327.

2. Write 2738 in the Roman Notation.
3. Write 1875 in the Roman Notation.
4. Write 4, 9, 17, 28, 51, 123, 571, 120, 115, 731.
5. Write 949, 494, 3489, 2974, 1740, 1620, 1492, 1776.

[NOTE.—Observe that 4 and 9 are the only digits whose values are represented by differences.]

6. Write all the numbers from 1 to 200.

31. It will be a matter of interest to observe a sort of tenfold ratio existing between the characters used in this notation. Thus,—

10 10 10

I. 5 V. 2 X. 5 L. 2 C. 5 D. 2 M.

V is 5 times I; X, 2 times V. Hence X is 10 times I, etc.

ADDITION

SECTION IV.

32. The **Sum** of two or more numbers is the number they make when united.

33. The sign + is called *plus*, and, when placed between two numbers, indicates that they are to be added. Hence it is called the sign of addition.

34. The sign = is called the sign of equality, and signifies that what is written before it is equal to that which is written after it.

Thus 5 + 4 = 9, is read "5 plus 4 equals 9." This means the same as "5 and 4 are 9," and "9 is the *sum* of 4 and 5."

1. Read the following: 6 + 3 = 9; 5 + 2 = 7; 8 + 4 = 12; 3 + 1 = 4; 7 + 3 = 10; 1 + 1 = 2; 9 + 8 = 17; 6 + 6 = 12.
2. What is the sum of 6 and 3?
3. What is the sum of 5 and 2?

35. ADDITION TABLE.[1]

$9+9=18$	$9+8=17$	$9+7=16$	$9+6=15$
$8+8=16$	$8+7=15$	$8+6=14$	$8+5=13$
$7+7=14$	$7+6=13$	$7+5=12$	$7+4=11$
$6+6=12$	$6+5=11$	$6+4=10$	$6+3=\ 9$
$5+5=10$	$5+4=\ 9$	$5+3=\ 8$	$5+2=\ 7$
$4+4=\ 8$	$4+3=\ 7$	$4+2=\ 6$	$4+1=\ 5$
$3+3=\ 6$	$3+2=\ 5$	$3+1=\ 4$	
$2+2=\ 4$	$2+1=\ 3$		
$1+1=\ 2$			

$9+5=14$	$9+4=13$	$9+3=12$	$9+2=11$
$8+4=12$	$8+3=11$	$8+2=10$	$8+1=\ 9$
$7+3=10$	$7+2=\ 9$	$7+1=\ 8$	
$6+2=\ 8$	$6+1=\ 7$		$9+1=10$
$5+1=\ 6$			

Exercises.

1. How many are 5 and 4? Then how many are 4 and 5?
2. How many are $4+3$? Then how many are $3+4$?
3. How many are $6+4$? Then how many are $4+6$?
4. How many are $8+7$? Then how many are $7+8$?

Does the order in which the numbers are named make any difference in their sum?

5. What pairs of digits give 0 as the units figure of their sum?

6. What give 1 as the units?

7. What give 2? 3? 4? 5? 6? 7? 8? 9?

8. What figure with 4 gives 2 as units? What 3? What 7? 4? 6? 8? 5? 1?

[1] This table is given in this order so that it may be learned in, say, eleven or twelve lessons as an arbitrary effort of memory, and without the hurtful associations into which the usual method leads. See *Manual.*

9. What figure with 7 gives 2 as units? What 5? What 9? What 6? 1? 4? 3? 8? 7?

10. What figure with 2 gives 8 as units? What 1? What 7? What 5? 2? 4? 3? 6? 8? 9?

---

11. 3 and what make 7? 3 and what make 9? 5? 8? 10? 12? 4? 11?

12. 7 and what make 12? 7 and what make 15? 13? 11? 8? 16? 14? 9?

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36. It will be seen that it is about equally easy to recognize the sum of two numbers, when one is represented by two digits and the other by one, as to recognize the sum of two digits.

2	2	2	2 etc.		4	4	4 etc.
3	13	23	33		5	15	25
8	8	8	8 etc.		7	7	7 etc.
7	17	27	37		6	16	26
5	5	5	5	5	5	5	5
8	18	28	38	48	58	68	78

Examples for Practice.

1. How many are 37 and 6? 35 and 4? 39 and 7?
2. How many are 45 and 8? 46 and 9? 48 and 3?
3. How many are 64 and 7? 61 and 8? 67 and 6?
4. How many are 89 and 6? 85 and 7? 84 and 9?
5. How many are 57 and 8? 53 and 9? 56 and 8?
6. How many are 17 and 6? 18 and 9? 19 and 7?
7. How many are 78 and 5? 79 and 4? 74 and 7?
8. How many are 16 and 8? 13 and 8? 13 and 6?
9. How many are 13 and 9? 85 and 7? 76 and 8?
10. How many are 23 and 7? 43 and 6? 55 and 5?
11. How many are 42 and 8? 54 and 6? 13 and 7?

12. How many are 13 and 2? 12 and 7? 15 and 4?

13. Copy the following on your slates, and fill the blanks without counting: —

$23+6=$ ——	$11+2=$ ——	$20+2=$ ——
$28+5=$ ——	$17+8=$ ——	$20+4=$ ——
$45+5=$ ——	$10+2=$ ——	$28+6=$ ——
$49+7=$ ——	$18+5=$ ——	$29+6=$ ——
$81+9=$ ——	$19+7=$ ——	$38+7=$ ——
$87+8=$ ——	$11+9=$ ——	$59+8=$ ——
$87+9=$ ——	$12+9=$ ——	$62+9=$ ——

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1. Count 100 by twos; thus, 2, 4, 6, 8, 10, etc.

2. Count 101 by twos, beginning with 3; thus, 3, 5, 7, 9, 11, etc.

3. Count 102 by threes; thus, 3, 6, 9, 12, etc.

4. Count 100 by threes, beginning with 1; thus, 1, 4, 7, 10, 13, etc.

5. Count 101 by threes, beginning with 2; thus, 2, 5, 8, 11, 14, etc.

6. Count 100 by fours. Count by fours from 1 to 101, from 2 to 102, from 3 to 103.

7. Count by fives from 1 to 101, from 2 to 102, from 3 to 103, from 4 to 104, from 5 to 100.

8. Count by sixes from 1 to 103, from 2 to 104, from 3 to 105, from 4 to 100, from 5 to 101, from 6 to 102.

9. Count by sevens from 1 to 106, from 2 to 100, from 3 to 101, from 4 to 102, from 5 to 103, from 6 to 104, from 7 to 105.

10. Count by eights from 1 to 105, from 2 to 106, from 3 to 107, from 4 to 100, from 5 to 101, from 6 to 102, from 7 to 103, from 8 to 104.

11. Count by nines from 1 to 100, from 2 to 101, from 3 to 102, from 4 to 103, from 5 to 104, from 6 to 105, from 7 to 106, from 8 to 107, from 9 to 108.

General Rule for Addition.

37. Rule.—I. *Write the numbers one under another, so that all the units shall fall in one column, all the tens in another, all the hundreds in another, etc.*

II. *Begin with the units column, and add each column in succession. Write the units of the sum of any column under that column, and add the tens to the next column. Write down the entire sum of the last column.*

[The process of adding the tens of the sum of any particular column to the next column is sometimes called "*carrying.*"]

REASONS.—I. We write the numbers so that units shall fall in one column, tens in another, etc., in order that we may more readily see what units there are in all the numbers, and then what tens there are, etc.; as we want to add the units first, then the tens, etc.

II. We begin to add with the units, or lowest order, and proceed regularly through the orders, so that, when we have added any one order, we may know whether there are any from the lower order to add in with the higher one which we are to add next.

III. When we have gone through all the orders in this way, we have the sum of the several numbers, since we have one number which is made up of all the others put together.

38. Addition is the process of finding the sum of two or more numbers by means of a knowledge of the *Addition Table.*

There are two ways of finding the sum of numbers; viz., by *counting* and by *adding.* The Addition Table gives the sum of each possible pair of the nine digits; and only when the pupil knows all these sums, and uses this knowledge in the process, do we admit that he *adds.*

Examples for Practice in Adding.

1. Add 427, 342, 856, and 728.

FORM OF SOLUTION.
427
342
856
728
2353

EXPLANATION.—We write the numbers so that all the units shall stand in one column, all the tens in another, and all the hundreds in another, so that we may readily add the *numbers in* each order separately.

We begin to add with the units column, since, having added

it, and found the sum 23, we can readily see that there are 3 units, and, writing this in units order of the sum, can add the 2 tens in with the column of tens.

In like manner, adding the 2 tens from the sum of the units column with the tens in the tens column, we have 15 tens, which is 1 hundred and 5 tens. Hence we write the 5 tens in tens order, and add the 1 hundred in with the hundreds in hundreds column.

This 1 hundred added in with the hundreds column makes 23 hundreds, which is 2 thousands and 3 hundreds. Writing these in their proper order, we have the sum 2353.

2. Add 5648, 3726, 8972, and 6025.

3. Add 38450, 64008, 56027, 37124, 73500, and 23478.

4. Add 642, 35827, 25, 634851, 3206, and 15432.

SUGGESTION. — When written for addition, these numbers stand as in the margin. First add a column from bottom to top, and then from top to bottom, repeating the process till the results agree.

```
   642
 35827
    25
 34851
 63206
 15432
──────
```

39. *When adding a column of figures, do not name each figure, but only name the sums.* Thus, in adding the units column in this example, do not say "2 and 6 are 8, and 1 are 9, and 5 are 14," etc.; but say, or rather *think*, 2, 8, 9, 14, etc.

5. What is the sum of 50802, 345, 289764, 30726, 29, 8, and 712800?

6. What is the sum of 8, 29, 347, 5284, 70504, 2536475, 976421, 24351, 5002, 437, 50, and 5?

7. Find the sum of 78206, 843, 964271, 1853, 2679, 570012, 8206143, 77899.

8. Add 749831, 8632, 54317, 48, and 432613. The following are the figures in the answer: 1445421. How should they be arranged?

9. 181 + 24 + 897156 + 881 + 71512 gives the following figures: 456799. How should they be arranged?

10. The sum of the following numbers is an important series of figures: 48, 5627, 82160, 3475, 426, 654136, 2485796, 92541643, 743260041, and 395534538.

11. What is the sum of 3241, 476, 84324, 18472, 31421, and 62066?

12. Add 18243, 32341, 7147, 165, 2342, and 50772.

13. $156890 + 34875 + 217006 + 1000 + 2005 + 406 + 342 =$ how many?

14. $2000 + 11001 + 801 + 5000 + 88 + 5764 + 872 + 99 + 447 =$ how many?

15. $88888 + 7777 + 6666 + 55555 + 4444 + 33333 + 222 + 11111 =$ how many?

16. $576843 + 5891476 + 438275 + 789642 + 12384 + 987640 =$ how many?

17. $870095 + 984573 + 642785 + 998877 + 679488 + 1257 =$ how many?

18. $576443 + 203 + 4703 + 56428 + 121 + 2546 + 70058 + 46 + 343 =$ how many?

19. $8192735742 + 2407643728 + 544337126 + 98724603 + 825473281 + 88116457263 =$ how many?

20. What is the sum of the following numbers: three thousand four hundred sixty-five, two thousand fifty-four, nine hundred six thousand two hundred forty-seven?

ANS., 911766.

21. What is the sum of the numbers, one hundred sixty-seven thousand, three hundred sixty-seven thousand, nine hundred six thousand, two hundred forty-seven thousand, ten thousand, seven hundred thousand, nine hundred seventy-six thousand, one hundred ninety-five thousand, ninety-seven thousand? ANS., 3665000.

22. What is the sum of two hundred seven, three hundred sixty-two, nine hundred forty-five, two thousand three hundred forty-three, fifteen thousand six hundred twenty-two, and forty-five thousand eight? ANS., 64487.

23. What is the sum of eighteen thousand three hundred twenty, seventy-four thousand five hundred six, two hundred seventeen thousand nine hundred twenty-one, fifty-three thou-

sand seven hundred eleven, five hundred seventy-six thousand three hundred four, and six hundred fifty thousand forty-four? ANS., 1590806.

24. What is the sum of 3 thousand 4 hundred 92, 1 thousand 4, 6 thousand 5 hundred seventy, and 42 hundred eleven? SUM, 15277.

25. Add 386 million 591, 546 million 311 thousand 122, 796 thousand 351, 84 hundred 1, 9 thousand, 86 thousand 5 hundred 21, 3 hundred fifty-eight thousand 6 hundred, 8 million 8 hundred 88 thousand eight hundred eighty-eight, and 1 hundred million. SUM, 550855074.

26. Add six hundred forty-two, three thousand one hundred twenty-four, seventy-nine thousand nine hundred six, eight hundred twenty-four, seven hundred five, and forty-seven thousand twenty-eight. SUM, 132229.

27. Find the sum of six million sixty thousand six, seven million nine hundred fifty thousand ninety-nine, ten million nine thousand eight hundred seven, and three hundred sixty-seven thousand forty-five. SUM, 24386957.

28. Add seventy thousand four hundred fifty-three, five million eight hundred six thousand twenty-eight, eighty million ninety-seven thousand nine, and twenty-five million seven hundred thousand. SUM, 111673490.

29. What is the sum of 81 million 80, 67 thousand 80, 46 hundred 80, forty-six, 90 million 90 thousand, 900 million 900 thousand?

30. Find the sum of 200 million 302 thousand, 200 thousand two hundred, 50 million 50 thousand 50, 25 million 860 thousand, 47 million 467 thousand, 202 million 63 hundred 67.

31. Add 41 million 278 thousand, 57 million 208 hundred, 50 hundred-thousand, 25 ten-thousand and 42, 600 hundred and 20, 500 tens and 5 units.

Applications.[1]

MENTAL EXERCISES.

1. Harry had 7 books, and his father gave him 4 more. How many books had he then?

SOLUTION.—If Harry had 7 books, and his father gave him 4 more, he then had the sum of 7 books and 4 books, which is 11 books.

2. If James rode 8 miles and walked 3, how far did he travel in all?

3. A merchant sold from a piece of cloth 2 yards at one time, 3 yards at another, and 6 yards at another. How much did he sell from the piece in all?

4. Jane is 7 years old, and her brother George is 6 years older than she is. How old is George?

SOLUTION.—If Jane is 7 years old, and George is 6 years older than she, George's age is $7 + 6$, or 13 years.

[NOTE.—Avoid stereotyped forms of solution.]

5. A farmer had 13 head of cattle, and bought 8 more. How many had he then?

6. Mr. A. had 37 sheep; Mr. B. had 9 more than Mr. A. How many had Mr. B.?

7. Henry earned 37 cents on Monday, 28 cents on Tuesday, 56 cents on Wednesday, 48 cents on Thursday, was idle Friday, and earned 19 cents on Saturday. How much did he earn during the week?

8. Mr. Jones bought a horse for 250 dollars, and sold it for 37 dollars more than he gave for it. For how much did he sell it?

9. James bought an orange for 8 cents, and a melon for 5 cents more than the orange cost. How much did he pay for both?

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[1] In such exercises as the following, the main purpose is, not to give practice in adding, but to develop the ability to see *when* and *why* we must add; i.e., to notice what operations the conditions of a problem require. Let this be remembered in class explanation, and a good form of solution be always required.

10. There are 6 boys and 3 girls in one class, and 7 girls and 4 boys in another class. How many pupils are there in both classes?

11. John bought a knife for 23 cents, and sold it for 3 cents more than he gave for it. How much did he receive for it?

12. There are 30 days in June, and 31 each in July and August. How many days in these three summer months?

13. Henry gave 85 cents for a sled, and 9 cents more for a pair of skates than for the sled. How much did the skates cost him?

14. There are 7 days in a week. In July there are 4 weeks and 3 days. How many days in July?

15. John bought some nuts for 37 cents, and some flowers for 8 cents. He sold the nuts for 5 cents more than they cost him, and sold the flowers for 6 cents. How much did he receive for the nuts and the flowers?

16. Henry bought 2 tops for 7 cents each, and sold one for 2 cents more than it cost him, and the other for 3 cents more than he sold this. How much did he receive for the tops?

17. Mary bought a paper of needles for 6 cents, a comb for 18 cents, and had 8 cents left. How much did she spend for the comb and needles?

18. John attended school 27 days, staid out and worked 8 days, was sick 9 days, and played truant 7 days. How many days was he out of school?

19. 4 boys had 7 marbles each, 2 had 3 each, and one had 8. How many marbles had the 7 boys?

20. If it is 2 feet from the ground to the top of the foundation of my house, and the first story is 11 feet, the second 10, the ridge of the roof 8 feet above the upper ceiling, the chimney top 4 feet above the ridge, and the lightning-rod extends 3 feet above the chimney, how high is the top of the lightning-rod above the ground?

Written Exercises.

21. A farmer has 47 acres in wheat, 36 in corn, 52 meadow-land, 18 in oats; his house and barn yards and garden contain 2 acres; he has 43 acres of pasture, 17 acres occupied as an orchard and with vegetables, and 81 acres of woodland. How many acres in his farm?

22. A farmer's stock consists of 27 cattle, 126 sheep, 7 horses, and 15 hogs. How many animals has he in all?

23. It is 30 miles from Detroit to Ypsilanti, 8 miles from Ypsilanti to Ann Arbor, 38 miles from Ann Arbor to Jackson, 20 miles from Jackson to Albion, 12 miles from Albion to Marshall, 13 miles from Marshall to Battle Creek, 23 miles from Battle Creek to Kalamazoo, 47 miles from Kalamazoo to Niles, 37 miles from Niles to Michigan City, and 56 miles from Michigan City to Chicago, — these places occurring in order along the Michigan Central Railroad. How far is it from Detroit to Chicago?

How far from Ann Arbor to Kalamazoo?

How far from Ypsilanti to Jackson?

How far from Jackson to Niles?

24. In a certain house there were 2 parlors, one of which required 37 yards of carpeting, and the other 42 yards; a sitting-room requiring 28 yards, 2 bed-rooms requiring 16 yards each, and 2 other bed-rooms requiring 12 yards each. How much carpeting was required for all these rooms?

25. A drover paid 17428 dollars for 530 head of cattle, 7689 dollars for 125 head, and 63850 for 1225 head. How much did he pay for all? How many cattle did he buy?

26. A gentleman is 15 years older than his wife, and she is 20 years older than their eldest son, who is 29 years of age. Required the gentleman's age, and the age of his wife.

ANS. — The gentleman's age is 64 years; his wife's, 49.

27. A farmer bought three plantations for 3750 dollars

each, and sold them again so as to make 1000 dollars on the whole. For what sum did he sell the three? Ans., 12250.

28. Several persons contributed towards the establishment of a library. A gave 200 dollars, and B 50 dollars more than A; C gave 300 dollars, and D 25 dollars more than C. What was the whole amount contributed?

Ans., 1075 dollars.

29. At the battle of Moscow there were 13000 Russians killed, 5000 taken prisoners, about 27000 wounded, and 40 generals either killed, wounded, or taken prisoners; 2500 of Napoleon's army were killed, 7500 wounded, and 15 generals either killed or wounded. What was the total loss?

Ans., 55055.

30. At the battle of Waterloo the French lost 40000 men, the Prussians 38000, the Belgians and Dutch 8000, the Hanoverians 3500, and the English about 12000. How many men were killed in all?

31. A merchant bought cloth for 375 dollars, and silk for 95 dollars. In selling he gained 50 dollars on the cloth, and 45 dollars on the silk. For what sum did he sell the whole?

Ans., 565 dollars.

32. A provision-dealer bought a load of potatoes in the morning containing 48 bushels, out of which he sold 12 bushels to one man, and 5 to another. He then bought a load of 37 bushels, and put them in with what were left of the former load. After this he sold to one man 7 bushels, to another 16, and to another 8. Finally he bought 63 bushels, and put them into the same bin with the others. How many potatoes did he buy? How many did he sell?

33. The mariner's compass was invented in China 1120 years before Christ; America was discovered by Columbus 1492 years after Christ; and steam was first applied by Fulton to propelling boats 315 years after the discovery of America. How many years after the invention of the mariner's compass was steam first applied to propelling boats?

Federal Money.

40. Federal Money is the money of the United States. It is commonly reckoned in Dollars and Cents.

41. One hundred cents make a dollar.

42. In writing dollars and cents together in one number, the cents are written at the right of the dollars, and a . called a DECIMAL POINT is placed between them.

43. There must always be two places allowed for cents, so that if the cents are less than 10 a 0 must be placed in the left-hand place, or just after the decimal point.

44. The character $ indicates dollars.

45. In adding Federal money, the numbers are written so that the decimal points all fall in the same column. The adding is the same as in common addition, and the decimal point in the sum is placed under that in the numbers added.

1. Write and add 12 dollars 57 cents, $8 nine cents, five dollars, one hundred dollars 50 cents, and $40.17.

$12.57
8.09
5.00
100.50
40.17
$166.33

2. Bought a barrel of flour for $6.50, a quarter of beef for $18.57, a load of wood for $5.25, and a ton of hay for $11. What did all cost? ANS., $41.32.

3. What is the sum of thirty dollars and fifteen cents, eight dollars and seven cents, three hundred and fifty dollars, two hundred dollars and seventy cents, eighty-five cents, ten dollars and three cents, and twenty-seven dollars and sixty-four cents?

4. A gentleman bought in the market one morning as follows: Two pounds of butter, $0.50; three pounds of tenderloin, $0.60; one quart of new potatoes, $0.25; 5 pounds of sweet potatoes, $0.50; a bunch of radishes, $0.10; two quarts

of strawberries, $0.60. How much did his marketing come to? Ans., $2.55.

5. A lady bought of a grocer three pounds of raisins for $0.75, fifteen pounds of sugar for $1.50, one pound of tea for $1.00, a jar of pickles for $0.75, two pounds of starch for $0.25. How much did the lady expend in all?

Ans., $4.25.

6. A little girl bought a jumping-rope for $0.20, a doll for $3.50, a story-book for $0.50, a tea-set for $1.50, and a box of dominoes for $1.75. How much did all the playthings cost? Ans., $7.45.

7. A lady went to a dry-goods store and purchased sixteen yards of calico for $2.00, twelve yards of domestic for $3.00, six spools of thread for $0.60, four papers of needles for $0.25, four bunches of tape for $0.60, three dozen pearl buttons for $0.45. How much did all the articles cost?

Ans., $6.90.

8. A boy bought eight oranges for $0.45, a pound of candy for $0.25, four lemons for $0.20, half a bushel of apples for $0.30, and a pound of almonds for $0.50. How much did all cost? Ans., $1.70.

9. A lady went to a drug-store and bought one bottle of hair-oil for $1.00; six cakes of toilet-soap, $1.50; a bottle of cologne, $2.00; a comb, $0.75; a tooth-brush, $0.40; two ounces of arnica, $0.20. How much did all cost?

Ans., $5.85.

10. Add 12 dollars 8 cents, 1 dollar 25 cents, 3 dollars 7 cents, thirty cents, seventy-five cents, 9 cents, eight dollars, 7 dollars 2 cents, 15 dollars ten cents, nine dollars 2 cents, 47 cents, one dollar eighteen cents, 5 dollars three cents, 2 dollars, 11 dollars, 37 cents, 7 cents, 3 dollars 4 cents, and 25 dollars sixty cents.

11. Find the sum of $7.48, $13.42, $87, $150 and 6 cents, 39 dollars and 70 cents, $520 and 8 cents, $7 and forty cents,

eighty cents, $90.18, $400, $376, 78 dollars and twenty-seven cents, $150.50, $11.21, $5.60, $13,20, $0.96, $420, $34.81, $57.83, forty cents, nine cents, $5.73, $340.18, $67, $41.20, $16.85, $158.47, $340, $18.50, and $200 and four cents.

Accountants are required to add long columns of figures with rapidity and accuracy. The following are specimens. Let them be added with care.

12.	13.	14.
$ cts.	$ cts.	$ cts.
8.37	.78	673.28
4.33	.47	597.84
7.62	.53	3426.87
.48	2.75	219.48
.97	1.20	8.37
2.50	4.37	167.84
6.19	8.29	5986.32
10.00	13.85	6749.31
4.28	2.00	4863.27
8.07	.62	7542.35
4.37	.25	2986.28
9.48	1.37	379.87
4.21	9.83	2.59
13.26	6.75	69.80
1.20	8.43	4060.75
.57	20.48	309.71
3.08	6.00	124.87
4.96	1.00	8520.06
.85	1.50	2493.28
4.00	7.69	48.75

15. A quarter of a dollar is 25 cents, and a half-dollar is 50 cents. What part of a dollar is 2 quarters? 3 quarter-dollars make how many cents? 3 quarters and 3 halves?

16. A drover bought 4 horses, paying for one $175; for two, $160 each; and for the other, $97. He sold them so as to make by the bargain as much as the cheapest and most expensive horses cost him. How much did he receive for all?

17. A merchant bought 5 pieces of cloth: 2 for $37.75 each, 1 for $7.93, and the other 2 for $11.25 and $13 respectively. He sold the first three pieces at a profit of $3.25 each; the fourth he sold for $11, and the fifth for $9.75. How much did he receive for all?

18. Mr. A. hired 4 men by the month. To the first he agreed to pay $17.50 per month; to the second and third, $3.25 each per month more than to the first; and to the fourth, as much as to the first and second. What would be the amount of 2 months' wages for the 4 men?

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46. *Making Change.*

1. Having bought 37 cents' worth of goods, I hand the clerk a $2 bill. How will he count the change due me?

Ans.—He will say "37," and, laying out 3c., say "40." Then he will lay out a 10c. piece, and say "50;" then a 50c. piece, and say "$1;" and then a dollar bill, and say "$2." Thus he counts on to the amount to be taken out by filling out the even parts of a dollar, and then counting on the remaining dollar.

2. Having quarter-dollars and $1 bills, how will the change for $1.25 be counted out of $5? How, if I have only 5c., 10c., and $1 pieces?

Ans. to Last. "$1.25, 30, 40, 50, 60, 70, 80, 90, $2, $3, $4, $5." The pieces handed out being *one* 5c., *seven* 10c. pieces, and *three* $1 bills.

3. Having 1c., 2c., 5c., and half-dollar pieces, how will the change for 27c. be counted out of $1? What pieces will be given?

Ans. — The pieces given will be *one* 1c., *one* 2c., *four* 5c., and *one* 50c. The counting will be, "27, 28, 30, 35, 40, 45, 50, $1."

4. Having $5, $2, and $1 bills, and 1c., 5c., and quarter-dollar pieces, how will the change for $2.13 be counted out of a $10 bill?

5. Having 1c., 10c., and 25c. pieces, how will the change for 7c. be counted out of a 50c. piece?

6. Having 2c., 3c., 10c., and 50c. pieces, and $1, $2, and $10 bills, how will the change for $3.17 be counted out of a $20 bill?

Ans. "$3.17, 20, 30, 40, 50, $4, $6, $8, $10, $20;" the pieces given in change being *one* 3c., *three* 10c., *one* 50c., *three* $2 bills, and *one* $10 bill.

7. With the same pieces as in the last, how will the change be made for $11.26 out of a $50 bill?

8. With 1c., 2c., and 10c. pieces, how will the change for 15c. be made out of 50c.? How 45c. out of $1? How 8c. out of 25c.?

9. With 1c., 2c., 10c., and 25c. pieces, and $2 bills, how will the change for 62c. be made out of a $5 bill?

The counting is, "62, 63, 65, 75, $1, $3, $5."

The pieces used are *one* 1c., *one* 2c., *one* 10c., *one* 25c., and *two* $2 bills.

10. With 1c., 10c., and 50c. pieces, how will the change be made for 32c. out of a $2 bill? How for 37c.? How for 12c. out of a $1 bill?

11. With quarters, 10c., and 1c. pieces, how will the change for 37c. be counted out of $1? How for 16c.? For 52c.? For 45c.? For 23c.? For 81c.? For 47c.?

12. With $1 and $5 bills, and half-dollars, 10c., 5c., and 1c. pieces, how will the change for $2.36 be counted out of a $20 bill? For $7.13? For $11.51?

Subtraction

SECTION V.

47. **Subtraction** is a process of taking one number from (i.e., out of) another.

The number to be subtracted is called the **Subtrahend.**

The number from which the Subtrahend is to be taken is called the **Minuend.**

What is left of the *Minuend* after the Subtrahend has been taken out is called the **Remainder.**

48. The sign — is called *minus*, and indicates that the number after it is to be subtracted from the number before it; thus $11 - 6$ means that 6 is to be subtracted from 11.

This sign is read *minus*, which means less; so that $11 - 6$ is read "11 minus 6."

1. What is $9 - 4$?

What number with 4 makes 9?

2. What is $7 - 5$? Why?

Ans., $7 - 5$ is 2, because $5 + 2 = 7$.

3. What is $15 - 8$? Why? What is $16 - 7$? Why?

4. If 3 is one of two parts of 11, what is the other? $11 - 3 =$ what?

5. If 7 is one of two parts of 15, what is the other? $15 - 7 =$ what?

Three dots placed thus ∴ are read "therefore."

6. Copy on your slate and fill out the table on the following page: —

49. SUBTRACTION TABLE.

When 1 is one part.

$1 + 1 = \text{—}. \therefore \{\, 2 - 1 = \text{—}.$

$2 + 1 = \text{—}. \therefore \begin{cases} 3 - 1 = \text{—}. \\ 3 - 2 = \text{—}. \end{cases}$

$3 + 1 = \text{—}. \therefore \begin{cases} 4 - 1 = \text{—}. \\ 4 - 3 = \text{—}. \end{cases}$

$4 + 1 = \text{—}. \therefore \begin{cases} 5 - 1 = \text{—}. \\ 5 - 4 = \text{—}. \end{cases}$

$5 + 1 = \text{—}. \therefore \begin{cases} 6 - 1 = \text{—}. \\ 6 - 5 = \text{—}. \end{cases}$

$6 + 1 = \text{—}. \therefore \begin{cases} 7 - 1 = \text{—}. \\ 7 - 6 = \text{—}. \end{cases}$

$7 + 1 = \text{—}. \therefore \begin{cases} 8 - 1 = \text{—}. \\ 8 - 7 = \text{—}. \end{cases}$

$8 + 1 = \text{—}. \therefore \begin{cases} 9 - 1 = \text{—}. \\ 9 - 8 = \text{—}. \end{cases}$

$9 + 1 = \text{—}. \therefore \begin{cases} 10 - 1 = \text{—}. \\ 10 - 9 = \text{—}. \end{cases}$

When 2 is one part.

$2 + 2 = \text{—}. \therefore \{\, 4 - 2 = \text{—}.$

$3 + 2 = \text{—}. \therefore \begin{cases} 5 - 2 = \text{—}. \\ 5 - 3 = \text{—}. \end{cases}$

$4 + 2 = \text{—}. \therefore \begin{cases} 6 - 2 = \text{—}. \\ 6 - 4 = \text{—}. \end{cases}$

$5 + 2 = \text{—}. \therefore \begin{cases} 7 - 2 = \text{—}. \\ 7 - 5 = \text{—}. \end{cases}$

$6 + 2 = \text{—}. \therefore \begin{cases} 8 - 2 = \text{—}. \\ 8 - 6 = \text{—}. \end{cases}$

$8 + 2 = \text{—}. \therefore \begin{cases} 10 - 2 = \text{—}. \\ 10 - 8 = \text{—}. \end{cases}$

$9 + 2 = \text{—}. \therefore \begin{cases} 11 - 2 = \text{—}. \\ 11 - 9 = \text{—}. \end{cases}$

When 3 is one part.

$3 + 3 = \text{—}. \therefore \{\, 6 - 3 = \text{—}.$

$4 + 3 = \text{—}. \therefore \begin{cases} 7 - 3 = \text{—}. \\ 7 - 4 = \text{—}. \end{cases}$

$5 + 3 = \text{—}. \therefore \begin{cases} 8 - 3 = \text{—}. \\ 8 - 5 = \text{—}. \end{cases}$

$6 + 3 = \text{—}. \therefore \begin{cases} 9 - 3 = \text{—}. \\ 9 - 6 = \text{—}. \end{cases}$

$7 + 3 = \text{—}. \therefore \begin{cases} 10 - 3 = \text{—}. \\ 10 - 7 = \text{—}. \end{cases}$

$8 + 3 = \text{—}. \therefore \begin{cases} 11 - 3 = \text{—}. \\ 11 - 8 = \text{—}. \end{cases}$

$9 + 3 = \text{—}. \therefore \begin{cases} 12 - 3 = \text{—}. \\ 12 - 9 = \text{—}. \end{cases}$

When 4 is one part.

$4 + 4 = \text{—}. \therefore \{\, 8 - 4 = \text{—}.$

$5 + 4 = \text{—}. \therefore \begin{cases} 9 - 4 = \text{—}. \\ 9 - 5 = \text{—}. \end{cases}$

$6 + 4 = \text{—}. \therefore \begin{cases} 10 - 4 = \text{—}. \\ 10 - 6 = \text{—}. \end{cases}$

$7 + 4 = \text{—}. \therefore \begin{cases} 11 - 4 = \text{—}. \\ 11 - 7 = \text{—}. \end{cases}$

$8 + 4 = \text{—}. \therefore \begin{cases} 12 - 4 = \text{—}. \\ 12 - 8 = \text{—}. \end{cases}$

$9 + 4 = \text{—}. \therefore \begin{cases} 13 - 4 = \text{—}. \\ 13 - 9 = \text{—}. \end{cases}$

When 5 is one part.

$5 + 5 = \text{—}. \therefore \{\, 10 - 5 = \text{—}.$

$6 + 5 = \text{—}. \therefore \begin{cases} 11 - 5 = \text{—}. \\ 11 - 6 = \text{—}. \end{cases}$

$7 + 5 = \text{—}. \therefore \begin{cases} 12 - 5 = \text{—}. \\ 12 - 7 = \text{—}. \end{cases}$

$8 + 5 = \text{—}. \therefore \begin{cases} 13 - 5 = \text{—}. \\ 13 - 8 = \text{—}. \end{cases}$

$9 + 5 = \text{—}. \therefore \begin{cases} 14 - 5 = \text{—}. \\ 14 - 9 = \text{—}. \end{cases}$

When 6 is one part.

$6 + 6 = \text{—}. \therefore \{\, 12 - 6 = \text{—}.$

$7 + 6 = \text{—}. \therefore \begin{cases} 13 - 6 = \text{—}. \\ 13 - 7 = \text{—}. \end{cases}$

$8 + 6 = \text{—}. \therefore \begin{cases} 14 - 6 = \text{—}. \\ 14 - 8 = \text{—}. \end{cases}$

$9 + 6 = \text{—}. \therefore \begin{cases} 15 - 6 = \text{—}. \\ 15 - 9 = \text{—}. \end{cases}$

When 7 is one part.

$7 + 7 = \text{—}. \therefore \{\, 14 - 7 = \text{—}.$

$8 + 7 = \text{—}. \therefore \begin{cases} 15 - 7 = \text{—}. \\ 15 - 8 = \text{—}. \end{cases}$

$9 + 7 = \text{—}. \therefore \begin{cases} 16 - 7 = \text{—}. \\ 16 - 9 = \text{—}. \end{cases}$

When 8 is one part.

$8 + 8 = \text{—}. \therefore \{\, 16 - 8 = \text{—}.$

$9 + 8 = \text{—}. \therefore \begin{cases} 17 - 8 = \text{—}. \\ 17 - 9 = \text{—}. \end{cases}$

$9 + 9 = \text{—}. \therefore \{\, 18 - 9 = \text{—}.$

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1. Take 425 from 768.

SUGGESTIONS.—As we want to take units from units, tens from tens, and hundreds from hundreds, *if we can*, it will be convenient to write the numbers so that the orders of the subtrahend shall fall under the corresponding orders of the minuend.

768
425

Will it make any difference in this case whether we subtract the 4 hundreds first, or the 2 tens, or the 5 units?

5 units from 8 units leave how many units?

2 tens from 6 tens leave how many tens?

4 hundreds from 7 hundreds leave how many hundreds?

Then 425 from 768 leaves how many?

2. From 856 take 324. From 743 take 531.

3. From 9872 take 5040. From 80460 take 3000.

4. From 3739 take 516. From 7892 take 321.

SUGGESTIONS.—After we have subtracted the 6 units from the 9 units, the 1 ten from the 3 tens, and the 5 hundreds from the 7 hundreds, there is nothing to take from the 3 thousands: hence 3 thousands remain.

3739	*Min.*
516	*Sub.*
3223	*Rem.*

5. From 562 take 237.

SUGGESTIONS.—Write the numbers as heretofore. How can we take the 7 units from the 562? How many units are there in the units order of 562? If we take one of the 6 tens, how many units will it make together with the 2 units? Now, taking 7 units from 12, how many remain? Again, how many tens were we to subtract? How many have we left from which to take the 3 tens? Why only 5 tens? 3 tens from 5 tens leave how many tens? What remains to be taken out? 2 hundreds from 5 hundreds leave how many?

562
237
325

6. From 4327 take 1563. From 53546 take 7364.

SUGGESTION.—When in this example we come to subtract the 3 hundreds from the 4 hundreds (one hundred having been taken from the 5 in order that we might subtract the 6 tens), we find 1 hundred left. Now, instead of thinking of the 5 hundreds as diminished by 1, suppose we had *added* 1 to the 3 hundreds, and then taken 4 hundreds from 5 hundreds: would the *result* have been different?

53546
7364
46172

General Rule for Subtraction.

50. Rule. — I. *Write the subtrahend under the minuend, so that each figure shall fall under one of the same order in the minuend.*

II. *Begin the work of subtraction with units, and subtract each figure in the subtrahend from the one directly over it.*

III. *When the figure in the subtrahend exceeds in value the one over it, add 10 to the upper, and then subtract. Having done this, consider the next digit in the minuend as diminished by 1, or the next in the subtrahend as increased by 1, and then proceed with the subtraction.*

Reasons. — The reason for writing units under units, tens under tens, etc., is that we can take units from units only, tens from tens, etc., and by placing them thus we can see at a glance how many there are in the minuend of the particular order which we are subtracting. There is no reason, except custom, for putting the subtrahend *under* the minuend. It is often more convenient to put it above.

The reason for beginning the subtraction with the units, and proceeding in succession through the higher orders, is, that we may have to go over our work but once. Thus, in taking 158 from 473, if we take out the 1 hundred and the 5 tens before we do the 8 units, we shall have 3 hundreds and 2 tens left; and, having written these in their places, we would have to take 1 of the 2 tens to put with the 3 units, in order to be able to get the 8 units out. This would oblige us to change the 2 tens to 1 ten.

$$\begin{array}{r} 473 \\ \underline{158} \\ 3\cancel{2} \\ \underline{15} \\ 315 \end{array}$$

When a figure in the subtrahend exceeds in value the one over it, we have to take 1 of the next higher order of which there are any in the minuend, in order that we may be able to take out the number required. This, of course, diminishes this next figure by 1, which we must not forget when we subtract the next figure. But, as it would leave the same remainder to *add* 1 to the next *lower* figure as to *subtract* one from the *upper*, we may do the latter, instead of the former, if we choose.

Ex. — From 5426 take 2834.

Explanation. — We write the subtrahend, 2834, under the minuend, 5426, because it is customary to do so.	Form of Operation.
We put units under units, tens under tens, etc., because we can take units from units only, tens from tens only, etc.	5426 2834
Now, 4 units from 6 units leave 2 units.	
As we cannot take 3 tens from 2 tens, we take one of the	2592

4 hundreds, which is 10 tens, and the 2 tens make 12 tens. Now, 3 tens from 12 tens leave 9 tens.

8 hundreds we cannot take from 3 hundreds: so we take 1 of the 5 thousands, which makes 10 hundreds, and, adding it to the 3 hundreds, have 13 hundreds. 8 hundreds from 13 hundreds leave 5 hundreds.

Finally, 2 thousands from 4 thousands leave 2 thousands.

51. Note. — When a figure in the minuend is less than the corresponding one in the subtrahend, most people simply call the upper figure 10 more, and, having subtracted, *add* 1 to the next higher figure in the subtrahend. Then, when they subtract this increased figure, they take the 1 out of the corresponding order in the minuend.

Examples for Practice.

1. What is 856432—648571? 32564—1768?
2. From 205643 take 32589. From 70542 take 30256.
3. From 72382 take 165. From 46537 take 2006.
4. From 523654 take 82465. From 10125 take 908.
5. From 132406 take 65348. From 2007 take 407.
6. From 34652 take 15836. From 548300 take 83.
7. From 82654 take 34271. From 8001 take 750.
8. From 10000 take 546. From 1000 take 999.
9. Minuend, 78206; subtrahend, 35264. What is the remainder?
10. Subtrahend, 10956; minuend, 235043. What is the remainder?
11. What is 564023—234560? 782005—12758?
12. What is 30012—15461? 65430—718?
13. Subtract 5056 from 100000. From 601 take 307.

14. Take 3 from 1000. Take 208 from 10000.

15. From 7003 subtract 1815. From 8111 subtract 88.

16. From 15 thousand 5 hundred seventy-six take 8 thousand eight.

17. From 2 million take 50 thousand.

18. From 3 hundred thousand five hundred take 27 thousand sixty-six.

19. From 65 million take 650 thousand 980.

20. From 9 hundred thousand take 5 hundred and 50.

21. From 1 million take 960 thousand.

22. From 460 million take 920 thousand 750.

23. From 640 thousand take 14.

Federal Money.

52. In order to subtract Federal money, we have to remember that the two orders at the right of dollars are cents, and that both places must always be filled. Write the numbers for subtraction the same as in addition, so that the decimal points shall fall in the same column.

Examples.

1. From $15.25 take $9.75.

2. Subtract $38 and 7 cents from $100 and 4 cents.

Rem., $62.03.

3. Take seventeen dollars and six cents from eighty-one dollars and seventy cents.

4. From thirty-six dollars take $8 and 9 cents.

5. From $1 take 37 cents.

6. From $5 take $1 and 20 cents.

7. Take 87 cents from $2.

8. Subtract $5 and 67 cents from $10.

9. From $5 take $3 and 35 cents.

10. From $20 take $11.42.

Proof of Subtraction.

1. If one part of 13 is 7, what is the other part?

2. A man has a farm of 400 acres. Part is woodland, and part is cultivated. The former part is 125 acres. How much is the latter?

3. If the remainder is what is left of the minuend after taking the subtrahend out, what do the remainder and subtrahend, when added together, make?

4. As the remainder is one part of the minuend, and the subtrahend the other, what will you obtain by taking the remainder out of the minuend?

53. Any device by which we may test the accuracy of an operation in arithmetic by some other operation is called a **Proof** of the work.

54. *To prove subtraction, add the remainder to the subtrahend; and, if the work is right, the sum will equal the minuend.*

Or take the remainder from the minuend; and, if the work is right, what is left will be equal to the subtrahend.

[The pupil should give the reasons.]

4 to 7. Solve the following examples, and prove them in both the above ways: —

(4)	(5)	(6)	(7)
528643	500608	182005	517000
216805	23471	37050	134056

Applications.

1. Harry had 8 rabbits, and sold 5 of them. How many had he left?

If Harry had 8 rabbits, and sold 5 of them, he had remaining 8 — 5, or 3 rabbits.

2. There were 11 eggs in the nest, and Mary took out 7. How many remained?

3. A farmer raised 347 bushels of potatoes, and, selling 259 bushels, reserved the remainder for his own use. How many bushels did he reserve?

4. If I borrow 17 dollars, and afterward pay 9 dollars on the debt, how much do I still owe?[1]

5. A well was sunk through sand and clay to the depth of 30 feet. 6 feet was sand. How much was clay?

6. A drover left Texas with 2782 head of cattle. On the way to Chicago 547 of them died. How many remained?

7. A person on a journey of 735 miles has travelled 93 miles of the distance. What distance has he yet to travel?

8. From a farm which contained 2350 acres, 1234 acres were sold. How many acres remained?

9. A young man received from his father $5325, of which he paid $2500 for a house. What remained?

10. A merchant deposited $5800 in bank, but afterwards made a draft upon it for $3270. What sum remained?

11. Suppose a farmer, who has 4000 bushels of wheat in his granary, should take out 2100 bushels to be sent to market. How many bushels would remain?

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12. In one bunch there are 22 grapes, and in another 15. If you were to pick as many from the larger bunch as there are in the smaller, how many would remain? How many more are there in the larger bunch than in the smaller?

What is the *Difference* between the number in the larger and that in the smaller?

55. The Difference *between two numbers is what is left of the larger after the smaller is subtracted.*[2] *If the numbers are equal, there is no difference, or the difference is 0.*

---

[1] When the numbers are small, the slate should not be used.

[2] In a more enlarged sense, the *difference* between two numbers is the number of *units which lie between them.* Thus the difference between 25 degrees north lati

13. John is 15 years old, and Mary is 9. What is the difference between their ages?

14. A merchant sold 25726 dollars' worth of goods one year, and 34718 dollars' worth the next. What was the difference between the sales of the two years?

15. America was discovered in 1492, and the British Colonies declared their independence in 1776. How long after the discovery before the Declaration of Independence?

16. Sir Isaac Newton was born in 1642, and died in 1727. How old was he at his death?

17. The telescope was invented in 1608. How many years since that time?

18. Benjamin Franklin died in 1798, and was 84 years old at his death. When was he born?

19. The art of printing was invented in 1449. How many years since its invention?

20. The area of the Chinese Empire is 4695334 square miles, and of the United States is 3578392. How much greater is the Chinese Empire than the United States?

21. If I buy a horse for 137 dollars, and sell it for 225 dollars, how much more do I get for it than I paid?

56. When we sell an article for more than we paid, what we get for it more than what we paid is called *Gain*. If we sell it for less than we paid, what the selling price lacks of being as much as we paid is called *Loss*.

22. John bought a sleigh for 87 cents, and sold it for 65 cents. Did he gain, or lose? How much?

23. Bought a cow for 58 dollars, and lost 17 dollars in selling her. How much did I sell her for?

24. Bought a horse for 139 dollars, and sold it for 250 dollars. What was my gain?

25. Bought cloth for a coat which cost 17 dollars, and

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tude and 10 degrees south latitude is 35 degrees. But the conception given in the text is as broad as is consistent with our present purpose.

handed the salesman a 50-dollar bill. How much change must he give me?

26. If I buy a horse for $175, and sell it for $225, do I gain, or lose? How much?

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27. Bought 2 pieces of cloth for $112 each, and sold them for $320. Did I gain, or lose? How much?

28. Bought 2 yards of cloth for $7 a yard, and gave the salesman a $20 bill. How much change must he give me?

29. How old is a man in 1875 who was born in 1832? One who was born in 1827?

30. How long is it since the Declaration of Independence by the United States?

31. How old are you? From this how do you find in what year you were born?

32. It is now 8 o'clock in the morning. How long is it to noon? How long since 3 o'clock this morning?

33. Borrowed of my neighbor at one time $175; at another, $340; and at another, $520. Having paid him $685, what balance have I yet to pay? Ans., $350.

34. Put in store at one time 500 pounds of hemp; at another time, 3800 pounds; and at another, 2005 pounds. Having withdrawn 3473 pounds, what quantity remains?

35. A merchant bought flour at one time for $325, and at another time for $460. Having become damaged, the whole was sold at a loss of $184. For what sum was it sold?

36. A man's annual income is $3700. His family expenses are $2500; and he bestows for benevolence $370 a year, and invests the remainder. How much does he invest?

37. A man owed $2000, and he made 3 payments of $375, $580, and $260, respectively. How much remained unpaid?

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38. William has 75 cents more than James, and 125 cents less than Henry, who has 420 cents. What is the number of *cents which they* all have? Ans., 935.

39. Jane went marketing with $10. She bought steak for 62 cents, radishes for 10 cents, sugar for $1.10, eggs for 45 cents, potatoes for 87 cents. How much money should she have had when she returned?

40. A coat that cost me $9.30 was sold for $15.87. How much did I gain?

41. A coat that cost me $17.40 was sold for $11.50. How much did I lose?

42. A horse that cost me $138 I sold so as to make $57.63. For how much did I sell it?

43. I sold a horse for $391.81, and by so doing made $114. What did it cost?

44. A man bought a village-lot for $347. After paying $11.30 for taxes, he sold it so as to make $125.30. For how much did he sell it?

45. Bought a horse, harness, and buggy for $254.50. Paid for repairs on the harness, $1.75; and on the buggy, $2.37. Sold the horse and harness for $175, and the buggy for $125. Did I gain, or lose? How much?

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46. The sum of 3 numbers is 5208. Two of the numbers are 1250 and 2340. What is the third?

47. A man paid $5347 for a farm, implements, and stock. The implements cost $500; and the stock, $2100. What was the cost of the land?

48. The difference between two numbers is 117, and the less number is 375. What is the greater?

49. A gentleman bought a 1000-mile ticket on a railroad for the use of his wife, his daughter, his son, and himself. His wife rode 285 miles; his daughter, 225 miles; his son, 235 miles; and he himself rode the remainder. How many miles did the gentleman ride?

50. How many years since Columbus discovered America in 1492?

Multiplication

SECTION VI.

Ex. — If a man, in counting eggs, takes from a basket 3 eggs at a time, and takes 4 times, how many eggs has he?

ANS., 12.

In this example 3 is *multiplied* by 4, and 12 is the *product.*

What *number* did the man take each time?

How many times did he take it?

What was the number he obtained by taking *three* 4 times?

57. The number which is taken a certain number of times is called the **Multiplicand.**

The number which tells how many times the Multiplicand is to be taken is called the **Multiplier.**

The number which tells how many a certain number of times a given number makes is called the **Product.**

Thus, when I say "3 times 5 are 15," 5 is the *Multiplicand*, 3 is the *Multiplier*, and 15 is the *Product.*

58. The sign $\times$ is called the **Sign of Multiplication.**

Thus 4×3 may be read "4 times 3," or "4 multiplied by 3," or "3 times 4."

We may always find the product of one number by another by *addition.* Thus 4 times 3 is $3+3+3+3$, or 12. But, when the multiplier is large, this would be very tedious. Thus, to find how many 782 times 3487 are, we should have to write 3487 no less than 782 times, and then add! To obviate this, a plan has been devised of getting the product of any two numbers from a knowledge of the products of the digits. A table giving the product of each possible pair of the nine digits is called a *Multiplication Table.* This table is commonly extended to 12×12.

59. Multiplication is the process of finding the product of two numbers by means of a knowledge of the *Multiplication Table.*[1]

Let the pupil fill out the following by *Addition*, as explained above, in order that he may thoroughly understand its nature: —

60. MULTIPLICATION TABLE.

1 × [2]1 = — 1 × 2 = — 1 × 3 = — 1 × 4 = — 1 × 5 = — 1 × 6 = — 1 × 7 = — 1 × 8 = — 1 × 9 = — 1 × 10 = — 1 × 11 = — 1 × 12 = —	2 × 2 = — 2 × 3 = — 2 × 4 = — 2 × 5 = — 2 × 6 = — 2 × 7 = — 2 × 8 = — 2 × 9 = — 2 × 10 = — 2 × 11 = — 2 × 12 = —	3 × 3 = — 3 × 4 = — 3 × 5 = — 3 × 6 = — 3 × 7 = — 3 × 8 = — 3 × 9 = — 3 × 10 = — 3 × 11 = — 3 × 12 = —	4 × 4 = — 4 × 5 = — 4 × 6 = — 4 × 7 = — 4 × 8 = — 4 × 9 = — 4 × 10 = — 4 × 11 = — 4 × 12 = —
5 × 5 = — 5 × 6 = — 5 × 7 = — 5 × 8 = — 5 × 9 = — 5 × 10 = — 5 × 11 = — 5 × 12 = —	6 × 6 = — 6 × 7 = — 6 × 8 = — 6 × 9 = — 6 × 10 = — 6 × 11 = — 6 × 12 = —	7 × 7 = — 7 × 8 = — 7 × 9 = — 7 × 10 = — 7 × 11 = — 7 × 12 = =	8 × 8 = — 8 × 9 = — 8 × 10 = — 8 × 11 = — 8 × 12 = —
9 × 9 = — 9 × 10 = — 9 × 11 = — 9 × 12 = —	10 × 10 = — 10 × 11 = — 10 × 12 = —	11 × 11 = — 11 × 12 = —	12 × 12 = —

[1] It is to be observed that there are *three* ways of finding how many a certain number of times a given number makes; viz., by counting, by adding, and by the process we call Multiplication. The Multiplication Table gives the product of each possible pair of the nine digits; and it is using these facts so as to find the product of any two numbers that we call Multiplication.

[2] This may be read in every one of the ways indicated in (58). For the purpose of *forming the table readily*, the second number may be read as the multiplier. The pupil should understand that it is the same each way.

CONDENSED MULTIPLICATION TABLE.

61. The following neat arrangement gives all the combinations between 2 times 2 and 9 times 9. It will be seen that there are but 36 in all, and only 31 *different* products. This is one of the most rational forms in which to have these products learned. (See Manual, and also foot-note, p. 25.)

$2 \times 2 = 4$	$3 \times 6 = 18$	$6 \times 6 = 36$
$2 \times 3 = 6$	$4 \times 5 = 20$	$5 \times 8 = 40$
$2 \times 4 = 8$	$3 \times 7 = 21$	$6 \times 7 = 42$
$3 \times 3 = 9$	$3 \times 8 = 24$	$5 \times 9 = 45$
$2 \times 5 = 10$	$4 \times 6 = 24$	$6 \times 8 = 48$
$2 \times 6 = 12$	$5 \times 5 = 25$	$7 \times 7 = 49$
$3 \times 4 = 12$	$3 \times 9 = 27$	$6 \times 9 = 54$
$2 \times 7 = 14$	$4 \times 7 = 28$	$7 \times 8 = 56$
$3 \times 5 = 15$	$5 \times 6 = 30$	$7 \times 9 = 63$
$2 \times 8 = 16$	$4 \times 8 = 32$	$8 \times 8 = 64$
$4 \times 4 = 16$	$5 \times 7 = 35$	$8 \times 9 = 72$
$2 \times 9 = 18$	$4 \times 9 = 36$	$9 \times 9 = 81$

62. The Factors of a number are those numbers which, multiplied together, produce it. Thus 2 and 3 are the factors of 6, because $2 \times 3 = 6$.

1. Of what number are 5 and 3 the factors? 7 and 2?
2. What are the factors of 8? Of 10? 12? 9? 6? 18?
3. If 3 is one of the factors of 12, what is the other? 3 times what number makes 12?
4. If 4 is one of the factors of 28, what is the other? 4 times what number makes 28?

63. Principle. — *The product of two factors is the same, whichever is considered the multiplier.* Thus 4 times 3 gives the same product as 3 times 4.

Hence, when the pupil has learned the above table, he knows the *product of any* two digits, whichever be taken as the multiplier.

To Multiply when the Multiplier is represented by One Figure.

64. Rule. — *Write the multiplier under the units figure of the multiplicand, and, beginning with the units, multiply each figure of the multiplicand successively by the multiplier, carrying as in addition.*

REASONS.[1] — There is no reason except custom for writing the multiplier *under* the multiplicand. It would be just as well to write it *over*. We do, however, want them so near each other that we can see them both at a glance; and so it is convenient to write one of them under the other.

It is not necessary that the multiplier be written under the units of the multiplicand.

The process of multiplying gives the correct product, because we multiply all the parts of the multiplicand by the multiplier, and add the resulting products.

We begin at units to multiply, because, by multiplying the lower orders first, we can discern how many the product of any lower order by the multiplier will make of the next higher order, and thus add it in as we go along, and not have to change our work.

Ex. 1. — Multiply 347 by 6.

FORM OF OPERATION.

$$\begin{array}{r} 347 \\ 6 \\ \hline 2082 \end{array}$$

EXPLANATION. — We write the multiplier, 6, under the units of the multiplicand, 347, simply as matter of custom. 6 times 7 units are 42, or 4 tens and 2 units. We hence write the two units in units' place in the product, and reserve the 4 tens to be added to the tens of the product.

6 times 4 tens are 24 tens; and the 4 tens we had to carry make 28 tens in all, or 2 hundreds and 8 tens. Hence we write the 8 tens in tens' place in the product, and reserve the 2 hundreds to be added to the hundreds of the product.

6 times 3 hundreds are 18 hundreds; to which adding the 2 hundreds we had to carry makes 20 hundreds. As this completes the multiplication, we write the 20 hundreds.

Thus we have taken 6 times 7 units, 4 tens, and 3 hundreds, which is 347, and find that the product is 2082.

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[1] If preferred by the teacher, these condensed statements of reasons may be omitted on first going over the subject, and taken in review.

Examples for Practice.

2.	Multiply 811 by 3.	PROD., 2433.
3.	Multiply 923 by 5.	PROD., 4615.
4.	Multiply 1816 by 4.	PROD., 7264.
5.	Multiply 3061 by 7.	PROD., 21427.
6.	Multiply 763 by 8.	PROD., 6104.
7.	Multiply 1412 by 6.	PROD., 8472.
8.	Multiply 769 by 6.	PROD., 4614.
9.	Multiply 3501 by 5.	PROD., 17505.
10.	Multiply 7108 by 7.	PROD., 49756.
11.	Multiply 3009 by 8.	PROD., 24072.

12. Multiply 5428 by 3. By 5. By 7. By 9.
13. Multiply 308057 by 6. By 4. By 2. By 8.
14. Multiply 78500079 by 3. By 7. By 4. By 6.
15. Multiply 1080808 by 9. By 8. By 7. By 5.
16. Multiply 809075 by 6. By 4. By 8. By 3.
17. Multiply 7898763 by 2. By 4. By 7. By 9.
18. Multiply 678954321 by each of the nine digits.

To Multiply when the Multiplier is represented by 1, with any number of 0's annexed.

65. Rule. — *Annex as many 0's to the multiplicand as there are in the multiplier.*

Ex. 1. — Multiply 785 by 10.

EXPLANATION. — We multiply 785 by 10 by simply annexing one 0, and the product is 7850.

The reason that this multiplies 785 by 10 is, that it removes each figure in 785 to the next higher order, thus making it represent 10 times as much as it did before.

2. Multiply 84 by 100. PROD., 8400.

QUERIES. — What was the 4 in the multiplicand? What is it in the product? How many times as much does the 4 in the product represent as it did in the multiplier?

What was the 8 in the multiplicand? What is it in the product? 8 thousands is how many times 8 tens?

3. Multiply 183 by 10. By 100. By 1000.
4. Multiply 7846 by 100. By 1000. By 10000.
5. Multiply 64834, 583, 17, 5, each by 10.
6. Multiply 8, 70, 600, 4, 108, each by 100.

To Multiply by using the Factors of the Multiplier.

66. Principle. — *One number may be multiplied by another by multiplying successively by all the factors of the multiplier; that is, by multiplying the multiplicand by one of the factors, and this product by another, and so on.*

Ex. 1. — Multiply 27 by 15 by multiplying successively by the factors of 15.

OPERATION.

	27
	5
5 times 27 is	135
	3
3 times 5 times or 15 times 27 is .	405

QUERIES. — How many times a number are 4 times 3 times the number? How many times a number are 5 times 7 times?

2. Multiply 285 by 42 by multiplying by the digits which are factors of 42.

What are the digits which are factors of 42? 6 times 7 times 285 are how many times 285?

3. Multiply 428 by 36 by using the factors 4 and 9. Also by using 6 and 6.

4. Multiply 376 by 30 by using the factors 3 and 10.

SUGGESTIONS. — How much is 3 times 376? If 3 times 376 is 1128, how much is 10 times 3 times 376? How do you multiply by 10?

5. Multiply 5836 by 70 by using the factors 7 and 10.

6. Multiply 2356 by 300 by using the factors 3 and 100.

As above, perform the following, making 10, 100, or 1000, one of the factors: —

7. 4206 × 60 = 252360.	10. 267 × 7000 = 1869000.
8. 241 × 500 = 120500.	11. 58 × 800 = 46400.
9. 5863 × 4000 = 23452000.	12. 389 × 80 = 31120.

General Rule for Multiplication.

67. Rule. — I. *Write the multiplier under the multiplicand, so that the orders in the multiplier shall fall under like orders in the multiplicand.*

II. *Beginning with the units of the multiplier, multiply the multiplicand by each of the figures of the multiplier in succession, observing to write the first figure in each product under the one by which you are multiplying.*

III. *Add these products.*

Reasons.[1] — We write the multiplier *under* the multiplicand as a matter of custom. It would do just as well to write it *above*; but we want both multiplier and multiplicand where we can see them at a glance.

We multiply first by the units also because it is customary. It is just as convenient to use the highest order in the multiplier first.

When we multiply by the tens figure, we get as many times the multiplicand as this figure indicates; and then, by moving this product *one* place to the right, we multiply it by 10: thus we multiply the multiplicand successively by the factors of this part of the multiplier.

In like manner we multiply by the factors of the hundreds part of the multiplier, &c.

Finally, having multiplied the multiplicand by the parts of the multiplier, we add these products together, and so have a number which is as many times the multiplicand as are indicated by the multiplier.

[1] If deemed best by the teacher, quite young or immature pupils may omit these general statements of the reasons for the rule until reviewing.

Ex. 1. — Multiply 87 by 34.

EXPLANATION. — To multiply 87 by 34 we take 87 4 times and 30 times; and then, adding these products, we have 34 times 87. 4 times 87 is 348. Then 30 times 87 is 10 times 3 times 87. 3 times 87 is 261, and 10 times 261 is 261 (**65**). But we need not write the 0 if we are careful to remove this product one place to the left.

OPERATION

$$\begin{array}{r} 87 \\ \underline{34} \\ 348 \\ \underline{261} \\ 2958 \end{array}$$

2. Multiply 3587 by 5462.

These are often called partial products:—

$$\begin{array}{r} 3587 \\ \underline{5462} \\ \left\{\begin{array}{r} 7174 \\ 21522 \\ 14348 \\ 17935 \end{array}\right. \\ \hline 19592194 \end{array}$$

Examples for Practice.

1. Multiply 3456 by 74. PROD., 255744.
2. Multiply 345 by 34.
3. Multiply 234 by 26.
4. Multiply 345 by 789. PROD., 272205.
5. Multiply 1357908 by 144. PROD., 195538752.
6. Multiply 543 by 254. PROD., 137922.
7. Multiply 3407 by 682. PROD., 2323574.
8. Multiply 781 by 23.
9. Multiply 5807 by 19. PROD., 110333.
10. Multiply 7005 by 37. PROD., 259185.
11. Multiply 850407 by 3789. PROD., 3222192123.
12. Multiply 23456789 by 14. PROD., 328395046.
13. Multiply 65432 by 15.
14. Multiply 8429638 by 7294. PROD., 61485779572.
15. Multiply 7364951 by 888. PROD., 6540076488.
16. Multiply 653842 by 18. PROD., 11769156.
17. Multiply 603040 by 19.
18. Multiply 364812 by 71. PROD., 25901652.
19. Multiply 482436 by 81.

20. Multiply 2468 by 91. PROD., 224588.
21. Multiply 6739542 by 346. PROD., 2331881532.
22. Multiply 72926495 by 4567.
23. Multiply 123456 by 16. PROD., 1975296.
24. Multiply 437426 by 17. PROD., 7436242.
25. Multiply 89764267 by 999. PROD., 89674502733.
26. Multiply 46371674 by 49684. PROD., 2303930251016.
27. Multiply 4364369 by 51. PROD., 222582819.
28. Multiply 6937845 by 61. PROD., 423208545.
29. Multiply 36598674 by 432. PROD., 15810627168.
30. Multiply 46354897816 by 56843.
31. Multiply 6847 by 207.

SUGGESTION. 207 is 200 and 7. Hence we are to take 7 times 6847, which is 47929, and 100 times 2 times 6847; that is, 13694 multiplied by 100, or written two places to the left. Hence we see that the 0 in the multiplier makes no exception to the rule that we are to write the first figure of each partial product under the figure in the multiplier by which we multiply to produce it. As we have *no* tens in this example, we skip that order in multiplying.

```
   6847
    207
  47929
 13694
1417329
```

32. Multiply 30257 by 2305. PROD., 69742385.
33. Multiply 587640 by 4008. PROD., 2355261120.
34. Multiply 380900 by 301. PROD., 114650900.
35. Multiply 4008 by 4008. PROD., 16064064.
36. Multiply 808058 by 808058. PROD., 652957731364.

37.	657 × 408 =	268056.	47.	67853 × 8765 =	594731545.
38.	6258 × 346 =	2165268.	48.	3678543 × 4567 =	16799905881.
39.	5679 × 507 =	2879253.	49.	492 × 625 =	307500.
40.	7856 × 658 =	5169248.	50.	1312 × 335 =	439520.
41.	9008 × 784 =	7062272.	51.	603456 × 94 =	56724864.
42.	3207 × 2345 =	7520415.	52.	1357908 × 144 =	195538752.
43.	6579 × 3506 =	23065974.	53.	2368689 × 190 =	450050910.
44.	8579 × 4078 =	34985162.	54.	8432 × 6350 =	53543200.
45.	7058 × 6007 =	42397406.	55.	27496 × 1658 =	45588368.
46.	35768 × 3456 =	123614208.	56.	82488 × 555 =	45780840.

To Multiply when there are 0's at the right of either the Multiplier or Multiplicand, or of both.

68. Rule. — *Neglect the 0's at first, and multiply as though there were none. To the product thus obtained annex as many 0's as there are at the right of both multiplier and multiplicand.*

1. Multiply 38400 by 260.

```
  38400
    260
   2304
  768
9984000
```

EXPLANATION. 38400 may be considered as 384 hundreds, and 260 as 10 times 26. Hence we may first take 260 times 384. This we can do by taking 26 times 384, which is 9984, and then 10 times this product. This gives 99840, which is 260 times 384 hundreds, or 99840 hundreds. This is written 9984000.

QUERIES. — Why could we neglect the two 0's in 38400? ANS. — Because we could remember that the 384 was hundreds without them. Why could we neglect the 0 at the right of 260? ANS. — Because, as we wished to multiply 384 hundreds by 260, we could do it by multiplying successively by the factors 26 and 10; and neglecting the 0 gives us the factor 26 to multiply by. Why do we annex three 0's to the product of 384 by 26? ANS. — We annex one 0 to multiply by the factor 10 of 260, and the other two 0's because the result is hundreds.

2. Multiply 75800 by 5600. PROD., 424480000.
3. Multiply 308000 by 1280. PROD., 394240000.

4.	35100 × 720 = ?	9.	111000 × 111000 = ?
5.	58000 × 1900 = ?	10.	2200 × 2200 = ?
6.	9870 × 3470000 = ?	11.	8080 × 8080 = ?
7.	4863000 × 5600 = ?	12.	17001700 × 17001700 = ?
8.	67300 × 820000 = ?	13.	7000 × 7000 = ?

Applications.

1. George bought 7 lemons at 6 cents each. How much did they cost?

If each lemon costs 6 cents, 7 lemons cost 7 times 6 cents, which is 42 cents.

2. If I pay $7 a cord for wood, how much will 3 cords cost me?

3. If I pay $6 a cord for wood, how much will 12 cords cost me? 23 cords?

If each cord costs $6, 23 cords will cost 23 times $6. But, since 6 times 23 is the same as 23 times 6, I will use the 6 as the multiplier, it being more convenient.

Or, at $1 a cord, 23 cords cost $23; but at $6 a cord they cost 6 times as much, or 6 times $23, which is $138.

4. At $8 a barrel, what do 5 barrels of flour cost? 7 barrels? 10 barrels? 30 barrels? 256 barrels? 3428 barrels?

[Use the slate for the last two only.]

5. How much will 14 cords of wood cost at 550 cents a cord?

6. How much will 7 acres of land cost at $128 an acre? How much will 37 acres cost at the same rate?

7. A boy lived 4 miles from the village, and used to go there and back every day in the week except Sunday. How far did he travel in so doing in 1 week? ANS., 48 miles.

8. Mary bought 13 yards of calico at 17 cents a yard, and 5 spools of thread at 6 cents a spool. How much did she pay for both? ANS., 251 cents.

9. How many hundreds in 251? A dollar is 100 cents. How many dollars in 251 cents? How many cents besides?

2 dollars and 51 cents are written $2.51. As the $2 is 2 hundred cents, we can write $2.51, 251 cents. What is $5.31? How many cents? $15.45 are how many cents? 2347 cents are how many dollars and cents? How written? What is $341.20?

Since dollars are merely *hundreds* of cents, we can multiply numbers representing dollars and cents the same as other numbers; and the tens and units of the product will be cents, and the other figures dollars.

10. At $5.37 a barrel, how much will 24 barrels of flour cost?

We perform this thus: —

$$\begin{array}{r} \$5.37 \\ 24 \\ \hline 2148 \\ 1074 \\ \hline \$128.88 \end{array}$$

11. At $3.75 a cord, how much will 58 cords of wood cost? 20 cords? 200 cords?

Ans. to the last, $750.

[Use the slate for the first of these only.]

12. If it cost on an average $2527 a mile to build a certain railroad, how much will it cost to build 288 miles?

Ans., 727776.

13. At $5.25 a yard, how much will 11 yards of cloth cost? 13 yards? 5 yards? 250 yards? Ans. to last, $1312.50.

14. I bought a work in 4 volumes, which cost $8.64 a volume. How much did the work cost?

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15. There are 24 hours in one day; i.e., a day and night. How many hours in a week? How many in 4 weeks? How many hours in a month of 30 days? A month of 31 days?

16. There are 60 minutes in an hour. How many minutes in a day (24 hours)?

17. There are 12 inches in a foot. How many inches in a yard, a yard being 3 feet?

18. There are 2 pints in a quart, and 8 quarts in a peck. How many pints in a peck? There are 4 pecks in a bushel. How many pints in a bushel? Ans. to last, 64 pints.

19. At 15 cents a quart, how much does a bushel of strawberries cost?

20. There are 16 ounces in 1 pound. How many ounces in 4 pounds? In 3 pounds? In 7? In 10? In 30? In 576?

21. There are 2000 pounds in 1 ton. How many pounds in 23 tons? In 70 tons? In 158 tons?

22. There are 3 feet in a yard, and 1760 yards in a mile? How many feet in a mile?

23. Twelve things make a dozen. How many eggs in a barrel containing 87 dozens?

24. There are 24 sheets of paper in a quire, and 20 quires in a ream. How many sheets in a ream?

25. A Western township contains 36 square miles. How many square miles in a county of 25 townships?

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26. How many hills of corn in a field that contains 36 rows, and 185 hills in each row?

27. A wholesale dealer in watches sold 48 watches at $125 each. How much did he receive for them? ANS., $6000.

28. In the year 1865 there had been 16 Presidents, whose united terms of office amounted to 76 years. How much had their salaries amounted to at $25000 a year? ANS., $1900000.

29. At the rate of 1400 words an hour, how many words can be sent over a telegraph-line in 24 hours?

30. How much will 94 passenger-cars cost at $2475 each?

31. If 97 tons of railroad-iron are required for one mile of track, how many tons will be required for a road 359 miles long? ANS., 34823.

32. What will be the weight of the wire for a line of telegraph 207 miles long, if one mile of wire weighs 326 pounds? ANS., 67482 pounds.

33. Show that 40 bushels of corn, at 75 cents per bushel, costs $30.

34. Show that 800 bushels of wheat, at a dollar and a quarter per bushel, is worth $1000.

35. At seven dollars and a half per ton, what cost a carload of twelve tons of coal?

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36. Find the amount of the following bills: —

MRS. JANE SMITH

BOUGHT OF WILLIAM THOMAS:

4 lb. coffee, @ 28¢ per lb. . . . . .
1 lb. tea, @ 75¢ per lb. . . . . .
10 lb. sugar, @ 11¢ per lb. . . . . .
6 lb. starch, @ 8¢ per lb. . . . . .

37. MR. HENRY JONES

BOUGHT OF C. H. MILLEN:

12 yd. calico, @ 8¢ per yard . . . . .
31 yd. sheeting, @ 9¢ per yard . . . .
1 hat, @ $2.50 . . . .
3 pr. socks, @ 38¢ per pair . . . . .

38. Find the balance due on the following account: —

MR. HENRY SMITH

TO JAMES TAYLOR,

1878.	*Dr.*		
March 20.	To 738 yd. of muslin,	@ $0.14,	$
" "	" 75 bbl. of sugar,	@ 12.50,	
April 16.	" 518 yd. of cloth,	@ 2.62,	
May 24.	" 234 pr. of boots,	@ 3.75,	
			$———
1878.	*Cr.*		
April 3.	By 156 bu. of potatoes,	@ $0.35,	$
" 17.	" 128 bu. of corn,	@ .45,	
May 28.	" 650 bu. of wheat,	@ .68,	
July 19.	" 340 bu. of oats,	@ .28,	
	Balance due,		$2628.08

39. Find the balance due on the following bill, and to whom it was due: —

MR. A. FARMER

BOUGHT OF THE MERCHANT:

Dr.

To 20 lb. rice, @ 8¢ . . . . . .
" 96 lb. sugar, @ 13¢ . . . . . .
" 12 lb. raisins, @ 22¢ . . . . . .

Cr.

By 100 bu. potatoes, @ 62¢ . . . . .
" 2 bu. beets, @ 80¢ . . . . . .
" 12 doz. eggs, @ 17¢ . . . . . .
" 5 bu. turnips, @ 40¢ . . . . .

40. A drover bought 256 horses at an average of $125 each, 347 oxen at $87 each, and 250 sheep at $3 each. How much did all cost him? ANS., $62939.

---

41. A has 395 acres of land, worth $27 an acre; and B has 493 acres, worth $19 an acre. What is the value of both of their farms? ANS., $20032.

42. A merchant bought 12 boxes of linen, each containing 25 pieces, and each piece containing 36 yards, at 65 cents a yard. How many pieces did he buy? How many yards? How many dollars did it all cost him? ANS. TO LAST, $7020.

43. A merchant bought 25 pieces of broadcloth, each piece containing 48 yards, at 9 dollars a yard. How much did he pay for the whole?

44. If a steamship can sail 18 miles in 1 hour, how far can she sail in 34 days of 24 hours each?

45. John bought 3 pencils at 7 cents each, and handed the salesman 25 cents. How much change must he receive?

46. Bought 3 oranges at 8 cents each, and a melon at 12 cents, and handed the grocer 50 cents. How much change must he give me back?

47. A woman brought 27 dozen eggs and 37 pounds of butter to market. She sold her eggs at 17 cents a dozen, and her butter at 32 cents a pound. She bought 1 pound of tea at $1.25, 5 pounds of sugar at 15 cents a pound, and 8 pounds of coffee at 39 cents. How much more did she get for her eggs and butter than her tea, sugar, and coffee cost her? ANS., $11.31.

48. There is an orchard consisting of 126 rows of trees, and in each row are 109 trees. How many apples in the orchard, allowing an average of 1007 on a tree?

ANS., 13830138 apples.

49. A certain state contains 50 counties; each county, 35 towns; each town, 300 houses; and each house, 8 persons. What is the population of the state? ANS., 4200000.

50. A man bought a farm of 450 acres for $75 an acre, and spent $2500 in improvements. He then sold it at $80 an acre. Did he gain, or lose? How much?

Ans., He lost $250.

51. A farmer purchased a farm of 325 acres at 45 dollars per acre, and made a payment of 875 dollars. In order to make another payment, he sold 75 acres at 58 dollars per acre. How much did he owe on his farm after the second payment was made? Ans., 9400 dollars.

52. If a young man's salary is 600 dollars per year, of which he takes 45 dollars to purchase books, and 300 dollars for board and other expenses, how much money will he have at the end of 7 years? Ans., 1785 dollars.

53. At 55 dollars per ton, what will the rails for a railroad cost, if the road is 75 miles long, and it takes 112 tons per mile? Ans., 462000 dollars.

54. It has been found by experiment that a sheep which is fed in the open air consumes 1912 pounds of turnips from Nov. 18 to March 9; and that a sheep of the same size, fed under a shed, consumes, during the same time, 1394 pounds of turnips. How many pounds of turnips would a farmer save in a single winter by feeding 345 sheep under a shed, instead of in the open air? Ans., 178710 pounds.

55. A man owned 4376 acres of land: he sold 468 acres at $19 per acre. What is the remainder worth at $20 per acre? Ans., $78160.

56. Henry earned $46 at one time, and $27 at another time. His father then gave him three times as much as he had earned. How much did his father give him?

Ans., $219.

57. A hardware merchant bought at one time 250 pounds of tin, at another 345 pounds, and at another 562 pounds, and paid for it all $462.80. He afterward sold it all at 45 cents a pound. How much did he receive for it?

Ans., $520.65.

58. A merchant bought 275 yards of cloth at 6 dollars per yard, of which he has sold 133 yards at 10 dollars per yard. What would he gain on the whole by selling the remainder at 9 dollars per yard? ANS., 958 dollars.

59. If a wagon cost $48, a yoke of oxen 3 times as much, lacking $54, and a span of horses as much as the wagon and oxen together, what was the cost of the oxen and horses respectively, and of all?

60. A passenger-train and a freight-train of cars start from a given station, and run in the same direction. The passenger-train moves at the rate of 37 miles an hour, and the freight-train 19 miles an hour. How far apart will they be at the end of 13 hours? ANS., 234 miles.

How far apart had they run in different directions?

ANS., 728 miles.

61. Bought 12 cows at $27 each, 9 horses at $95 each, and 250 sheep at $3 each. Sold all for $2000. What was the gain?

62. A man travels from Chicago to New York, 911 miles. He travels 10 hours a day at the rate of 6 miles an hour. How far will he be from New York at the end of 15 days?

63. Two vessels are 4500 miles apart, and travel toward each other, — one at the rate of 91 miles a day, and the other at the rate of 85 miles a day. How far apart are they at the end of 24 days? ANS., 276 miles.

64. Two ships start from the same port, and travel in opposite directions, — one at the rate of 75 miles a day, the other at the rate of 85 miles a day. How far apart will they be at the end of 15 days? ANS., 2400 miles.

65. If a cow cost $27, a horse 5 times as much, and a farm 8 times as much as the cow and horse together, less $109, how much more will the farm cost than 4 horses and 10 cows, at the same rate? ANS., $377.

DIVISION

SECTION VII.

69. **Division** is a process of finding how many times one number is contained in another.

Thus, when we find how many 9's there are in 36, we divide 36 by 9. This is expressed by saying, "9 is contained in 36 *four* times."

Division enables us also to separate a number into any number of equal parts, and find how many there are in one of these parts.

Thus, when we separate 12 apples into 4 equal groups to find how many there will be in each, we divide 12 by 4.

70. The **Dividend** is the number to be divided.

The **Divisor** is the number by which we divide.

The **Quotient** is the number which tells how many times the divisor is contained in the dividend, or what is one of the required parts of the dividend.

71. The sign ÷ is the sign of division, and is read "divided by." Thus 24 ÷ 6 is read "24 divided by 6." The dividend is written before the sign, and the divisor after it.

72. As multiplication is finding the product when the two factors are given, so division is finding one of the factors when the product and the other factor are given. The Dividend is the Product of the Quotient and Divisor.

Hence, to find how many times 6 is contained in 24, we have only to ask by what number 6 must be multiplied to make 24. Thus, as $6 \times 4 = 24$, 6 is contained in 24, 4 times; and 4 is contained in 24, 6 times. In this way fill out the following

DIVISION TABLE.

When 2 is one factor.

$2 \times 2 = \text{—}. \therefore \begin{cases} 4 \div 2 = \text{—}. \end{cases}$

$2 \times 3 = \text{—}. \therefore \begin{cases} 6 \div 2 = \text{—}. \\ 6 \div 3 = \text{—}. \end{cases}$

$2 \times 4 = \text{—}. \therefore \begin{cases} 8 \div 2 = \text{—}. \\ 8 \div 4 = \text{—}. \end{cases}$

$2 \times 5 = \text{—}. \therefore \begin{cases} 10 \div 2 = \text{—}. \\ 10 \div 5 = \text{—}. \end{cases}$

$2 \times 6 = \text{—}. \therefore \begin{cases} 12 \div 2 = \text{—}. \\ 12 \div 6 = \text{—}. \end{cases}$

$2 \times 7 = \text{—}. \therefore \begin{cases} 14 \div 2 = \text{—}. \\ 14 \div 7 = \text{—}. \end{cases}$

$2 \times 8 = \text{—}. \therefore \begin{cases} 16 \div 2 = \text{—}. \\ 16 \div 8 = \text{—}. \end{cases}$

$2 \times 9 = \text{—}. \therefore \begin{cases} 18 \div 2 = \text{—}. \\ 18 \div 9 = \text{—}. \end{cases}$

When 3 is one factor.

$3 \times 3 = \text{—}. \therefore \begin{cases} 9 \div 3 = \text{—}. \end{cases}$

$3 \times 4 = \text{—}. \therefore \begin{cases} 12 \div 3 = \text{—}. \\ 12 \div 2 = \text{—}. \end{cases}$

$3 \times 5 = \text{—}. \therefore \begin{cases} 15 \div 3 = \text{—}. \\ 15 \div 5 = \text{—}. \end{cases}$

$3 \times 6 = \text{—}. \therefore \begin{cases} 18 \div 3 = \text{—}. \\ 18 \div 6 = \text{—}. \end{cases}$

$3 \times 7 = \text{—}. \therefore \begin{cases} 21 \div 3 = \text{—}. \\ 21 \div 7 = \text{—}. \end{cases}$

$3 \times 8 = \text{—}. \therefore \begin{cases} 24 \div 3 = \text{—}. \\ 24 \div 8 = \text{—}. \end{cases}$

$3 \times 9 = \text{—}. \therefore \begin{cases} 27 \div 3 = \text{—}. \\ 27 \div 9 = \text{—}. \end{cases}$

When 4 is one factor.

$4 \times 4 = \text{—}. \therefore \begin{cases} 16 \div 4 = \text{—}. \end{cases}$

$4 \times 5 = \text{—}. \therefore \begin{cases} 20 \div 4 = \text{—}. \\ 20 \div 5 = \text{—}. \end{cases}$

$4 \times 6 = \text{—}. \therefore \begin{cases} 24 \div 4 = \text{—}. \\ 24 \div 6 = \text{—}. \end{cases}$

$4 \times 7 = \text{—}. \therefore \begin{cases} 28 \div 4 = \text{—}. \\ 28 \div 7 = \text{—}. \end{cases}$

$4 \times 8 = \text{—}. \therefore \begin{cases} 32 \div 4 = \text{—}. \\ 32 \div 8 = \text{—}. \end{cases}$

$4 \times 9 = \text{—}. \therefore \begin{cases} 36 \div 4 = \text{—}. \\ 36 \div 9 = \text{—}. \end{cases}$

When 5 is one factor.

$5 \times 5 = \text{—}. \therefore \begin{cases} 25 \div 5 = \text{—}. \end{cases}$

$5 \times 6 = \text{—}. \therefore \begin{cases} 30 \div 5 = \text{—}. \\ 30 \div 6 = \text{—}. \end{cases}$

$5 \times 7 = \text{—}. \therefore \begin{cases} 35 \div 5 = \text{—}. \\ 35 \div 7 = \text{—}. \end{cases}$

$5 \times 8 = \text{—}. \therefore \begin{cases} 40 \div 5 = \text{—}. \\ 40 \div 8 = \text{—}. \end{cases}$

$5 \times 9 = \text{—}. \therefore \begin{cases} 45 \div 5 = \text{—}. \\ 45 \div 9 = \text{—}. \end{cases}$

When 6 is one factor.

$6 \times 6 = \text{—}. \therefore \begin{cases} 36 \div 6 = \text{—}. \end{cases}$

$6 \times 7 = \text{—}. \therefore \begin{cases} 42 \div 6 = \text{—}. \\ 42 \div 7 = \text{—}. \end{cases}$

$6 \times 8 = \text{—}. \therefore \begin{cases} 48 \div 6 = \text{—}. \\ 48 \div 8 = \text{—}. \end{cases}$

$6 \times 9 = \text{—}. \therefore \begin{cases} 54 \div 6 = \text{—}. \\ 54 \div 9 = \text{—}. \end{cases}$

When 7 is one factor.

$7 \times 7 = \text{—}. \therefore \begin{cases} 49 \div 7 = \text{—}. \end{cases}$

$7 \times 8 = \text{—}. \therefore \begin{cases} 56 \div 7 = \text{—}. \\ 56 \div 8 = \text{—}. \end{cases}$

$7 \times 9 = \text{—}. \therefore \begin{cases} 63 \div 7 = \text{—}. \\ 63 \div 9 = \text{—}. \end{cases}$

When 8 is one factor.

$8 \times 8 = \text{—}. \therefore \begin{cases} 64 \div 8 = \text{—}. \end{cases}$

$8 \times 9 = \text{—}. \therefore \begin{cases} 72 \div 8 = \text{—}. \\ 72 \div 9 = \text{—}. \end{cases}$

When 9 is one factor.

$9 \times 9 = \text{—}. \therefore \begin{cases} 81 \div 9 = \text{—}. \end{cases}$

73. When the entire divisor has been taken out of the dividend as many times as possible, if any thing is left it is called the **Remainder.**

If there is no remainder, the division is said to be *exact.*

How many times is 3 contained in 17? and what is the remainder? Does it take all of 17 to contain 3 *five* times?

---

74. Principle. — *If a given divisor is contained in any dividend a certain number of times with a certain remainder, it is contained in 10 times as great a dividend 10 times as many times with 10 times as great a remainder, in 100 times as great a dividend 100 times as many times with 100 times as great a remainder, etc.*

Thus 3 is contained in 17 *five* times, with 2 remainder: hence it is contained in 10 times 17 *ten* times 5, or 50 times, with 10 times 2, or 20, as a remainder. So, again, 3 is contained in 100 times 17 *one hundred* times 5, or 500 times, with 100 times 2, or 200 remainder.

SHORT DIVISION.

When the Divisor is represented by a Single Digit.

1. Divide 536 by 2.

$$2\overline{)536}$$
$$\overline{268}$$

EXPLANATION. — In order that we may see both at once conveniently, we write the divisor on the left of the dividend.

2 is contained in 5 *two* times, with 1 remainder. Hence it is contained in 5 hundreds 2 hundreds times, with 1 hundred remainder. The 1 hundred remainder is 10 tens, which, with the 3 tens, make 13 tens. 2 is contained in 13 *six* times, with 1 remainder: hence it is contained in 13 tens 6 tens times, with 1 ten remainder. The 1 ten remaining is 10 units, which, with the six units, make 16 units. 2 is contained in 16 units 8 times.

To Divide by the Method called Short Division.

75. Rule. — I. *Write the divisor on the left of the dividend. Begin with the highest order or orders, which, regarded*

as units, will contain the divisor. Divide, and write the quotient figure underneath the lowest order thus used.

II. ***Prefix the remainder to the next lower order, and divide as before.***

III. ***If, at any time after the first quotient figure has been written, the divisor is not contained in the next lower order, together with what is brought to it from the higher, write 0 in the quotient, and proceed to the next lower order.***

Reasons. — The divisor is written at the left of the dividend, simply that we may be able to see both at once conveniently.

We begin at the highest order to divide, because by so doing we can put what remains after each division into the next lower order, and divide it; and thus we get all there is of any order in the quotient as we go along.

We write the quotient figures under the orders from whose division they arise, because they are of the same orders.

The process ascertains how many times the divisor is contained in the dividend by finding how many times it is contained in the parts of the dividend, and adding the results. This can be readily illustrated by an example. For this purpose let us divide 1547 by 4.

Analysis of Operation.

1547 =	12 hds.	In this 4 is contained 3 hds., or	300 times.
	32 tens.	In this 4 is contained 8 tens, or	80 times.
	24 } units.	In this 4 is contained 6 units, or	6 times.
	and 3 } units.	In this 4 is contained *no* times.	
∴ In 1547	.	4 is contained . .	386 times,

with a remainder of 3.

2. Divide 7683 by 5. What is the quotient? What the remainder?

Operation.

$$5)\underline{7683}-3$$
$$1536$$

Perform the following, explaining the process: —

3.	4.	5.	6.	7.
2)1349	3)736	5)2483	6)3486	7)238905

8. Divide 4238 by 7.

SUGGESTION. — We observe that 7 is contained in 42 (hundreds) 6 (hundreds) times, with no remainder. Now, 7 is not contained in 3 (tens) any *tens* times: so we write a 0 in the tens place in the quotient to mark the vacant order, and unite the 3 tens with the 8 units, making 38 units. In this, 7 is contained 5 times, with a remainder 3.

$$7\overline{)4238} - 3$$
$$605$$

Perform the following, explaining the process: —

9.	10.	11.	12.
5)25200	9)723856 — 4	8)5462	8)1768162
5040	80428		

Examples for Practice.

1. 5462 ÷ 2; by 5; by 7.
2. 1256 ÷ 3; by 4; by 9.
3. 10702 ÷ 5; by 6; by 8.
4. 78520 ÷ 7; by 9; by 4.
5. 653426 ÷ 2; by 3; by 5.
6. 827001 ÷ 4; by 6; by 7.
7. 10000 ÷ 2; by 4; by 6; by 8.
8. 10000 ÷ 3; by 5; by 7; by 9.
9. 20003 ÷ 5; by 8; by 6; by 2.
10. 102504 ÷ 3; by 7; by 8; by 9.
11. 101010 ÷ 8; by 4; by 2; by 6.
12. 202020 ÷ 4; by 5; by 7; by 3.

Two Ways of disposing of the Remainder.

76. First Method. — *In dividing when the division is not exact, we may write the divisor under the remainder, thus forming a fraction, and annex this to the integral part of the quotient.*[1]

Ex. 1. — Divide 38 by 5.

EXPLANATION. 5 is contained in 38 *seven* times, with 3 remainder. To divide 3 by 5, we consider that 1 divided by 5 gives $\frac{1}{5}$. Hence 3 divided by 5 will give 3 times $\frac{1}{5}$, or $\frac{3}{5}$. Thus 38 divided by 5 is $7\frac{3}{5}$ (read 7 and $\frac{3}{5}$).

OPERATION.
$$5\overline{)38}$$
$$7\tfrac{3}{5}$$

2. Show that $128 \div 3 = 42\frac{2}{3}$. Explain the division.
3. Show that $51 \div 4 = 12\frac{3}{4}$. $17 \div 5 = 3\frac{2}{5}$.
4. Show that $11 \div 6 = 1\frac{5}{6}$. $38 \div 3 = 12\frac{2}{3}$.
5. Show that $7 \div 5 = 1\frac{2}{5}$. $58 \div 11 = 5\frac{3}{11}$.
6. Show that $9 \div 2 = 4\frac{1}{2}$. $87 \div 8 = 10\frac{7}{8}$.

---

[1] The pupil is supposed to have learned in the Primary course what a fraction is. If this is not the case, the teacher will need to explain it.

Perform the following, disposing of the remainder as above, and giving the explanation: —

7. $39 \div 4 = 9\frac{3}{4}$.	11. $417 \div 5$.	15. $1000 \div 7$.
8. $52 \div 5 =$?	12. $3428 \div 7$.	16. $2015 \div 4$.
9. $18 \div 7 =$?	13. $5000 \div 3$.	17. $18276 \div 5$.
10. $143 \div 2 = 71\frac{1}{2}$.	14. $20 \div 7$.	18. $41287 \div 8$.

---

77. Principle. — *If the divisor and dividend are both multiplied, or both divided by the same number, the quotient is not altered.*

Ex. 1. — If you have 12 apples to divide equally among 4 boys, how many apples can you give to each?

If there are *twice* as many boys, how many times as many apples must you have in order to give each boy just as many as before?

If there are only half as many boys, how many apples will be needed in order to give each as many as at first?

2. 8 is contained in 128 *sixteen* times. Then how many times is it contained in $\frac{1}{2}$ of 128? In $\frac{1}{4}$ of 128?

3. The expression $\frac{27}{9}$ means that 27 is to be divided by 9. What difference would it make in the quotient if you were to multiply both numbers by 3 before dividing? If you were to divide both by 3?

4. Show that $\frac{6}{9} = \frac{2}{3}$. That $\frac{4}{8} = \frac{1}{2}$. That $\frac{15}{75} = \frac{3}{25}$. That $\frac{18}{48} = \frac{3}{8}$.

78. Principle. — *The numerator and denominator of a fraction may be both divided by the same number without altering the value of the fraction.*

(*a.*) Whenever a fraction occurs in the quotient in division, it should be *Reduced to its Lowest Terms*; i.e., its numerator and denominator should be made the smallest whole number possible by such division as above.

5. Show that $876 \div 9 = 97\frac{1}{3}$. That $436 \div 8 = 34\frac{1}{2}$. That $172 \div 6 = 28\frac{2}{3}$. That $2341 \div 8 = 292\frac{3}{4}$.

6. Show that $\frac{504}{756} = \frac{168}{252} = \frac{84}{126} = \frac{28}{42} = \frac{14}{21} = \frac{2}{3}$.

7. Show that $\frac{462}{714} = \frac{231}{357} = \frac{77}{119} = \frac{11}{17}$.

---

79. Second Method. — *When the division is not exact, we can place a point after the integral part of the quotient, annex a 0 to the remainder, and divide this result. To this remainder annex a 0, and divide again, till the work terminates, or till we have gone as far as we wish. The orders thus obtained at the right of units are tenths, hundredths, thousandths, etc.*

Ex. 1. — Divide 347 by 8.

$$\begin{array}{r} 8\underline{)347} \\ 43.375 \end{array}$$

EXPLANATION. — Having divided 347 by 8, we find a remainder 3. Now, if we divide each of these 3 into 10 equal parts, or tenths, there will be 30 of them. 30 tenths divided by 8 gives 3 tenths, and 6 tenths remaining. Again: dividing each of the 6 tenths into 10 equal parts, the parts will be hundredths, and there will be 60 of them. Dividing 60 hundredths by 8, we have 7 hundredths, and 4 hundredths remaining. 4 hundredths make 40 thousandths, and 40 thousandths divided by 8 gives 5 thousandths. These tenths, hundredths, and thousandths we separate from the integral part of the quotient by a dot (.) called a *Decimal Point.*

In this process we multiply each remainder by 10, and this is done by annexing a 0 (65).

It will be seen that the numbers thus obtained by annexing 0's are a kind of *Fractions*. They are called **Decimal Fractions**, or simply *Decimals*. We shall have more about them by and by.

80. .3 is 3-tenths; .4 is 4-tenths; .05 is 5-hundredths; .006 is 6-thousandths, etc. .37 is 37-hundredths; since it is 3-tenths and 7-hundredths, and 3-tenths make 30-hundredths.

.375 is 375-thousandths, because 3-tenths and 7-hundredths make 37-hundredths, and 37-hundredths are 370-thousandths, which, with the 5-thousandths, make 375-thousandths.

2. Read the following, and tell what they mean: .3; .05; .38; .072; .135; 4.2; 5.23; 42.26; 12.346.

Perform the following, carrying out the division to three places of decimals, if it does not terminate before: —

3. $26 \div 8 = 3.25$.	9. $1342 \div 4$.
4. $132 \div 8 = 16.5$.	10. $521 \div 5$.
5. $346 \div 7 = 49.428+$.[1]	11. $1376 \div 9$.
6. $3471 \div 2 =$?	12. $8156 \div 8$.
7. $581 \div 6 =$?	13. $417 \div 2$.
8. $5312 \div 3 =$?	14. $1826 \div 3$.

To be taken up on Review.

81. It is desirable that pupils somewhat advanced learn to divide by short division when the divisor is less than 13, as by 10, 11, and 12, in addition to the digits. This requires a knowledge of the Multiplication Table up to 12 times 12. Some will carry the process farther, as to 25 times 25.

Ex. 1. — Divide 789634 by 12 by short division.

OPERATION.

$$12\overline{)789634}-2$$
$$74136$$

EXPLANATION OF PROCESS. 12 is contained in 78 *seven* times, with 4 remainder. 12 is contained in 49 *four* times, with 1 remainder. 12 is contained in 16 once, with 4 remainder, etc. (The reasoning is omitted because it is just the same as when the divisor is represented by a single digit.)

Solve the following by short division, putting the quotient in each form: —

---

[1] This sign means that the division does not terminate.

2. 865732 ÷ 11; by 12.	6. 74600 ÷ 12; by 11.
3. 3462 ÷ 12; by 11.	7. 100000 ÷ 11; by 12.
4. 10764 ÷ 12; by 11.	8. 300700 ÷ 12; by 11.
5. 8000 ÷ 11; by 12.	9. 44444 ÷ 11; by 12.

To Divide by 1 with any number of O's annexed.

82. Rule. — *Cut off as many figures at the right of the dividend (including the units), by placing the decimal point before them, as there are O's in the divisor.*

Ex. 1. — Divide 74826 by 100.

OPERATION.
748.26

EXPLANATION. — Since removing each figure two orders to the left multiplies a number by 100 (**65**), removing each two places to the right divides a number by 100. There is, therefore, nothing to be done but to cut off the units and tens, thus making what was hundreds (the 8) units; what was thousands, tens, etc.

In this manner divide the following, each by 10, 100, 1000, and read the quotients, telling what each means (**80**).

2. 43726, 50000, 840056, 48000, 100000, 500400, 681243, 441111, 5867, 10045, 7000.

3. Divide 7856 by 1000; that is, tell how many thousands there are in it, and how many over. In like manner divide 17541 by 1000.

4. Speak the quotients and remainders in the following: 58 ÷ 10; 736 ÷ 10; 34568 ÷ 10; 12057 ÷ 1000; 19027 ÷ 100; 34567 ÷ 1000; 30456 ÷ 100; 750263 ÷ 1000.

LONG DIVISION.

To Divide when the Divisor is more than 10, or more than 12.

1. Divide 73482 by 214.

OPERATION.

$$
\begin{array}{l}
214\ )\ 73482\ (\ 343\ \textit{Quot.} \\
\quad\ \ \underline{642} \\
\qquad \ 928 \\
\qquad \underline{856} \\
\qquad\quad 722 \\
\qquad\quad \underline{642} \\
\qquad\qquad 80\ \textit{Rem.}
\end{array}
$$

EXPLANATION. — The principles used in this process are just the same as those in *Short Division*, the only difference being that the divisor is so large that it is not convenient to find out the several remainders without writing down the product of the divisor by each quotient figure as we obtain it, and performing the subtraction. Hence we write the quotient at the right of the dividend, that it may not be in our way.

We now say "214 is contained in 734[1] 3 times, and write the 3 as the highest order in the quotient. Then, multiplying 214 by 3, we get the product, 642, and, writing it under the 734, subtract, and find a remainder of 92. Then, as 214 is contained in 734 3 times, with a remainder 92, it is contained in 734 *hundreds* 3 *hundreds* times, with a remainder 92 *hundreds*; and the quotient figure 3 is hundreds.

Again: 92 hundreds and 8 tens make 928 tens. 214 is contained in 928 *four* times, with a remainder 72: hence it is contained in 928 *tens* 4 *tens* times, with a remainder 72 *tens*.

Finally, 72 tens and 2 units make 722 units; and 214 is contained in 722 *three* times, with a remainder 80.

Thus we have found how many times 214 is contained in 73482 by finding how many times it is contained in the parts of 73482, and adding the quotients thus obtained. This may be exhibited at one view, thus: —

$$
73482 = \left\{
\begin{array}{rl}
642 & \textit{hds.}\ \text{In this 214 is contained 3 } \textit{hds.}\text{, or 300 times.} \\
856 & \textit{tens.}\ \text{In this 214 is contained 4 } \textit{tens}\text{, or 40 times.} \\
642 & \textit{units.}\ \text{In this 214 is contained 3 } \textit{units}\text{, or 3 times.} \\
\underline{80} & \textit{units.}\ \text{In this 214 is contained } \textit{no}\text{ times.}
\end{array}
\right.
$$

∴ In 73482 . 214 is contained . 343 times, with a remainder 80.

[1] Is 214 contained in the first figure of the dividend? In the first two, or 73? In the first three, or 734?

To Divide by the Method called Long Division.

83. Rule. — I. *Write the divisor at the left of the dividend, and the quotient as it is obtained at the right.*[1]

II. *Seek how many times the divisor is contained in the fewest of the left-hand figures of the dividend which will contain it, and write the number of times as the highest order in the quotient.*

III. *Multiply the divisor by this quotient figure, and, writing the product under the part of the dividend used, subtract it therefrom. Annex to this remainder the figure of the next lower order of the dividend. Divide the number so formed by the divisor, writing the number of times it is contained as the second figure in the quotient. Continue the process till the dividend is exhausted, or until the remainder will not contain the divisor.*

IV. *If at any time a remainder, with the next figure of the dividend annexed, will not contain the divisor, write 0 in the quotient, and bring down the next figure of the dividend.*

Reasons. — The reasons for this rule are the same as for the rule for Short Division; and, instead of repeating them, we will give a series of questions, which the pupil can answer if he has learned well what goes before.

Questions. — 1. Why do we write the divisor on the left and the quotient on the right of the dividend?[2] 2. Why do we begin to divide with the highest order or orders, and proceed through the lower orders in succession? 3. How do we find out how many times the divisor is contained each time? 4. On what principle do we determine what the order of any quotient figure is? 5. On what principle are we able to take part of the dividend at a time? 6. Finally, how does it appear that this process determines how many times the divisor is contained in the dividend?

---

[1] For other forms of writing the work, see Ex. 4, p. 80.

[2] A better form is to write the first figure of the quotient directly *over* the first figure of the product of the *units* of the divisor by this figure. See second form, p. 80, Ex. 4, and Decimal Fractions (172). Some write the divisor at the *right* of the dividend, and the quotient under it. See third form, p. 80, Ex. 4.

Examples for Practice.

1. Divide 82756 by 234, and afterward put the work in form to exhibit the principles at one view, as on page 78.

REM., 154.

2. Divide 7854 by 96, and explain as directed in the last.

REM., 78.

3. Divide 346827 by 271, and then multiply the quotient by the divisor, and to this product add the remainder. What ought the result to be?

84. *Division may be* **Proved** *by multiplying divisor and quotient together, and to the product adding the remainder. The result should be the dividend.* Why?

4. Divide 17856 by 39, and prove the process as above.

OPERATION.	SECOND FORM.	THIRD FORM.		PROOF.
	457			
39) 17856 (457	39) 17856	17856	39	457
156	156	156	457	39
225	225	225		4113
195	195	195		1371
306	306	306		17823
273	273	273		33
33 *Rem.*	33 *Rem.*	33 *Rem.*		17856

QUERY. — Why is it, that, if you add the remainder and several subtrahends just as they stand in the work, their sum will make the dividend? Try it.

This, then, is another method of proof.

Perform the following divisions, and prove the process in each case; also write the complete quotient in both forms (76), (79).

5. 2592 ÷ 63. REM., 9.
6. 7776 ÷ 108. REM., 0.
7. 6750 ÷ 15. REM., 0.
8. 437639 ÷ 42. REM., 41.
9. 1893312 ÷ 2076.
10. 84764367 ÷ 431.
11. 4683579 ÷ 234.
12. 2686211248 ÷ 296.
13. 99424788962 ÷ 978.
14. 347628 ÷ 84.

15. 57432168 ÷ 8762.
16. 134007502 ÷ 34007.
17. 88888888 ÷ 9999.
18. 100000001 ÷ 785.
19. 2000000 ÷ 691.
20. 4360000 ÷ 436.
21. 84200 ÷ 421.
22. 13230000 ÷ 735.
23. 46521 ÷ 43.
24. 8762435 ÷ 872.
25. 1000000 ÷ 777.
26. 62050 ÷ 75.
27. 173248 ÷ 256.
28. 4000404 ÷ 388.
29. 7430055 ÷ 82649.
30. 1234567890 ÷ 456.
31. 9876543210 ÷ 12345.
32. 7810000 ÷ 427.
33. 85432 ÷ 125.
34. 14732560 ÷ 275.
35. 171426 ÷ 987.
36. 70000 ÷ 333.
37. 500500 ÷ 1001.
38. 878787 ÷ 999.
39. 1524630 ÷ 738.
40. 400000 ÷ 8888.
41. 436 ÷ 158.
42. 64827 ÷ 377.
43. 508508 ÷ 99.
44. 7642382 ÷ 98764.
45. 10000 ÷ 777.
46. 1234549380 ÷ 12345.

The *complete quotients* to some of the above are, $827\frac{25}{75}$, $676\frac{192}{256}$, 683.456, $4138\frac{36}{84}$, 200, 18000.

The quotients of some, extended to three orders of decimals, are, 53572.945+, 2894.356+, 879.666+.

47. Divide 39333933 by 437.
48. Divide 184000 by 92.
49. Divide 87612 by 873.
50. Divide 552160000 by 937.
51. Divide 80000 by 40.
52. Divide 49419533647761876 by 9876.

To Divide by using the Factors of the Divisor.

85. Rule. — *Divide the given dividend by one of the factors, and the quotient thus arising by the other.*[1]

REASONS. — To divide a number by 5 shows how many 5's there

[1] Of course this process is capable of extension to cases of more than two factors; but we have no occasion to use such cases, and hence give only that of two factors.

are in it. Then, as every 7 of these 5's makes 35, if we find how many 7's there are in this first quotient, the result will show how many 7's containing 5 each, or how many 35's, there are in the given number.

This reasoning applies to dividing by any composite number.

Ex. 1. — Divide 5265 by 15, using the factors of 15.

SUGGESTION. — The factors of 15 are 3 and 5. Now, dividing by 3, we find that there are 1755 *threes* in 5265. Now, since 5 *threes* make 15, there are as many 15's in 5265 as there are 5's in 1755, which is 351. Hence there are 351 15's in 5262.

3) 5265	
5) 1755	Number of 3's in 5265.
351	Number of 5's containing 3 each, or of 15's in 5265.

Perform each of the following by resolving the divisor into two factors, and using these factors as divisors. Give the explanation as above in each case.

2. 5796 ÷ 21.	6. 9576 ÷ 28.	10. 18576 ÷ 72.
3. 19880 ÷ 35.	7. 7740 ÷ 45.	11. 21364 ÷ 28.
4. 1530 ÷ 6.	8. 4704 ÷ 42.	12. 3402 ÷ 63.
5. 8748 ÷ 12.	9. 2670 ÷ 30.	13. 3402 ÷ 54.

86. If there are remainders, it is easy to tell how many of the original dividend remain by noticing what the remainders are. The number of the original dividend which remains undivided is called the **True Remainder.**

14. Divide 5276 by 15, using the factors of 15, and find the true remainder.

SUGGESTION. — Dividing by 3, we find 2 of the 5276 remaining. Now, the 1758 are 3's of the 5276; that is, every 1 of the 1758 corresponds to 3 of the 5276. But, when we divide the 1758 by 5, we find 3 of it remaining. Hence, as each one of these 3 corresponds to 3 of the 5276, the 3 corresponds to 9 of that number. The remainder of the 5276 is, therefore, 9 + 2, or 11.

3) 5276 — 2
5) 1758 — 3
351

15–18. Perform the following, using the factors of the *divisors,* and find the true remainders as above: —

578 ÷ 21 ; 3412 ÷ 15 ; 12047 ÷ 28 ; 1879 ÷ 30.

To perform Division when there are 0's at the Right in the Divisor.

87. Rule. — *Cut off the 0's from the divisor, and a like number of figures from the right of the dividend. Divide the remaining figures in the left of the dividend by the significant figures of the divisor. The remainder after this division, prefixed to the figures of the dividend cut off, is the entire remainder.*

REASONS. — Cutting off the 0's from the divisor may be considered as pointing out its factors; and cutting off the figures at the right in the dividend is dividing by 10, 100, 1000, or by 1, with as many 0's annexed as there are figures cut off (**82**). Dividing by the significant figures is dividing the first quotient by the other factor of the divisor, according to (**85**). The remainder from the last division, being merely of the next higher orders than the first remainder, needs simply to be prefixed according to the principles of notation.

Ex. 1. — Divide 8264 by 230.

OPERATION.

```
23 | 0 ) 826 | 4 ( 35 Quot.
         69
         ---
         136
         115
         ---
         214 Entire rem.
```

EXPLANATION. — The factors of 230 may be considered as 23 and 10. To divide by 10 we cut off the units figure of the dividend, and thus have 826 and 4 remainder (**82**).

Now, dividing this quotient by 23, we have 35, and a remainder 21. As this remainder is *tens*, and the former, 4, is units, the *true remainder* is 214.

2 to 21. Perform the following according to this rule: —

(2.) 6285 ÷ 70.
(3.) 5786 ÷ 200.
(4.) 23478 ÷ 700.
(5.) 3005080 ÷ 12000.
(6.) 5200700 ÷ 2300.
(7.) 12305 ÷ 310.
(8.) 4300 ÷ 3700.

(9.) 504372 ÷ 500.
(10.) 142605 ÷ 12000.
(11.) 2800 ÷ 700.
(12.) 58276432 ÷ 13600.
(13.) 20075006 ÷ 3510.
(14.) 50707032 ÷ 25700.
(15.) 500504000 ÷ 32000.

(16.) 408208 ÷ 1260.
(17.) 437256 ÷ 387000.
(18.) 2654002 ÷ 810000.
(19.) 4000000 ÷ 343000.
(20.) 1000000 ÷ 75000.
(21.) 3754212 ÷ 346210.

Applications.

88. Since $\frac{1}{2}$ of any thing is one of the *two* equal parts into which it may be divided, we obtain $\frac{1}{2}$ of a number by dividing the number by 2. In like manner, $\frac{1}{3}$ of a number is found by dividing the number by 3; $\frac{1}{5}$, by dividing by 5; $\frac{1}{6}$, by dividing by 6, etc. (**69**, *a*).

1. If it require 3 yards of cloth to make a pair of pantaloons, how many pairs can be made from a piece containing 27 yards? From 18 yards? From 30 yards?

SOLUTION. — As every 3 yards will make 1 pair, 27 yards will make as many pairs as there are 3's in 27. $27 \div 3 = 9$. Hence 27 yards will make 9 pairs, if 3 yards make one pair.[1]

2. If 9 pairs of pantaloons of equal size are made from 27 yards of cloth, how many yards does it take for one pair?

SOLUTION. — If 27 yards make 9 pairs, one pair requires $\frac{1}{9}$ of 27 yards. $\frac{1}{9}$ of a number is found by dividing it by 9. $27 \div 9 = 3$. Hence 1 pair requires 3 yards, if 9 pairs require 27 yards.

[Notice the difference between this solution and the preceding.]

3. If apples cost $3 per barrel, how many barrels can be bought for $12? For $15? For $30? For $327? For $18756?

[When the numbers are small, the computation should be without writing, — mentally, as it is called.]

4. If 21 barrels of apples are bought for $63, how much is that per barrel? If 21 barrels cost $84, how much is it per barrel?

[1] Some such *form of solution* should be insisted on in the class; for let it be remembered, that, in the "*Applications*," the important thing is to tell *why we divide, multiply, add, or subtract*. These exercises are not designed to teach division, — *that is, how to divide*, — but to teach the *uses of division*.

5. At $7 per yard, how much cloth can be bought for $84? For $28? For $63? For $256? For $359?

6. If 6 bushels of potatoes cost $4.50 (450 cents), what is the price per bushel?

7. A drover bought 7 horses for $1274, paying the same price for each. What did one horse cost him?

8. If 5 oranges cost 35 cents, what is that per orange? If they cost 45 cents? If 40 cents?

9. If 46 pounds of sugar cost $5.98, what is that per pound?

10. What is the price of coal per ton when 6 tons cost $66? When 8 tons cost $60?

11. John's board bill for a term of 13 weeks was $78. What was that per week?

12. A laborer received $39.00 for a month's work of 26 days. How much was that per day?

13. How many stoves, at $47 each, can be bought for $893?

14. If 19 stoves are bought for $893, what is the price of one stove?

15. If one stove cost $47, what will 19 stoves cost?

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16. A yard is 3 feet. How many yards, and how many feet over, in 100 feet? In 83? In 60? In 11?

17. A foot is 12 inches. How many inches in a yard? In a half-yard? In a quarter-yard?

18. A pound, grocers' weight, is 16 ounces. How many pounds and ounces in 180 ounces? In 250? In 48?

19. How many ounces make a quarter of a pound? A half-pound?

20. Four quarts make a gallon, and 2 pints make a quart. How many gallons in 80 pints? In 64? In 320?

21. 320 rods make a mile. How many rods in a half-mile? In a quarter-mile?

22. 5280 feet make a mile. How many yards make a mile? How many inches?

23. In a bushel are 4 pecks, and in a peck 8 quarts? How many bushels in 800 quarts? In 1000?

24. If a certain rope weighs a pound for every 5 feet, how much will 200 feet of it cost at 12¢ per pound?

25. How many bushels in 5280 pints, if 2 pints make a quart, 8 quarts make a peck, and 4 pecks make a bushel?

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26. A man going to buy sheep took with him $1256. His expenses were $150. He bought 217 sheep at a uniform price, and had $21 left. How much did one sheep cost. ANS., $5.

27. If a man on horseback ride 53 miles each day, how much will he lack of 1000 miles in 16 days? ANS., 152 miles.

28. If a man pay $75 each for oxen, how many oxen can he buy for $2000? and how much money will he have left?

29. A young man attending college had $200 at the beginning of a term of 18 weeks. He spent $35 for books, and $23 for other purposes besides board, and had $34 left at the end of the term. How much did his board cost him per week?

SUGGESTION. — How much of the $200 did he *not* spend for board? How much *did* he spend for board? If he spent so much for 18 weeks' board, how much was that for one week? ANS., $6.

30. If a man had an income of $3742 a year (52 weeks), and spent $1500 for his family expenses, gave to charitable purposes $370, and saved the rest, how much did he save per week? ANS., $36.

31. A has 340 head of cattle worth $9860, and B has 760 acres of land worth $32680. Required the value of A's cattle per head, and of B's land per acre. ANS., $29 and $43.

32. Having a tract of land containing 540 acres, I wish to divide it into fields containing 45 acres each. What number *of fields* will it make? ANS., 12 fields.

33. A planter has 2280 dollars to lay out for mules and oxen, and wishes to purchase the same number of each. If he pay 65 dollars a head for mules, and 30 for oxen, how many of each can he buy?

SUGGESTION. — How much do 1 ox and 1 mule together cost? Then how many times can he buy 1 ox and 1 mule? ANS., 24.

34. How many yards of cloth, at 7 dollars a yard, 8 dollars a yard, and 9 dollars a yard, — the quantity of each kind to be the same, — can a merchant buy for 1800 dollars?

ANS., 75 yards of each kind.

35. A cistern, the capacity of which is 10000 gallons, is to be filled with water by 3 pipes discharging into it. The first pipe discharges 200 gallons per hour, the second and third each 150 gallons per hour. In what time will the cistern be filled by the three pipes running together?

ANS., 20 hours.

36. A farmer wishes to fill three kinds of sacks, containing 3 bushels, 4 bushels, and 5 bushels, and the same number of each kind, with 1728 bushels of corn. How many sacks can he fill? ANS., 144 of each kind.

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37 If 6 yards of calico can be bought for $1, how much will 12 yards cost? 24 yards? 48 yards? 60 yards?

38. If 8 melons cost $1, how many dollars will 3744 melons cost?

39. When 6 books are bought for $1, how many dollars must be paid for 1743 books?

40. If 7 days make 1 week, how many weeks in 21 days? 42 days? 365 days?

41. If there are 12 inches in 1 foot, how many feet in 578 inches, and how many inches over?

42 How many cents make $1? Then how many dollars in 5682 cents? In 586 cents? In 1280 cents? How many cents over in each case? How do you divide by 100?

43. Shingles are put up in bunches of 500 each. How many bunches does it take to make 1000? How many bunches will it take to shingle a roof which requires 8000 shingles?

44. If 3 pounds of coffee cost $1, how much will 27 pounds cost?

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45. If 3 gallons of molasses cost $2, how much will 6 gallons cost? How many times $2?

46. If 6 oranges can be had for 35 cents, how much will 36 oranges cost? How many times 35 cents?

47. If 7 lemons can be bought for 25 cents, how many can be had for $1, which is 100 cents? How many times as many can be had for 100 cents as for 25 cents?

48. If you can buy 5 slate-pencils for 3 cents, how many can you get for 18 cents? How much will 35 pencils cost?

49. If 2 horses can be bought for $350, how many can be bought for $8400 at the same rate?

SUGGESTION. — Observe that such problems can be solved in two ways. 1. How many times the price of 2 horses is $8400? Then how many times 2 horses can be bought? 2. How much does 1 horse cost? Then how many can be bought for $8400?

Sometimes one method involves fractions, while the other does not. In such cases take the latter.

50. If 7 sheep can be bought for $24.50, how much will a flock of 133 cost at the same rate?

51. If 4 bushels of apples can be bought for $3, how many can be bought for $51 at the same rate?

52. If 6 oranges can be bought for 42 cents, how many can be bought for 56 cents?

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53. If you divide 7 apples equally between 2 boys, how many will each boy have? If 9 apples among 4 boys? 13 apples among 3 boys? 27 apples among 5 boys?

ANS. TO LAST, $5\frac{2}{5}$ apples.

54. If a pound of sugar cost 8 cents, how much at that rate can be bought for 17 cents? For 25 cents?

55. At $7 a yard, how much cloth can be had for $45? For $105? For $23? For $63? For $5829?

56. If a man with a reaper can cut 8 acres of wheat in a day, how long will it take him to cut 25 acres? 30 acres? 270 acres? 372 acres? ANS. TO LAST, $46\frac{1}{2}$ days.

57. If a man with a reaper can cut 7 acres of wheat in a day, in how many days can 8 men with reapers cut 250 acres? How much will they cut in one day? Then how long will it take them to cut 250 acres?

58. There being 1760 yards in a mile, find how many miles there are in 66160 yards. ANS., $37\frac{13}{22}$ miles.

59. How many days would 21 horses subsist on an amount of food which would suffice one horse 300 days?

ANS., $14\frac{2}{7}$ days.

60. Allowing a steamboat to run 275 miles in a day, in what time would she make a trip of 5350 miles?

ANS., $19\frac{5}{11}$ days.

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61. A flour-merchant sold 108 barrels of flour for $9 a barrel, and gained on it $216. What price, then, must he have paid for it by the barrel? ANS., $7.

62. A provision-dealer bought 1000 bushels of potatoes at 50¢ per bushel, and sold the whole for $600. How much did he make per bushel?

63. At what price per barrel must a lot of 57 barrels of flour, which cost $399, be sold, so as to make 50¢ per barrel?

ANS., $7.50.

64. If I make a profit of 75¢ on a pair of boots by selling them at $10.50, how much shall I make on a lot which cost me $370?

SUGGESTION.—How many cents is $10.50? How many $370? Read the example, calling the several sums *cents*.

65. How much shall I lose on a piece of cloth containing 45 yards, which cost me $270, by selling it at the rate of 10 yards for $55? ANS., $22.50.

66. If I sell 3 dozen pairs of gloves, which cost me $45, for $48, how much do I make per pair?

67. If I make 60¢ per yard on cloth which cost me $240, how much must I sell to make $300? ANS., 500 yards.

68. A coal-dealer makes $3 profit on a car-load of 12 tons. How much does he make on 600 tons?

69. If a merchant sells gloves which cost him $18 per dozen pairs at a loss of 25¢ per pair, how much will he lose on a stock which cost him $126?

70. Bought 750 bushels of wheat for $787.50, and sold it at a profit of 10¢ per bushel. At what price per bushel did I sell it? ANS., $1.15.

71. If I buy corn at $3.15 for 5 bushels, and sell it at $4.90 for 7 bushels, how many bushels must I sell to make $56? ANS., 800 bushels.

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72. Eight quarts make a peck. Then what part of a peck is 4 quarts? 2 quarts?

73. Twelve inches make a foot. Then what part of a foot is 6 inches? 3 inches? 4 inches?

74. Eight-eighths make 1. Then what part of one is 4-eighths, or $\frac{4}{8}$? $\frac{2}{8}$?

75. 9-ninths make 1. Then what part of 1 is 3-ninths, or $\frac{3}{9}$?

76. What part of any thing is $\frac{3}{6}$ of it? $\frac{2}{6}$?

77. If flour is $8 a barrel, how much can a man buy for $4? For $2? How many pounds for $4? (196 pounds make a barrel.) How many pounds for $2?

78. A countrywoman brought to market 5 dozen eggs, for which she got 28 cents a dozen, and 28 pounds of butter, for which she got 32 cents a pound. She bought 12 yards of

calico at 15 cents a yard, a shawl for $7, and took the rest in sugar at 9 cents a pound. How much sugar did she get?

Ans., $17\frac{1}{3}$ pounds.

79. A man had an orchard of 48 plum-trees. 24 of them died. What part died? Eight of them bore fruit. What part bore fruit?

80. How many 80's in 320? Then what part of 320 is 80? How many 40's in 320? Then what part of 320 is 40?

81. What part of 144 is 72? 24? 36?

82. What part of 100 is 50? 25?

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83. Dividend 7826, quotient 18, and remainder 212. What is the divisor?

84. Quotient 371, divisor 168, and remainder 47. What is the dividend?

85. Divisor 564. What must the remainder be, in order that the dividend be 6210, and the quotient 11?

86. Subtrahend 5834, remainder 1276. What is the minuend?

87. The product is 875000, and one of the factors is 125. What is the other?

88. Two factors are 625 and 25. What is their product?

89. Subtrahend 3182, remainder 689. What is the minuend?

90. Divisor 187, quotient 321, and remainder 133. What is the dividend?

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91. Two men travel in the same direction, and from the same place. One goes 43 miles a day, and the other 52. How long will it be before they are a day's journey for the latter apart?

92. Two men start from the same point: one goes east at 3 miles per hour, and the other 4 miles. How many working days of 8 hours each before they will be 144 miles apart?

93. A courier starts after a man who has been on the way 4 hours. The courier travels 10 miles per hour, and the man 8. How long before the courier overtakes the man? How far has the man traveled after the courier started?

94. How many years does it take to make the difference between saving \$2 per month and \$6 per month amount to a saving of \$100?

95. Two young men start in life with the same capital; say, \$5000. One saves \$2 per month out of his income, and the other spends \$3 per month more than his income. What will each be worth in 20 years? How long would it be before the difference between them would be \$1000?

96. What is the difference, in 15 years of 365 days each, between spending 2¢ per day more than one earns, or 3¢ less, exclusive of 52 Sundays each year?

97. How long would it take, at the rates mentioned in Ex. 96, to make a difference of \$1000?

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98. A man earned in 6 months the following amounts: 18 dollars, 21 dollars, 12 dollars, 27 dollars, 30 dollars, and 36 dollars. What was the average monthly earnings?

Ans., 24 dollars.

By this question is meant, What would be the *uniform* wages per month, in order to earn the same amount in the same time?

99. The heights of 5 monuments are taken, and found to be 10 ft., 17 ft., 12 ft., 13 ft., and 15 ft. What is the average height according to these measurements?

100. A drover bought 100 horses. For 20 he paid \$125 each; for 30, \$100 each; for 5, \$400 each; for 40, \$150 each; for the remainder, \$85 each. At what average price must he sell them to make a profit of \$500 on the lot?

101. A, B, and C, working together, dig a ditch for \$378; A receiving \$6 a week, B \$7, and C \$8. Required the time they labored, and amount received by each.

Ans., A received \$108, B \$126, and C \$144.

SECTION VIII.

FACTORS.

89. An Integer is a *Whole Number* in distinction from a Fraction.

Whole numbers are also called *Entire Numbers.*

90. The Factors of a number are those numbers which, multiplied together, produce it. Thus 2 and 3 are the factors of 6, because $2 \times 3 = 6$.

A Factor of a number is also called a *Divisor*, or an *Exact Divisor.*

In arithmetic a *factor* is supposed to be integral, unless otherwise stated.

91. In arithmetic a **Composite Number** is the product of two or more integral factors each greater than 1.

Thus 6 is a composite number; for it is the product of 2 and 3. 15 is a composite number; for it is the product of 3 and 5.

1. Point out the composite numbers in the following, and tell what their factors are: 21; 12; 24; 36; 7; 8; 11; 13; 28; 23; 20; 30; 130; 300; 230; 1200.

2. Can there be a number, whose right-hand figure is 0, which is not composite?

92. Numbers which are not *composite* are called **Prime Numbers.**

3. Point out *all* the prime numbers between 1 and 100. Make a list of them on your slate.

4. Of what 3 factors is 12 composed? 30? 66? 60? 8? 27?

[See Appendix I. for the method of finding Prime Numbers, called Eratosthenes' Sieve.]

93. One number is said to be **Divisible** by another when the former contains the latter an integral number of times without a remainder.

94. An **Even Number** is a number which is divisible by 2.

95. An **Odd Number** is a number which is not divisible by 2.

What numbers do you name when you count by 2, beginning "2, 4, 6, 8," etc.?

What numbers do you name when you count by 2, beginning "1, 3, 5, 7," etc.?

96. All *numbers* ending in 0, 2, 4, 6, or 8, are *even*.

This is so, since, in dividing any such number as 354 by 2, when we come to units, we shall either have just the units to divide, or the units with 1 before them; as 10, 12, 14, 16, 18. Now, each of these numbers is exactly divisible by 2.

97. All *numbers* ending in 1, 3, 5, 7, or 9, are *odd*.

This is so, since, in dividing any such number as 247 by 2, when we come to units, we shall either have just the units to divide, or the units with 1 before them; as 11, 13, 15, 17, 19. Now, no one of these numbers is divisible by 2.

98. All numbers ending in 0 or 5 are divisible by 5.

The reason for this is, that any number represented by *two* digits, the right hand one being 0 or 5, is divisible by 5; as 10, 20, 30, etc., and 15, 25, 35, etc.

To Resolve a Number into its Prime Factors.

99. Rule. —*Divide the number by the least number which will divide it exactly. Treat this quotient in the same way, and continue the process until there is no exact divisor of the quotient last obtained other than itself and 1.*

The several divisors and the last quotient are the prime factors sought.

1. Resolve 60 into its prime factors.

OPERATION.

$$\begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

Therefore 2, 2, 3, and 5 are the prime factors of 60; for they are all prime numbers, and $2 \times 2 \times 3 \times 5 = 60$.

2 to 52. Resolve 108 into its prime factors. So also 56, 14, 27, 135, 42, 75, 45, 128, 63, 59, 43, 48, 160, 28, 32, 34, 49, 50, 70, 72, 76, 77, 4, 8, 9, 10, 91, 330, 420, 231, 462, 2310, 105, 39, 38, 36, 154, 133, 79, 37, 885, 912, 1380, 391, 1728, 1008, 5760, 1760, 44, 327.

COMMON DIVISORS.

100. **A Common Divisor** of two or more numbers is a common integral factor; that is, a whole number which exactly divides each of the numbers.

Thus 3 is a common divisor of 12 and 18. 5 is a common divisor of 10, 15, and 20. Why?

101. **The Greatest Common Divisor** of two or more numbers is the greatest whole number which will exactly divide each of them.

Thus 8 is the greatest common divisor of 16, 24, and 32. Why?

To Find the Greatest Common Divisor of Two or More Numbers.

102. **Rule.** — *Resolve the numbers into their* **Prime Factors,** *and take the product of all the factors which are common to the numbers.*

Reasons. — 1st, This product divides each of the numbers, because the product of any of the factors of a number is a divisor of it.

2d, It is the *greatest* common divisor, since, if any other factor be introduced, the product will not divide those numbers in which this factor does not occur.

1. What is a common divisor of 6 and 4? Of 10 and 15? Of 8 and 12?

2. How many common divisors of 8 and 12 can you name? What is the greatest common divisor of 8 and 12?

3. What is the greatest common divisor of 16 and 40? Of 42 and 63?

4. Find the greatest common divisor of 1512, 882, and 630.

$1512 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7$, $882 = 2 \times 3 \times 3 \times 7 \times 7$, and $630 = 2 \times 3 \times 3 \times 5 \times 7$. The prime factors common to all are 2, 3, 3, 7. Hence $2 \times 3 \times 3 \times 7 = 126$ is the G. C. D. of 1512, 882, and 630.

Examples for Practice.

5. 96, 120.	17. 144, 196, 324.	29. 17, 136.
6. 78, 130.	18. 780, 720, 1008.	30. 24, 120.
7. 62, 93.	19. 1100, 144, 960.	31. 31, 29.
8. 209, 253.	20. 216, 408, 740.	32. 143, 1001.
9. 252, 288.	21. 260, 416, 384.	33. 63, 217.
10. 1064, 1274.	22. 128, 256, 1280.	34. 114, 171.
11. 741, 1938.	23. 140, 308, 819.	35. 973, 1668.
12. 326, 807.	24. 210, 315, 420.	36. 744, 1680.
13. 336, 576.	25. 120, 210, 360.	37. 1560, 1008.
14. 1056, 1768.	26. 330, 495, 660.	38. 47, 83.
15. 1064, 1274.	27. 4485, 5980.	39. 151, 247.
16. 235, 445.	28. 3795, 345.	40. 864, 568.

[For the General Rule for finding the Greatest Common Divisor and its demonstration, as well as a specially economical and elegant method of applying it, see Appendix II.]

COMMON MULTIPLES.

103. A Multiple of a number is a number which contains that number as a factor. Hence a *Composite Number* is a *multiple of* each of its factors.

In speaking of multiples the number itself and 1 are considered as factors, and any number is its own least multiple; but fractions are excluded.

104. A **Common Multiple** of two or more numbers is a multiple of each of them.

105. The **Least Common Multiple** of two or more numbers is the least number which is a multiple of each of them, and hence cannot be less than the largest of the numbers.

Thus 15 is a multiple of 3 and 5; but it is not a multiple of 2 or 7.

48 is a common multiple of 2, 4, 6, 8, and 12; but it is not their *least* common multiple, since 24 is a multiple of each of these numbers.

106. Principle. — **The Product** *of two or more numbers is a* Common Multiple *of them all, since it contains each of them as a factor.*

To Find the Least Common Multiple of Two or More Numbers.

107. Rule. — I. *Resolve each of the numbers into its prime factors.*

II. *Multiply the largest number by all the prime factors found in the next smaller number, and not in it.*

III. *Treat this product and the next smaller number in the same way, and continue the process till all the numbers are used.*

Demonstration. — We take the largest number, because the least common multiple must contain it. Then we multiply this by all the prime factors contained in the next smaller, but not in the largest; because, if there is a component factor of this number lacking in the product, it will not contain this number: and so on of all the numbers. This product will contain each of the numbers, since it contains all its factors.

Again: the product thus obtained is the *least* common multiple, because no factor can be left out of it without preventing its containing some one of the numbers.

Ex. 1 —Find the least common multiple of 27, 42, and 36.

SOLUTION.

$42 = 2 \times 3 \times 7$. $36 = 2 \times 2 \times 3 \times 3$. $27 = 3 \times 3 \times 3$.

$\therefore 42 \times 2 \times 3 \times 3 = 756$, the least common multiple.

For, as the number must contain 42 as a factor, we write it as one of the factors of the multiple sought. Then, as 36 contains *two* factors 2, and *two* factors 3, and 42 has but *one* each of these, we write these factors as factors of the multiple, and have $42 \times 2 \times 3$. Now, this product has but *two* factors 3 in it; whereas 27 has 3 such factors. Hence we write another factor 3, and have $42 \times 2 \times 3 \times 3 = 756$ as the least common multipie. It is the *least* common multiple, because no factor can be omitted from it without preventing its containing some one of the numbers.

If we strike out one factor 2 from $42 \times 2 \times 3 \times 3$, which number will the product not contain? If we strike out one factor 3, which number will the product not contain? If the 7 (in the 42)?

Examples for Practice.

2. Show that the l. c. m. of 3, 8, 9, 12, is 144.
3. Show that the l. c. m. of 45, 63, 81, is 2835.
4. Show that the l. c. m. of 8, 36, 9, 17, is 1224.

5. 15, 20, 32, 75.	13. 5, 10, 13, 24.	21. 17, 11, 23.
6. 4, 6, 8, 10.	14. 6, 7, 2, 17.	22. 31, 57, 46, 1.
7. 9, 3, 12, 15.	15. 11, 4, 5, 19.	23. 19, 17, 53.
8. 21, 7, 4, 9.	16. 4, 6, 9, 14.	24. 57, 51, 159.
9. 6, 4, 12, 20.	17. 18, 27, 54, 45.	25. 100, 10, 1000.
10. 8, 7, 10, 14.	18. 7, 15, 21, 28, 35.	26. 200, 100, 300.
11. 15, 2, 10, 13.	19. 84, 100, 224.	27. 7, 5, 3, 11.
12. 24, 5, 6, 10.	20. 49, 56, 63, 84.	28. 70, 50, 30, 110.

29. Find the least number which each of the digits, 1, 2, 3, 4, 5, 6, 7, 8, 9, will exactly divide.

[See Appendix III. for a rule frequently given, and also for a method when the common factors are not easily detected.]

CANCELLATION.

108. Cancellation is rejecting equal factors which appear both as multipliers and divisors in an operation.

Cancelling such equal factors does not change the value of the result; because dropping a factor from a multiplier would divide the result, while dropping it from a divisor would multiply the result. Hence one operation compensates for the other.

Ex. 1.— Required the result if the numbers 12, 15, and 63 are multiplied together, and the product divided by the product of 4, 14, and 21.

OPERATION.

$$\frac{\overset{3}{\cancel{12}} \times \overset{5}{\cancel{35}} \times \overset{3}{\cancel{63}}}{\cancel{4} \times \underset{2}{\cancel{14}} \times \cancel{21}} = \frac{45}{2} = 22\tfrac{1}{2}.$$

EXPLANATION. — For convenience we *indicate* the operations to be performed by writing above a line the numbers to be multiplied together for the dividend; and below, those which are to be multiplied together for the divisor. Then, observing that 4 is a factor in 12, we cancel it, and write the remaining factor of 12 (viz., 3) above the 12. This is equivalent to dividing both dividend and divisor by 4, which does not alter the quotient. We then observe a factor 7 in 14 and 35, and, striking it out, have left the factors 2 and 5 respectively. So also we strike the factor 21 from divisor and dividend. There then remain the factors 3, 5, and 3 in the dividend, and 2 in the divisor. Hence the result is,

$$\frac{3 \times 5 \times 3}{2}, \text{ or } \frac{45}{2}, \text{ which equals } 22\tfrac{1}{2}.$$

Examples for Practice.

2. $(16 \times 24 \times 48) \div (32 \times 36 \times 8) = ?$ ANS., 2.
3. $(84 \times 12 \times 18) \div (21 \times 24 \times 9) = ?$ ANS., 4.
4. $(72 \times 18 \times 16) \div (24 \times 16 \times 9) = ?$ ANS., 6.
5. $(76 \times 34 \times 96) \div (17 \times 51 \times 32) = ?$ ANS., $8\frac{16}{17}$.
6. $(184 \times 145 \times 80) \div (23 \times 29 \times 60) = ?$
7. $(164 \times 96 \times 44) \div (8 \times 33 \times 48) = ?$
8. $(45 \times 60 \times 7) \div (49 \times 12 \times 9) = ?$

9. $(2 \times 3 \times 5 \times 8 \times 7) \div (6 \times 5 \times 2 \times 7) = ?$

10. $(5 \times 8 \times 12 \times 6) \div (20 \times 16 \times 2) = ?$

11. $(12 \times 60 \times 36 \times 35) \div (7 \times 30 \times 18 \times 24) = ?$

12. $(30 \times 49 \times 64 \times 25) \div (35 \times 15 \times 24) = ?$

13. $\dfrac{35 \times 39 \times 40}{26 \times 30 \times 42} = ?$

14. $\dfrac{26 \times 33 \times 35}{4 \times 9 \times 25} = ?$

15. $\dfrac{6 \times 9 \times 15 \times 21}{4 \times 6 \times 10 \times 14} = ?$

16. $\dfrac{21 \times 24 \times 28 \times 35}{14 \times 18 \times 20 \times 25} = ?$

17. $\dfrac{45 \times 49 \times 81}{35 \times 84 \times 63} = ?$

18. $\dfrac{360 \times 7 \times 8}{100 \times 12} = ?$

19. $\dfrac{360 \times 10 \times 5}{100 \times 12} = ?$

20. $\dfrac{360 \times 8 \times 4}{100 \times 12} = 9.6.$

The Use of the Signs +, —, ×, ÷, and ( ), in indicating Operations.

109. 1st, If no signs but +, —, ×, and ÷ are used, the operations indicated by × and ÷ are to be performed in succession, and before those indicated by + and —.

2d, When the ( ) is used, the indicated operations within are to be performed first, subject to the preceding direction.

Ex. 1. — What is the result of $5 \times 4 \div 2 + 6 \div 3 + 12 - 4 \div 2$?

Performing the operations indicated by × and ÷, we have

$10 + 2 + 12 - 2$, which is 22.

2. Find the result of the following operations: $(5 + 4 \div 2) + (3 \times 2 + 4 - 1) + 3 - (5 \times 2 + 12 \div 3.)$

Performing the operations indicated within the parentheses, we have

$7 + 9 + 3 - 14$, which is 5.

Examples for Practice.

Show the correctness of the following: —

3. $(6+8)\times 5=70.$
4. $6+8\times 5=46.$
5. $(8-3)\times 2=10.$
6. $8-3\times 2=2.$
7. $8+12\div 4=11.$
8. $(8+12)\div 4=5.$
9. $(2+1)\times(7-2)=15.$
10. $3\times 8\div 4\times 3=18.$
11. $3\times 8\div(4\times 3)=2.$
12. $14-\dfrac{3\times 4-2\times 3}{2}=11.$
13. $\dfrac{9+3}{2}+\dfrac{8-2}{2}=9.$
14. $4\times(5\times 6\div 3)+10=50.$
15. $(48\div 2+12)\times 3+(8\div 2+14\times 2)=?$
16. $48\div(2+12)\times(3+8)\div 2+14\times 2=?$

Applications.

1. A farmer sold 24 bushels of apples at 56 cents per bushel, and took his pay in codfish at 8 cents per pound. How many pounds did he receive?

First indicate the operations, and then cancel. Thus, —

$$\frac{56\times 24}{8}=56\times 3=168 \text{ pounds.}$$

2. If 7 books cost \$63, how much will 9 books cost?
3. How much will 4 times 9 bushels of apples cost, if 12 times 7 bushels cost \$63.84? Ans., \$27.36.
4. How many bushels of wheat, at \$1.20 a bushel, can be bought for 12 bushels of rye, at \$1.10 a bushel?
5. How many hats, at \$5 each, can be bought for 25 pounds of butter, at 40 cents a pound?
6. How many bushels of rye, at 84 cents a bushel, must be given for 13 sacks of corn, each containing 3 bushels, at 56 cents a bushel? Ans., 26 bushels.

7. How many vests, at \$6 each, can be bought for 24 yards of broadcloth, at \$2.50 a yard?

8. What is the length of the longest curb-stones that will exactly fit each of four strips of sidewalk,—the first being 273 feet long; the second, 294; the third, 567; the fourth, 651?

9. The junior class in a school consists of 132 students, and the senior of 99. How might each class be divided so that the whole school should be disposed in equal sections?

ANS., Into sections of 3, 11, or 33.

10. For what sum of money could a carpenter hire journeymen for one month at 15 dollars, 21 dollars, or 24 dollars each, allowing the whole sum to be thus expended?

11. A gentleman has a corner of ground, the sides of which measure 225 feet, 297 feet, and 369 feet. He wishes to inclose it with a fence having panels of uniform length. What must be the length of each panel? ANS., 9 feet.

12. An upholsterer has 125 yards of carpeting of one kind, 175 of another, and 225 of another. He wishes to divide the whole into pieces of equal length, and the longest that can be obtained. What must be the length of each piece?

13. What is the smallest sum of money for which I could purchase a number of ploughs at 14 dollars each, or a number of carts at 30 dollars each, or a number of wagons at 90 dollars each?

14. A has 413 dollars, B 531 dollars, and C 590 dollars; and they agree to purchase horses at the same price per head, provided each man can thus invest all his money. How many horses could each man purchase?

15. An island is 200 miles in circumference; and three persons, A, B, and C, start together, and travel the same way around it. A goes 20 miles per day; B, 25; and C, 40 miles per day. In what time would they all come together again at the same point from which they started?

First find the number of days it would require each person *to go around* the island. ANS., 40 days.

COMMON FRACTIONS

CHAPTER II.

SECTION I.

DEFINITIONS AND FUNDAMENTAL PRINCIPLES.

110. A Fraction is a number representing one or more of the equal parts into which a unit, or some number taken as a whole, is conceived to be divided.

111. The number which indicates into how many equal parts the number is to be divided is called the **Denominator.**

Denominator means *namer;* and it will be seen that the number of equal parts into which a thing is divided determines the *name* of one of those parts. Thus, if we divide a thing into 3 equal parts, the parts are called (denominated) *thirds;* if into 4, *fourths;* if into 13, *thirteenths*, etc.

112. The number which indicates how many of the equal parts are represented by the expression is called the **Numerator.**

Numerator means *numberer*, and hence the appropriateness of the name. If 2-thirds are taken, 2 tells the *number* of thirds, and hence is the *numerator*, or numberer; if 5-sevenths are taken, 5 is the *numerator*, or numberer, etc.

113. The Terms of a fraction are the *Numerator* and *Denominator*.

114. Fractions are of two kinds, *Common* and *Decimal*.

115. A Common Fraction is written in figures by writing the numerator above the denominator with a line between.

For the definition of a Decimal Fraction see next chapter.

1. What does $\frac{2}{3}$ signify? Into how many equal parts is the division to be made? What are such parts called? How many of them are indicated? What is the numerator? What the denominator? What are the terms of the fraction?

2. Explain, as above, the meaning of $\frac{3}{5}$, $\frac{1}{7}$, $\frac{3}{7}$, $\frac{4}{9}$, $\frac{3}{11}$, $\frac{2}{10}$, $\frac{3}{156}$, $\frac{127}{342}$.

3. Write in figures three-sevenths; four-ninths; one-thirteenth; 5-twelfths; 7-eighths; 11-seventeenths; 123-three hundred fifty-sixths; two-tenths; 203-thousandths; one-half.

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116. A Mixed Number is an *integer* (88) and a *Fraction* written in connection, and signifies that the two are to be taken together; as $4\frac{2}{3}$, $128\frac{1}{2}$.

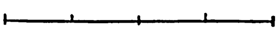
A mixed number is read by naming the whole number, and then the fraction. Thus $4\frac{2}{3}$ is read "four and two-thirds;" $136\frac{5}{6}$ is read "136 and $\frac{5}{6}$;" etc.

117. A Proper Fraction is a fraction whose numerator is less than its denominator.

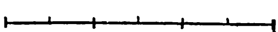
118. An Improper Fraction is a fraction whose numerator is equal to or greater than its denominator.

119. The Value of a fraction is the amount which it represents. Thus two fractions are said to have the same value when they represent the same amount of any thing.

Illustrate by this divided line that $\frac{4}{4} = \frac{2}{2}$; that $\frac{2}{4} = \frac{1}{2}$.



Illustrate by this line that $\frac{2}{6} = \frac{1}{3}$; that $\frac{4}{6} = \frac{2}{3}$; that $\frac{3}{6} = \frac{1}{2}$; $\frac{6}{6} = \frac{2}{2} = \frac{4}{4}$.



Fundamental Principles.

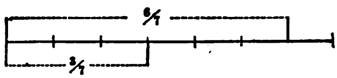
120. Principle I. — *Multiplying the numerator of a fraction, while the denominator remains the same, multiplies the value of the fraction.*

Demonstration.[1] — The reason for this is, that, as the numerator tells how many parts are represented, multiplying it multiplies the *number* of parts represented; and, as the denominator is not changed, the *size* of the parts remains the same.

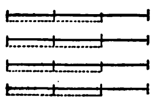
Ex. 1. — Show that multiplying the numerator of $\frac{3}{7}$ by 2 multiplies the value of the fraction by 2.

Explanation. — Multiplying the numerator of $\frac{3}{7}$ by 2, we have $\frac{6}{7}$. Now, $\frac{6}{7}$ is twice as much as $\frac{3}{7}$, since $\frac{6}{7}$ contains twice as many parts as $\frac{3}{7}$, and the size of the parts is the same in each case.

Illustrate, by a line divided as in the margin, that 2 times $\frac{3}{7}$ is $\frac{6}{7}$; i.e., that multiplying the numerator multiplies the fraction.



Illustrate, by 4 lines divided into thirds, that 4 times $\frac{2}{3}$ is $\frac{8}{3}$.



2–7. Perform the following, and give the explanation as above: $\frac{2}{9} \times 5$; $\frac{3}{7} \times 4$; 4 times $\frac{3}{10}$; 7 times $\frac{5}{12}$; 128 times $\frac{13}{179}$; 2145 times $\frac{314}{4371}$.

121. Principle II. — *Dividing the denominator of a fraction, while the numerator remains the same, multiplies the value of the fraction.*

Demonstration. — The reason for this is, that it multiplies the *size* of the parts represented, while the number represented remains the same.

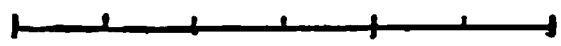
Ex. 1. — Show that dividing the denominator of $\frac{3}{8}$ by 2 multiplies the value of the fraction by 2.

---

[1] Let the teacher explain that a Demonstration is a condensed and formal presentation of the reasons for a statement. It is the argument which proves the statement true.

Explanation. — Dividing the denominator of $\frac{2}{6}$ by 2, we have $\frac{2}{3}$. Now, $\frac{2}{3}$ is twice as much as $\frac{2}{6}$, since a third of any thing is twice as much as a sixth, and there is the same *number* of parts in each case.

Illustrate, by a line divided into six equal parts, that $\frac{2}{3}$ is twice as much as $\frac{2}{6}$; i.e., that dividing the denominator multiplies the fraction.



2–6. Prove and illustrate, as above, that, if the denominator of $\frac{2}{9}$ be divided by 3, the result is 3 times $\frac{2}{9}$. In like manner, that $\frac{5}{12} \times 3 = \frac{5}{4}$. That $\frac{3}{16} \times 4 = \frac{3}{4}$. That $\frac{11}{200} \times 10 = \frac{11}{20}$. That $\frac{6}{35} \times 7 = \frac{6}{5}$.

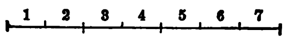
122. Principle III. — *Dividing the numerator of a fraction, while the denominator remains the same, divides the value of the fraction.*

Demonstration. — The reason for this is, that it divides the *number* of parts represented, while the size of the parts remains the same.

Ex. 1. — Show that dividing the numerator of $\frac{4}{7}$ by 2 divides the value of the fraction by 2.

Explanation. — Dividing the numerator of $\frac{4}{7}$ by 2, we have $\frac{2}{7}$. Now, $\frac{2}{7}$ is only $\frac{1}{2}$ as much as $\frac{4}{7}$, since in $\frac{2}{7}$ there are only $\frac{1}{2}$ as many parts as in $\frac{4}{7}$, while the *size* of the parts is the same in each case.

Illustrate, by a line divided into 7 equal parts, that $\frac{2}{7}$ is $\frac{1}{2}$ of $\frac{4}{7}$; i.e., that dividing the numerator divides the fraction.



2. Divide $\frac{10}{17}$ by 2. By 5. Give the reasons.
3. Why is $\frac{4}{7}$ $\frac{1}{2}$ of $\frac{8}{7}$? What part of $\frac{12}{7}$ is $\frac{4}{7}$?
4. Divide $\frac{6}{9}$ by 3. By 2. By 6.

123. Principle IV. — *Multiplying the denominator of a fraction, while the numerator remains the same, divides the value of the fraction.*

Demonstration. — The reason for this is, that multiplying the denominator divides the *size* of the parts, while the number remains the same.

Ex. 1. — Show that multiplying the denominator of $\frac{2}{3}$ by 2 divides the value of the fraction.

Explanation. — Multiplying the denominator of $\frac{2}{3}$ by 2, we have $\frac{2}{6}$. Now, $\frac{2}{6}$ is only $\frac{1}{2}$ of $\frac{2}{3}$, since in $\frac{2}{6}$ the parts are only $\frac{1}{2}$ as large as the parts in $\frac{2}{3}$, while the *number* is the same in each case.

Illustrate, by a line divided into 6 equal parts, that $\frac{2}{6}$ is $\frac{1}{2}$ of $\frac{2}{3}$; i.e., that multiplying the denominator divides the fraction.

2. Divide $\frac{3}{5}$ by 2. By 3. By 4. Give the reasons.
3. Divide $\frac{5}{7}$ by 4. By 6. By 12.

124. Principle V. — *The value of a fraction is not altered by multiplying both terms of the fraction by the same number, or by dividing both terms by the same number.*

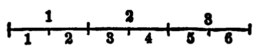
Demonstration. — Multiplying both terms of a fraction by the same number does not alter the value of the fraction; because multiplying the numerator *multiplies* the *number* of parts, while multiplying the denominator *divides* the *size* of the parts.

If we divide both terms, we *divide* the *number* of *parts*, and multiply the *size*, and hence do not change the value of the fraction.

Ex. 1. — Show that the value of $\frac{2}{3}$ is not altered by multiplying both terms by 2.

Explanation. — Multiplying both terms of $\frac{2}{3}$ by 2, we have $\frac{4}{6}$. Now, $\frac{4}{6}$ represents *twice* as many parts as $\frac{2}{3}$; but the parts are only $\frac{1}{2}$ as large. Hence $\frac{4}{6} = \frac{2}{3}$.

Illustrate, by a line divided into 6 equal parts, that $\frac{4}{6} = \frac{2}{3}$.



2. Show that the value of $\frac{8}{12}$ is not altered by dividing both terms by 4.

3. Why is $\frac{2}{7} = \frac{8}{28}$? Why is $\frac{10}{12} = \frac{5}{6}$? Why is $\frac{3}{5} = \frac{12}{20}$? Why is $\frac{18}{24} = \frac{3}{4}$?

4. Illustrate, as above, that $\frac{4}{12} = \frac{1}{3}$. That $\frac{2}{3} = \frac{8}{12}$. That $\frac{3}{7} = \frac{9}{21}$. That $\frac{16}{28} = \frac{4}{7}$.

125. Principle VI. — *The Numerator of a fraction may be considered as a Dividend, and the Denominator as a Divisor of it, the value of the fraction being the quotient.*

Ex. 1. — Show that $\frac{7}{11}$ may be considered as 7 divided by 11.

If we take 7 things, and divide each into 11 equal parts, and then take 1 part from each 11, what part of the 7 have we? How do we get $\frac{1}{11}$ of any thing? Then how many 11ths have we when we divide 7 by 11?

2. Show that $\frac{6}{5}$ may be considered as 6 divided by 5.

SECTION II.

REDUCTION OF FRACTIONS.

126. Reduction is changing the form of an expression without altering its value. Thus, changing $\frac{4}{8}$ into $\frac{1}{2}$ is a reduction.

To Lower or Lowest Terms.

127. A fraction is said to be in its lowest terms when there is no number greater than 1 which will exactly divide both terms of it.

To Reduce a Fraction to its Lowest Terms.

128. Rule. — *Reject all common factors greater than 1 from both terms; or divide both terms by their greatest common divisor.*

Demonstration. — To reject a factor is the same as to divide by that factor. Hence rejecting the common factors does not change the value of the fraction (*Principle V.*); and, when all the common factors are rejected, the fraction is in its lowest terms, according to *(127)*.

Mental Exercises.

Reduce the following to their lowest terms mentally, giving the reason in each case: —

Ex. 1. — Reduce $\frac{9}{12}$ to its lowest terms.

EXPLANATION. — Since 3 is a common divisor of both terms, we divide the terms of the fraction by 3, and have $\frac{3}{4}$. That $\frac{3}{4} = \frac{9}{12}$ is evident, since the parts (fourths) in $\frac{3}{4}$ are 3 times as large as the parts in $\frac{9}{12}$ (twelfths), and we have $\frac{1}{3}$ as many parts in $\frac{3}{4}$ as in $\frac{9}{12}$.

2. $\frac{5}{10}$.	8. $\frac{36}{54}$.	14. $\frac{20}{36}$.	20. $\frac{50}{60}$.
3. $\frac{6}{15}$.	9. $\frac{72}{81}$.	15. $\frac{16}{48}$.	21. $\frac{200}{300}$.
4. $\frac{8}{14}$.	10. $\frac{15}{45}$.	16. $\frac{24}{36}$.	22. $\frac{400}{1200}$.
5. $\frac{9}{12}$.	11. $\frac{37}{74}$.	17. $\frac{84}{120}$.	23. $\frac{600}{2000}$.
6. $\frac{27}{45}$.	12. $\frac{17}{85}$.	18. $\frac{26}{39}$.	24. $\frac{40}{280}$.
7. $\frac{13}{26}$.	13. $\frac{12}{15}$.	19. $\frac{44}{77}$.	25. $\frac{120}{200}$.

26. Reduce $\frac{294}{378}$ to its lowest terms.

SUGGESTION. — As it is not easy to see what is the greatest common divisor of the terms of this fraction we proceed thus: —

$$^{2)}\frac{294}{378} = {}^{7)}\frac{147}{189} = {}^{3)}\frac{21}{27} = \frac{7}{9}.$$

This process consists in dividing the terms successively by any number which will divide them both. These operations do not change the value of the fraction by (**124**); and, when there is no number which will exactly divide the terms, the fraction is in its lowest terms by (**127**).

Written Exercises.

Show the truth of the following: —

27. $\frac{25}{75} = \frac{1}{3}$.	31. $\frac{99}{225} = \frac{11}{25}$.	35. $\frac{225}{375} = \frac{3}{5}$.
28. $\frac{216}{360} = \frac{3}{5}$.	32. $\frac{180}{468} = \frac{5}{13}$.	36. $\frac{909}{2323} = \frac{9}{23}$.
29. $\frac{125}{150} = \frac{5}{6}$.	33. $\frac{210}{294} = \frac{5}{7}$.	37. $\frac{873}{1067} = \frac{9}{11}$.
30. $\frac{279}{403} = \frac{9}{13}$.	34. $\frac{80}{112} = \frac{5}{7}$.	38. $\frac{182}{196} = \frac{13}{14}$.

Reduce the following to their lowest terms: —

39. $\frac{665}{700}$.	48. $\frac{288}{360}$.	57. $\frac{123}{579}$.	66. $\frac{35}{1000}$.
40. $\frac{195}{299}$.	49. $\frac{216}{252}$.	58. $\frac{4862}{7896}$.	67. $\frac{45}{200}$.
41. $\frac{144}{240}$.	50. $\frac{960}{1536}$.	59. $\frac{1111}{7777}$.	68. $\frac{1700}{2300}$.
42. $\frac{352}{364}$.	51. $\frac{352}{384}$.	60. $\frac{111111}{999999}$.	69. $\frac{1430}{1320}$.
43. $\frac{171}{280}$.	52. $\frac{3172}{2196}$.	61. $\frac{3030}{9090}$.	70. $\frac{17860}{93725}$.
44. $\frac{291}{763}$.	53. $\frac{10212}{10656}$.	62. $\frac{1358}{7642}$.	71. $\frac{43}{976}$.
45. $\frac{1344}{1536}$.	54. $\frac{5643}{5940}$.	63. $\frac{57000}{93000}$.	72. $\frac{500}{700}$.
46. $\frac{133}{152}$.	55. $\frac{3060}{5940}$.	64. $\frac{1100}{4400}$.	73. $\frac{289}{340}$.
47. $\frac{336}{884}$.	56. $\frac{551}{4123}$.	65. $\frac{76200}{128600}$.	74. $\frac{23}{111}$.

To Reduce an Improper Fraction to a Whole or Mixed Number.

129. Rule. — *Divide the numerator by the denominator.*

DEMONSTRATION. — As the denominator shows into how many equal parts a unit is conceived to be divided, it shows how many such parts make a unit. Hence, dividing the number of parts represented by the fraction — i.e., the numerator — by the number of parts which it takes to make a unit, — i.e., by the denominator, — we find how many units are represented by the fraction, and how many parts (if any) remain.

1. Reduce $\frac{125}{13}$ to a whole or mixed number.

SOLUTION.[1] — Since 13 thirteenths make one unit, 125 thirteenths make as many units as 13 is contained times in 125. $125 \div 13 = 9$ and 8 remainder. Hence $\frac{125}{13} = 9\frac{8}{13}$.

Mental Exercises.

2–21. Reduce the following without writing, giving the solution as above: $\frac{7}{3}$; $\frac{8}{3}$; $\frac{9}{2}$; $\frac{10}{5}$; $\frac{13}{4}$; $\frac{28}{7}$; $\frac{80}{7}$; $\frac{63}{4}$; $\frac{71}{6}$; $\frac{20}{4}$; $\frac{11}{6}$; $\frac{17}{3}$; $\frac{10}{4}$; $\frac{36}{3}$; $\frac{400}{10}$; $\frac{40}{8}$; $\frac{57}{9}$; $\frac{78}{12}$; $\frac{93}{9}$; $\frac{56}{4}$.

[1] Teacher explain that a Solution is a formal statement of how an example is performed, *and the reasons* for *each step.* It is the *explanation.*

Written Exercises.

Reduce the following to whole or mixed numbers, giving the explanation as above: —

22. $\frac{346}{23} = 15\frac{1}{23}$.	27. $\frac{847}{15}$.	32. $\frac{3482}{412}$.	37. $\frac{58000}{700}$.
23. $\frac{747}{61} = 12\frac{15}{61}$.	28. $\frac{1246}{22}$.	33. $\frac{50082}{7185}$.	38. $\frac{42600}{300}$.
24. $\frac{456}{79} = 5\frac{61}{79}$.	29. $\frac{1312}{136}$.	34. $\frac{126}{10}$.	39. $\frac{12876}{100}$.
25. $\frac{444}{16} = 27\frac{3}{4}$.	30. $\frac{1312}{17}$.	35. $\frac{85}{10}$.	40. $\frac{7807}{10}$.
26. $\frac{111}{9} = 12\frac{1}{3}$.	31. $\frac{56789}{21}$.	36. $\frac{9372}{100}$.	41. $\frac{64}{30}$.

To Reduce a Whole or Mixed Number to an Improper Fraction.

130. Rule. — *Multiply the whole number by the denominator of the proposed fraction, and to this product add the numerator of the given fraction (if any), and write the result over the proposed denominator.*

[Let the pupil write out a demonstration.]

1. Reduce $7\frac{2}{11}$ to elevenths.

SOLUTION. — As there are 11 elevenths in 1, in 7 there are 7 times 11 elevenths, or 77 elevenths. Now, 77 elevenths and 2 elevenths make 79 elevenths. Hence $7\frac{2}{11} = \frac{79}{11}$.

Mental Exercises.

Reduce the following to improper fractions without writing the work. Give the solution in form as above.

2–20. $8\frac{2}{5}$; $5\frac{2}{3}$; $4\frac{3}{7}$; $9\frac{5}{6}$; $8\frac{5}{6}$; $7\frac{1}{2}$; $5\frac{1}{3}$; $12\frac{2}{7}$; $11\frac{5}{9}$; $4\frac{2}{3}$: $7\frac{1}{2}$; $6\frac{1}{3}$; $12\frac{3}{4}$; $14\frac{2}{5}$; $11\frac{7}{8}$; $9\frac{1}{10}$; $8\frac{2}{10}$; $4\frac{1}{100}$; $2\frac{1}{1000}$.

21–24. How many thirteenths in 5? In 8? In 11? In 3?

25–31. Reduce 7 to thirds; 3 to fifths; 6 to halves; 13 to 7ths; 8 to 4ths; 10 to 100ths; 11 to 11ths.

Written Exercises.

Reduce the following to improper fractions: —

32. $23\frac{3}{4}$.	38. $124\frac{1}{2}$.	44. $1\frac{34}{71}$.	50. $343\frac{1}{6}$.
33. $5\frac{2}{7}$.	39. $342\frac{2}{3}$.	45. $2\frac{128}{607}$.	51. $87\frac{5}{156}$.
34. $16\frac{3}{8}$.	40. $200\frac{5}{9}$.	46. $3\frac{17}{19}$.	52. $3\frac{231}{873}$.
35. $17\frac{5}{12}$.	41. $1256\frac{2}{7}$.	47. $5\frac{2}{10}$.	53. $10\frac{1}{100}$.
36. $57\frac{3}{11}$.	42. $48\frac{5}{13}$.	48. $7\frac{3}{100}$.	54. $9\frac{9}{10}$.
37. $81\frac{5}{9}$.	43. $2\frac{721}{935}$.	49. $13\frac{21}{1000}$.	55. $7\frac{77}{100}$.

Common Denominators.

131. Fractions are said to have a *Common Denominator* when their denominators are alike.

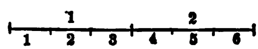
132. Principle. — *A fraction may be reduced to an equivalent fraction having a denominator which is any multiple of its own by multiplying both terms of the fraction by the other factor of the required denominator.*

This is evident, since it does not change the value of a fraction to multiply both its terms by the same number (**124**).

Ex. 1. — Reduce $\frac{2}{3}$ to 6ths.

Solution. — Multiplying the denominator by 2, the fraction represents 6ths; and, in order to preserve the value of the fraction as at first, we multiply the numerator also by 2, obtaining $\frac{4}{6}$ (**124**).

Illustrate by a line divided as in the margin.



Mental Exercises.

2. Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{2}{3}$, each to 12ths.
3. Reduce $\frac{3}{4}$, $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{7}{10}$, each to 20ths.
4. Reduce $\frac{5}{6}$, $\frac{2}{3}$, and $\frac{2}{9}$, each to 18ths.
5. Reduce $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$; $\frac{7}{20}$, and $\frac{3}{10}$, to 40ths.

To Reduce Fractions to Equivalent Ones having a Common Denominator.

133. Rule. — *Multiply both terms of each fraction by the denominators of all the other fractions.*

DEMONSTRATION. — This gives a common denominator, because each denominator is the product of all the denominators of the several fractions. The value of any one of the fractions is not changed, because both numerator and denominator are multiplied by the same number (**124**).

6. Reduce $\frac{5}{7}$, $\frac{2}{3}$, and $\frac{3}{5}$ to equivalent fractions having a common denominator.

SOLUTION. — Multiplying both terms of $\frac{5}{7}$ by 3 and 5, we have

$$\frac{5 \times 3 \times 5}{7 \times 3 \times 5} = \frac{75}{105}.$$

In like manner, $\frac{2 \times 7 \times 5}{3 \times 7 \times 5} = \frac{70}{105}.$

So also, $\frac{3 \times 7 \times 3}{5 \times 7 \times 3} = \frac{63}{105}.$

The denominators are all alike, because each is the product of 7, 3, and 5. $\frac{75}{105} = \frac{5}{7}$, because it arises from multiplying both terms of $\frac{5}{7}$ by 15, which does not change the value of the fraction according to (**124**).

In like manner, show that $\frac{70}{105} = \frac{2}{3}$, and $\frac{63}{105} = \frac{3}{5}$.

Reduce the following to equivalent sets of fractions having common denominators, and give the explanation as above: —

7. $\frac{5}{11}$, $\frac{9}{7}$, $\frac{2}{3}$.
8. $\frac{3}{5}$, $\frac{4}{9}$, $\frac{3}{13}$.
9. $\frac{4}{5}$, $\frac{5}{7}$, $\frac{2}{9}$.
10. $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{11}$.
11. $\frac{2}{13}$, $\frac{1}{12}$, $\frac{1}{7}$.
12. $\frac{5}{11}$, $\frac{1}{15}$, $\frac{2}{3}$.
13. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{7}$.
14. $\frac{12}{27}$, $\frac{10}{19}$.
15. $\frac{103}{257}$, $\frac{53}{125}$.
16. $\frac{1}{131}$, $\frac{1}{147}$.

Perform the following mentally: —

17. $\frac{1}{2}$, $\frac{5}{3}$.
18. $\frac{2}{5}$, $\frac{3}{7}$.
19. $\frac{5}{8}$, $\frac{1}{3}$.
20. $\frac{11}{2}$, $\frac{4}{5}$.
21. $\frac{3}{2}$, $\frac{4}{3}$.
22. $\frac{1}{7}$, $\frac{1}{9}$.
23. $\frac{2}{3}$, $\frac{1}{9}$.
24. $\frac{3}{4}$, $\frac{2}{3}$.

To Reduce Fractions to Equivalent Fractions having the Least Common Denominator.

134. Rule. — *Find the Least Common Multiple of all the denominators for the Least Common Denominator. Then multiply both terms of each fraction by the quotient of the Least Common Multiple divided by the denominator of that fraction.*

Demonstration. — The purpose for which we get the least common multiple is, that we may know what the *least* number is which can be produced by multiplying each denominator by some number. Then we divide this least common multiple by each of the denominators in turn to find by what both terms of each particular fraction must be multiplied, in order to reduce the fraction to one having this least common multiple for its denominator. That the values of the fractions are not changed is evident from (**124**).

N.B. — *This rule presumes all the fractions to be in their lowest terms.*

1. Reduce $\frac{5}{6}$, $\frac{3}{8}$, and $\frac{7}{12}$ to equivalent fractions having the least common denominator.

Solution. — The least common multiple of 6, 8, and 12 is 24 (**107**). Now, to make the denominator of $\frac{5}{6}$ 24, we must multiply it by 4; but, to preserve the value of the fraction, we must multiply the numerator also by 4. Hence we have $\frac{5 \times 4}{6 \times 4} = \frac{20}{24}$.

In like manner, $\frac{3 \times 3}{8 \times 3} = \frac{9}{24}$.

And $\frac{7 \times 2}{12 \times 2} = \frac{14}{24}$.

[The pupil tell why $\frac{5}{6} = \frac{20}{24}$, $\frac{3}{8} = \frac{9}{24}$, and $\frac{7}{12} = \frac{14}{24}$.]

Reduce the following fractions to equivalent sets of fractions having the least common denominators, performing mentally all that you can: —

2. $\frac{5}{8}$, $\frac{2}{6}$.
3. $\frac{7}{15}$, $\frac{9}{10}$, $\frac{4}{5}$.
4. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{8}{9}$, $\frac{3}{10}$.
5. $\frac{4}{15}$, $\frac{1}{30}$, $\frac{5}{6}$.
6. $\frac{2}{24}$, $\frac{5}{32}$, $\frac{7}{16}$.
7. $\frac{3}{5}$, $\frac{7}{10}$.
8. $\frac{1}{10}$, $\frac{1}{100}$.
9. $\frac{4}{5}$, $\frac{11}{3}$, $\frac{1}{12}$, $\frac{9}{16}$.
10. $\frac{3}{5}$, $\frac{1}{3}$, $\frac{7}{9}$, $\frac{8}{11}$.
11. $\frac{1}{6}$, $\frac{5}{12}$.
12. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$.
13. $\frac{3}{16}$, $\frac{13}{20}$, $\frac{25}{60}$.
14. $\frac{1}{30}$, $\frac{7}{40}$, $\frac{19}{100}$.
15. $\frac{3}{20}$, $\frac{1}{4}$, $\frac{11}{32}$, $\frac{5}{8}$.
16. $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$.

SECTION III.

ADDITION AND SUBTRACTION OF FRACTIONS.

To Add or Subtract Fractions.

135. Rule. — *Reduce the fractions to equivalent ones having a common denominator (if they have not such denominators at first); and then add or subtract the numerators as any other numbers, writing the result over the common denominator.*

DEMONSTRATION. — Reducing the fractions to forms having a common denominator does not change the values of the fractions, and hence does not change their sum or difference. When they have like denominators, their numerators represent parts of the same kind, and hence can be added or subtracted.

1. What is the sum of $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{12}$?

SOLUTION. — Reducing these fractions to forms having a common denominator, we have $\frac{2}{3}=\frac{8}{12}$, $\frac{5}{6}=\frac{10}{12}$. Hence, if we find the sum of $\frac{8}{12}$, $\frac{10}{12}$, and $\frac{7}{12}$, it will be the sum of $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{12}$, since the former are equivalent to the latter.

Now, $\frac{8}{12}+\frac{10}{12}+\frac{7}{12}=\frac{25}{12}$: for they represent 8, 10, and 7 things all of the same kind; viz., 12ths. Finally the improper fraction $\frac{25}{12}=2\frac{1}{12}$ by (**129**).

Perform the following additions, reducing the results to mixed or whole numbers when improper fractions arise, and to their lowest terms when they are proper fractions: —

2. $\frac{3}{5}+\frac{2}{7}+\frac{4}{15}$.	SUM, $1\frac{16}{105}$.
3. $\frac{5}{32}+\frac{1}{6}+\frac{1}{24}$.	SUM, $\frac{35}{96}$.
4. $\frac{3}{10}+\frac{2}{11}+\frac{3}{5}+\frac{3}{4}$.	SUM, $1\frac{183}{220}$.
5. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$.	SUM, $2\frac{3}{4}$.
6. $\frac{3}{4}+\frac{5}{6}+\frac{2}{3}+\frac{4}{9}$.	SUM, $2\frac{25}{36}$.

7. $\frac{5}{8}+\frac{3}{5}+\frac{7}{10}+\frac{1}{6}$.	12. $\frac{2}{3}+\frac{1}{7}+\frac{4}{5}$.
8. $\frac{3}{7}+\frac{4}{5}+\frac{9}{20}+\frac{5}{18}$.	13. $\frac{5}{6}+\frac{2}{3}+\frac{1}{2}$.
9. $\frac{3}{2}+\frac{12}{5}+\frac{8}{3}+\frac{11}{6}$.	14. $\frac{12}{5}+\frac{13}{7}+\frac{5}{11}$.
10. $\frac{9}{4}+\frac{17}{3}+\frac{23}{6}+\frac{41}{2}+\frac{3}{4}$.	15. $\frac{14}{7}+\frac{20}{5}+\frac{12}{3}$.
11. $\frac{111}{5}+\frac{42}{15}+\frac{105}{3}$.	16. $\frac{128}{4}+\frac{408}{2}+\frac{40}{5}$.

17. Subtract $\frac{3}{5}$ from $\frac{11}{15}$.

SUGGESTION. $\frac{3}{5}=\frac{9}{15}$. Hence $\frac{11}{15}-\frac{3}{5}=\frac{11}{15}-\frac{9}{15}=\frac{2}{15}$.

Perform the following subtractions. Most of them should be done without writing.

18. $\frac{5}{6}-\frac{1}{3}$.	24. $\frac{4}{7}-\frac{2}{5}$.	30. $\frac{7}{10}-\frac{4}{9}$.
19. $\frac{7}{8}-\frac{1}{2}$.	25. $\frac{11}{12}-\frac{2}{3}$.	31. $\frac{12}{13}-\frac{9}{10}$.
20. $\frac{4}{5}-\frac{2}{3}$.	26. $\frac{13}{127}-\frac{11}{427}$.	32. $\frac{15}{4}-\frac{3}{4}$.
21. $\frac{11}{12}-\frac{4}{7}$.	27. $\frac{43}{17}-\frac{2}{112}$.	33. $\frac{17}{3}-\frac{51}{9}$.
22. $\frac{11}{3}-\frac{2}{5}$.	28. $\frac{57}{80}-\frac{41}{207}$.	34. $\frac{222}{4}-\frac{1}{2}$.
23. $\frac{14}{9}-\frac{1}{2}$.	29. $\frac{17}{111}-\frac{2}{17}$.	35. $\frac{1145}{137}-\frac{27}{115}$.

Perform the following mentally:—

36. $\frac{1}{2}+\frac{2}{3}$.	40. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$.	44. $\frac{1}{5}+\frac{1}{7}$.
37. $\frac{5}{6}-\frac{1}{2}$.	41. $\frac{11}{12}-\frac{5}{6}$.	45. $\frac{1}{5}-\frac{1}{7}$.
38. $\frac{1}{5}+\frac{1}{4}$.	42. $\frac{43}{15}-\frac{2}{3}$.	46. $\frac{11}{4}-\frac{6}{8}$.
39. $\frac{3}{4}-\frac{1}{2}$.	43. $\frac{2}{3}+\frac{3}{2}+\frac{5}{6}$.	47. $\frac{13}{6}+\frac{91}{42}$.

To Add Mixed Numbers or Whole Numbers and Fractions.

136. Rule.—*Add the fractions first, reducing the result, if an improper fraction, to a mixed number; and, writing the fraction thus arising, prefix to it the sum of the integers and the integer (if any) arising from adding the fractions.*

Mixed numbers may be reduced to improper fractions, and then added; but this is not a desirable method.

1. Add $4\frac{2}{3}$ and $5\frac{2}{3}$.

SUGGESTION. — Adding the fractions, we have $\frac{2}{3}+\frac{2}{3}=\frac{4}{3}=1\frac{1}{3}$. Writing the $\frac{1}{3}$ as a fraction, and adding the integer 1 to the integers 4 and 5, we have $10\frac{1}{3}$.

$$\begin{array}{r} 4\frac{2}{3} \\ 5\frac{2}{3} \\ \hline 10\frac{1}{3} \end{array}$$

2. Add $5\frac{1}{2}$, $11\frac{3}{4}$, and $24\frac{1}{6}$.

What is the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{1}{6}$?

3. Add $214\frac{1}{3}$, $517\frac{7}{48}$, and $145\frac{5}{12}$.

4. Add $3\frac{2}{3}$, $17\frac{3}{4}$, $28\frac{5}{12}$. Also $4\frac{1}{5}$, $1\frac{2}{3}$, $2\frac{3}{10}$, $\frac{3}{5}$.

5. Add $3\frac{5}{6}$, $17\frac{2}{7}$, and $4\frac{3}{21}$. Also $3\frac{1}{7}$, $2\frac{1}{3}$.

6. Add 126, $13\frac{2}{3}$, $40\frac{5}{6}$, and $7\frac{1}{2}$. Also $5\frac{2}{7}$ and $\frac{3}{5}$.

7. Add 35, $126\frac{2}{3}$, $15\frac{5}{6}$, $58\frac{4}{5}$, $9\frac{3}{10}$, $861\frac{3}{5}$.

The annexed is a convenient form for such operations. Having written the numbers in the ordinary way for addition, write the least common denominator to which all are to be reduced (30 in this case) at the right, and the new numerators in order for addition. In this case we see that the sum of the fractions is $\frac{96}{30}$, or $3\frac{1}{5}$, etc.

$$\begin{array}{rrl} 35 & & \\ 126\frac{2}{3}, & 20 & \\ 15\frac{5}{6}, & 25 & \\ 48\frac{4}{5}, & 24 & \text{30ths.} \\ 9\frac{3}{10}, & 9 & \\ 861\frac{3}{5}, & 18 & \\ \hline 1097\frac{1}{5} & \frac{96}{30} & =3\frac{1}{5}. \end{array}$$

To Perform Subtraction when both Whole Numbers and Fractions are involved.

137. Rule. — *First take the fraction in the subtrahend from that in the minuend, or, if the latter fraction be the smaller, from it increased by 1; and then subtract the remaining integers, prefixing the latter remainder to the former.*

8. From $256\frac{5}{6}$ take $117\frac{1}{3}$.

SUGGESTION. — Since $\frac{1}{3}=\frac{2}{6}$, we have $256\frac{5}{6}-117\frac{2}{6}$. $\frac{2}{6}$ from $\frac{5}{6}$ leaves $\frac{3}{6}$ or $\frac{1}{2}$; and 117 from 256 leaves 139. Hence $256\frac{5}{6}-117\frac{2}{6}=139\frac{1}{2}$.

$$\begin{array}{r} 256\frac{5}{6} \\ 117\frac{2}{6} \\ \hline 139\frac{1}{2} \end{array}$$

9. From $34\frac{3}{4}$ take $16\frac{1}{2}$.

10. From $83\frac{1}{12}$ take $27\frac{2}{3}$.

11. From $157\frac{2}{5}$ take $68\frac{5}{7}$.

SUGGESTION. — As $\frac{14}{35}$ is less than $\frac{25}{35}$, we take $1=\frac{35}{35}$ from the 7, which with the $\frac{14}{35}$ makes $\frac{49}{35}$. Then taking $\frac{25}{35}$ from $\frac{49}{35}$, and 68 from 156, there remains $88\frac{24}{35}$.

$$\begin{array}{rr} 157\frac{2}{5}= & 157\frac{14}{35} \\ 68\frac{5}{7}= & 68\frac{25}{35} \\ \hline & 88\frac{24}{35} \end{array}$$

12. From $7\frac{2}{3}$ take $3\frac{3}{4}$. From $1\frac{3}{5}$ take $\frac{8}{9}$.

13. From 13 take $9\frac{2}{3}$.

SUGGESTION. — As there is no fraction in the minuend, we take one of the 3 units from which to subtract the $\frac{2}{3}$ in the subtrahend. $\frac{2}{3}$ from 1 ($\frac{3}{3}$) leaves $\frac{1}{3}$, and 9 from 12 leaves 3. Hence the remainder is $3\frac{1}{3}$.

$$\begin{array}{r} 13 \\ 9\frac{2}{3} \\ \hline 3\frac{1}{3} \end{array}$$

14. From 286 take $156\frac{7}{11}$. From 5 take $\frac{3}{7}$.

15. From 11 take $\frac{3}{5}$. From 11 take $3\frac{2}{7}$.

16. From $6\frac{3}{5}$ take 4. From 1 take $\frac{4}{11}$.

17. From $81\frac{5}{7}$ take 43. From $10\frac{2}{5}$ take $\frac{5}{6}$.

Examples for Practice.

1. $12\frac{3}{5} + 28\frac{5}{6} + 15\frac{2}{3}$?
2. $45\frac{2}{9} + 56\frac{3}{4} + 38\frac{1}{6}$?
3. $15 + \frac{2}{7} + 4\frac{5}{14}$?
4. $\frac{3}{11} + 18\frac{3}{4} + \frac{71}{6}$?
5. $346\frac{2}{3} + 48\frac{5}{123}$?
6. $\frac{5}{2} + \frac{13}{6} + \frac{128}{4}$?
7. $7\frac{1}{2} + 8\frac{2}{5} + 4\frac{2}{3}$?
8. $4\frac{5}{8} + 1\frac{5}{9} + 2\frac{3}{4}$?
9. $7\frac{1}{3} + 9\frac{1}{6} + 8\frac{3}{5}$?
10. $6\frac{4}{5} + 8\frac{8}{9} + \frac{10}{15}$?
11. $6\frac{1}{2} + 12\frac{7}{22} + 9\frac{3}{11}$?
12. $\frac{6}{14} + 5\frac{1}{2} + 4\frac{11}{13}$?
13. $24\frac{1}{2} - 8\frac{2}{5}$?
14. $42\frac{7}{8} - 6\frac{3}{5}$?
15. $54\frac{1}{4} - 21\frac{2}{7}$?
16. $55\frac{1}{2} - 38\frac{1}{6}$?
17. $36\frac{2}{7} - 18\frac{2}{5}$?
18. $39\frac{2}{3} - 33\frac{1}{8}$?
19. $5\frac{3}{8} + 2\frac{1}{8} - 3\frac{1}{4}$?
20. $4\frac{3}{4} - 2\frac{1}{5} + 1\frac{1}{3}$?
21. $14 + 6\frac{1}{2} - 9\frac{3}{4}$?
22. $2\frac{3}{4} - 5\frac{1}{2} + 3\frac{7}{8}$?
23. $6\frac{3}{4} + 3\frac{1}{5} - 6\frac{1}{10}$?
24. $7\frac{1}{2} - 2\frac{1}{5} - \frac{1}{2}$?
25. $\frac{1}{5} + \frac{3}{7} + \frac{5}{8} - \frac{6}{7}$?
26. $\frac{2}{7} + \frac{4}{14} + \frac{3}{8} + \frac{5}{9}$?
27. $\frac{4}{3} + \frac{6}{7} - \frac{3}{8} + \frac{6}{7}$?
28. $4\frac{3}{7} + \frac{6}{7} - \frac{2}{9} + 6\frac{3}{8}$?
29. $4\frac{3}{7} + \frac{6}{7} - (\frac{3}{7} + \frac{6}{7})$?
30. $24 - \frac{3}{7} + \frac{5}{9} - (\frac{3}{7} + 9)$?
31. $\frac{6}{7} + 113 + \frac{3}{14} - (6 + 9)$?
32. $\frac{5}{7} + 12 - 11 + 3 - 2$?
33. $\frac{4}{7} + 13 - (6\frac{7}{8} - \frac{3}{8}) + \frac{4}{3}$?
34. $8\frac{3}{7} + 11 - (3 - 2) + \frac{5}{7}$?
35. $(14 + \frac{3}{9} + \frac{6}{7} - \frac{3}{4}) - 4\frac{3}{7}$?
36. $\frac{5}{8} + (16\frac{3}{7} - \frac{6}{8}) - \frac{3}{7} + \frac{4}{6}$?
37. $\frac{3}{7} + (\frac{5}{8} - \frac{2}{7}) + (\frac{3}{4} + \frac{6}{7})$?
38. $5\frac{2}{9} + \frac{3}{4} - \frac{6}{7} + (\frac{3}{8} + 13)$?
39. $6 \times 5 + \frac{3}{4} - 4\frac{8}{7}$?
40. $\frac{3}{4} + \frac{5}{6} + \frac{5}{8} - \frac{3}{7}$?
41. $\frac{3}{5} + \frac{6}{7} + \frac{3}{9} + \frac{5}{9} + \frac{3}{7}$?
42. $\frac{4}{7} + \frac{1}{3} + \frac{5}{7} - \frac{6}{7} + \frac{1}{11}$?
43. $\frac{3}{8} + 6\frac{4}{7} - \frac{3}{8} + \frac{6}{7} + \frac{5}{14}$?
44. $4 + \frac{3}{8} + 6\frac{3}{4} - (\frac{3}{4} + \frac{6}{7})$?

45. $\frac{6}{7} + 14 - (12\frac{2}{3} - \frac{3}{4})$?	50. $\frac{5}{8} + \frac{3}{7} - \frac{2}{3} - \frac{3}{7} + \frac{4}{7}$?
46. $(\frac{4}{7} + \frac{3}{8}) + \frac{3}{7} + (\frac{6}{7} + 9)$?	51. $(13 - 1\frac{7}{8}) + \frac{6}{7} + \frac{3}{7}$?
47. $5\frac{3}{7} + \frac{4}{7} - \frac{3}{8} + (\frac{4}{7} - \frac{6}{7})$?	52. $(11 + \frac{3}{4}) + \frac{5}{9} - \frac{3}{8}$?
48. $\frac{4}{7} + 16 - \frac{3}{11} + (\frac{3}{7} - \frac{2}{14})$?	53. $5 + \frac{3}{7} + \frac{3}{8} - \frac{2}{3}$?
49. $(\frac{2}{9} + 12\frac{1}{2}) - \frac{3}{7} + \frac{5}{8}$?	54. $(8 \div 11) + (3 \div 9)$?

SECTION IV.

MULTIPLICATION OF FRACTIONS.

To Multiply a Fraction by an Integer.

138. Rule. — *Multiply the numerator, or divide the denominator, by the integer.*

This is the same as Principles I. and II. (**120, 121**). If necessary, review those, and solve the following, giving the explanations as in those articles: —

1. $\frac{5}{7} \times 3 =$ what?	5. $\frac{121}{347} \times 264.$	9. $\frac{7}{16} \times 4.$
2. $\frac{3}{8} \times 2 =$ what?	6. $\frac{71}{189} \times 81.$	10. $\frac{28}{125} \times 25.$
3. $\frac{11}{13} \times 7 =$ what?	7. $\frac{134}{182} \times 91.$	11. $\frac{43}{17} \times 12.$
4. $\frac{5}{23} \times 12 =$ what?	8. $\frac{251}{1000} \times 500.$	12. $\frac{1412}{3471} \times 286.$

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13. Multiply $\frac{3}{16}$ by 40, using the factors of 40; viz., 8 and 5.

SUGGESTION. $\frac{3}{16} \times 8 = \frac{3}{2}$, and $\frac{3}{2} \times 5 = \frac{15}{2} = 7\frac{1}{2}$. Hence $\frac{3}{16} \times 40 = 7\frac{1}{2}$. What principles are used? What in multiplication? What *two* in fractions?

Solve as above, and explain the following: —

14. $\frac{9}{56} \times 24.$	17. $\frac{12}{25} \times 15.$	20. $\frac{124}{343} \times 147.$
15. $\frac{11}{81} \times 27.$	18. $\frac{8}{9} \times 6.$	21. $\frac{51}{144} \times 60.$
16. $\frac{37}{125} \times 50.$	19. $\frac{4}{15} \times 21.$	22. $\frac{31}{121} \times \frac{7}{11}.$

[Let the pupils *write* a rule for this case; that is, when the multiplier contains a factor which is also a factor of the denominator of the multiplicand.]

23. Multiply $12\frac{3}{7}$ by 9.

SUGGESTION. 9 times $\frac{3}{7}$ are $\frac{27}{7}$, which is 3 and $\frac{6}{7}$. 9 times 12 are 108; to which adding the 3, we find that 9 times $12\frac{3}{7}$ are $111\frac{6}{7}$.

$$\begin{array}{r} 12\frac{3}{7} \\ 9 \\ \hline 111\frac{6}{7} \end{array}$$

Solve, as above, the following, performing such as you can mentally: —

24. $23\frac{2}{3} \times 11.$	28. $3472\frac{3}{4} \times 7.$	32. $482\frac{2}{7} \times 126.$
25. $137\frac{1}{43} \times 12.$	29. $126\frac{1}{4}\frac{2}{1}\frac{3}{1} \times 23.$	33. $1\frac{2}{5} \times 10.$
26. $2\frac{1}{2} \times 5.$	30. $4\frac{3}{7} \times 8.$	34. $10\frac{1}{2} \times 6.$
27. $5\frac{1}{3} \times 6.$	31. $5\frac{3}{8} \times 4.$	35. $16\frac{1}{2} \times 8.$

[Let the pupils *write* the rule, and state the case.]

36. Multiply $\frac{3}{5}$ by 5.

SUGGESTION. — If we divide the denominator by 5, according to the rule, we have $\frac{3}{1}$, or 3. Or, if we multiply the numerator, we have $\frac{15}{5}$, or 3. Hence we see, that,

139. *To multiply a fraction by a number equal to its denominator, we simply drop the denominator.*

37. What is $\frac{5}{7} \times 7$? $\frac{2}{3} \times 3$? $\frac{12}{13} \times 13$? $4\frac{1}{2} \times 2$?

To Multiply by a Fraction.

140. Rule. — *To multiply by a fraction, divide the multiplicand by the denominator of the multiplier, and multiply the result by the numerator.*

DEMONSTRATION. — Since to multiply by any number is to take the multiplicand as many times as is indicated by the multiplier, to multiply by $\frac{3}{5}$ is to take $\frac{3}{5}$ of the multiplicand; i.e., to take $\frac{1}{5}$ 3 times. Hence we divide by the denominator to get the part of the multiplicand to be repeated, and then multiply by the numerator to repeat this part the required number of times.

Ex. 1. — Multiply 12 by $\frac{5}{6}$.

SOLUTION. — To multiply 12 by $\frac{5}{6}$ is to take $\frac{5}{6}$ of 12. Now, $\frac{1}{6}$ of 12 is 2: hence $\frac{5}{6}$ is 5 times 2, or 10.

2. Multiply $\frac{2}{3}$ by $\frac{5}{7}$.

SOLUTION. — To multiply $\frac{2}{3}$ by $\frac{5}{7}$ is to take $\frac{5}{7}$ of $\frac{2}{3}$. Now, $\frac{1}{7}$ of $\frac{2}{3}$ is $\frac{2}{21}$:[1] hence $\frac{5}{7}$ is 5 times $\frac{2}{21}$, or $\frac{10}{21}$.

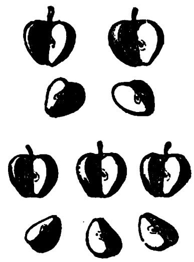
How is a fraction divided by 7? (See **122.**)

How is a fraction multiplied by 5? (See **120.**)

3. Multiply 17 by $\frac{5}{6}$.

SOLUTION. — To multiply 17 by $\frac{5}{6}$ we would divide by 6, and multiply the result by 5, but, as the order of operations is immaterial, we may multiply *first*, and then divide. 5 times 17 is 85, and $\frac{1}{6}$ of 85 is $14\frac{1}{6}$.

This method may also be analyzed on the principle that $\frac{5}{6}$ of a number is the same as $\frac{1}{6}$ of 5 times that number.



Show from the annexed cut that $\frac{2}{3}$ of 1 is the same as $\frac{1}{3}$ of (each of) 2.

Show also that $\frac{3}{4}$ of 1 is the same as $\frac{1}{4}$ of 3.

If you divide each of two lines into 4 equal parts, and take 3 parts from each, how many parts have you? If you divide each of 6 lines into 4 equal parts, and take 1 part from each, how many have you? Show, by dividing lines in this way, that $\frac{3}{4}$ of 2 is the same as $\frac{1}{4}$ of 6.

Mental Exercises.

Solve the following mentally, performing the operations by *division* as far as practicable. Give the explanations as above in each case.

4. $12 \times \frac{3}{4}$.	PROD., 9.	10. $\frac{5}{8} \times \frac{2}{3}$.	16. $\frac{3}{7} \times \frac{4}{5}$.
5. $13 \times \frac{5}{6}$.	PROD., $10\frac{5}{6}$.	11. $\frac{4}{9} \times \frac{3}{5}$.	17. $\frac{8}{11} \times \frac{3}{2}$.
6. $\frac{12}{35} \times \frac{5}{6}$.	$\frac{4}{9} \times \frac{3}{2}$.	12. $\frac{14}{5} \times \frac{4}{7}$.	18. $\frac{5}{13} \times \frac{4}{3}$.
7. $\frac{11}{13} \times \frac{5}{6}$.	$\frac{12}{25} \times \frac{5}{6}$.	13. $\frac{35}{12} \times \frac{6}{7}$.	19. $\frac{4}{7} \times \frac{1}{2}$.
8. $28 \times \frac{7}{4}$.	$16 \times \frac{3}{4}$.	14. $15 \times \frac{3}{5}$.	20. $20 \times \frac{1}{4}$.
9. $17 \times \frac{2}{3}$.	$27 \times \frac{5}{9}$.	15. $31 \times \frac{2}{7}$.	21. $11 \times \frac{1}{3}$.

[1] Read "2-twenty-firsts," etc.

141. The rule for multiplying one fraction by another is sometimes given thus: ***Multiply the numerators together for the numerator of the product, and the denominators together for the denominator of the product.***

Can you give the reasons for this rule? They are just the same as for the rule (**140**). How does multiplying the numerator of the multiplicand affect the multiplicand? How does multiplying the denominator of the multiplicand affect the multiplicand?

22. Multiply $\frac{5}{7}$ by $\frac{3}{4}$ by this rule, and give the explanation.

OPERATION.

$\frac{5}{7} \times \frac{3}{4} = \frac{15}{28}.$

REASONS. — Multiplying the denominator of $\frac{5}{7}$ by 4 divides $\frac{5}{7}$ by 4 (**123**), and hence gets $\frac{1}{4}$ of it, which is the part to be repeated. Then multiplying the numerator by 3 repeats this part the required number of times (**120**).

Perform and explain the following: —

23. $\frac{8}{13} \times \frac{3}{10}$.	27. $300 \times \frac{5}{3}$.	31. $1000 \times \frac{3}{500}$.
24. $\frac{10}{11} \times \frac{3}{20}$.	28. $72 \times \frac{2}{9}$.	32. $\frac{17}{43} \times \frac{11}{57}$.
25. $\frac{5}{7} \times \frac{4}{5}$.	29. $53 \times \frac{127}{43}$.	33. $\frac{121}{19} \times \frac{7}{183}$.
26. $184 \times \frac{2}{3}$.	30. $256 \times \frac{342}{576}$.	34. $\frac{427}{1025} \times \frac{376}{41}$.

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142. *By Cancellation.*

Ex. 1. — Multiply $\frac{10}{21}$ by $\frac{28}{15}$.

SOLUTION. $\frac{10}{21} \times \frac{28}{15} = \frac{10 \times 28}{21 \times 15}$. Now, we can see a factor 5 in the 10 in the numerator, and also in the 15 in the denominator. Rejecting this factor from the numerator divides it by 5, and rejecting it also from the denominator divides *it* by 5. Hence, if we reject it from both, we shall not alter the value of the fraction (**124**). Thus we have $\frac{10 \times 28}{21 \times 15} = \frac{2 \times 28}{21 \times 3}$. Again: we see a factor 7 in the 28 in the numerator, and also a factor 7 in the 21 in the denominator, which can be rejected for like reason with the 5. (What is the reason?) Thus we have $\frac{2 \times 28}{21 \times 3} = \frac{2 \times 4}{3 \times 3} = \frac{8}{9}$.

2. Multiply together the three fractions $\frac{3}{4}$, $\frac{12}{35}$, and $\frac{7}{24}$.

OPERATION.

$$\frac{3}{4}\times\frac{12}{35}\times\frac{7}{24}=\frac{3\times\cancel{12}\times\cancel{7}}{4\times\underset{5}{\cancel{35}}\times\underset{2}{\cancel{24}}}=\frac{3}{40}.$$

EXPLANATION. — Observing the factor 12 in the numerator, and a like factor in the 24 in the denominator, we cancel it, leaving a factor 2 in place of 24 in the denominator. This does not change the value of the result. (Why?) In like manner we cancel the 7 in the numerator, and the same factor in 35 in the denominator. (Why?) As there are now no common factors, we perform the indicated multiplications, and have $\frac{3}{40}$ as the product.

3. Perform by cancellation $\frac{10}{11}\times\frac{3}{20}\times\frac{1}{6}\times\frac{5}{7}\times\frac{21}{5}$.

OPERATION.

$$\frac{10}{11}\times\frac{3}{20}\times\frac{1}{6}\times\frac{5}{7}\times\frac{21}{5}=\frac{\cancel{10}\times\cancel{3}\times1\times\cancel{5}\times\overset{3}{\cancel{21}}}{11\times\underset{2}{\cancel{20}}\times\underset{2}{\cancel{6}}\times\cancel{7}\times\cancel{5}}=\frac{3}{44}.$$

Pupil trace the process, and give the reasons.

4. What is $\frac{3}{5}$ of $\frac{4}{7}$?

Observe that to multiply by $\frac{3}{5}$ is to take $\frac{3}{5}$ of a number, or to take $\frac{1}{5}$ 3 times. Hence $\frac{3}{5}$ of $\frac{4}{7}$ is $\frac{4}{7}\times\frac{3}{5}$; or, since in multiplying it is immaterial which of the factors we use as the multiplier, $\frac{3}{5}$ of $\frac{4}{7}$ is the same as $\frac{3}{5}\times\frac{4}{7}$.

143. *Fractions connected by the word "of" are sometimes called* **Compound Fractions.** This "of" is simply equivalent to the sign $\times$.

Perform the following by cancellation: —

5. $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{3}{5}$ of $\frac{8}{9}=\frac{8}{27}$.
6. $\frac{3}{4}\times\frac{2}{5}\times\frac{15}{18}\times\frac{8}{21}\times\frac{9}{5}$.
7. $8\times\frac{13}{5}\times\frac{5}{39}\times\frac{3}{10}\times\frac{8}{3}$.
8. $15\times\frac{2}{5}\times\frac{5}{3}\times\frac{3}{2}\times\frac{7}{10}$.
9. $\frac{3}{5}$ of $\frac{35}{27}$ of $\frac{6}{21}$ of $\frac{14}{27}$.
10. $\frac{1}{2}$ of $\frac{1}{3}$ of 12.
11. $\frac{1}{3}$ of $\frac{1}{4}$; $\frac{1}{5}$ of $\frac{1}{6}$; $\frac{1}{2}$ of $\frac{1}{7}$.
12. $30\times\frac{2}{5}\times\frac{1}{6}\times\frac{1}{12}$.

Perform the following, first reducing the mixed numbers to improper fractions: —

13. $5\frac{3}{8}\times4\frac{3}{4}=25\frac{17}{32}$.
14. $8\times2\frac{3}{5}\times\frac{5}{39}\times\frac{3}{10}\times2\frac{2}{3}=$
15. $8\frac{2}{3}\times2\frac{3}{13}\times3\frac{3}{4}\times16\frac{2}{3}=$
16. $4\frac{2}{7}\times5\frac{3}{5}=$
17. $2\frac{3}{5}\times3\frac{3}{4}=$
18. $7\frac{1}{2}\times4\frac{1}{3}$.

Two mixed numbers may be multiplied as in the margin. See if you can trace the process.

$$\begin{array}{r} 4\frac{2}{7} \\ 5\frac{3}{5} \\ \hline 20 + \frac{10}{7} + \frac{12}{5} + \frac{6}{35} = 24 \end{array}$$

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19. Multiply $\frac{3}{35}$ by $\frac{7}{3}$ by cancellation.

SUGGESTION. — We have $\frac{\cancel{3} \times \cancel{7}}{\underset{5}{\cancel{35}} \times \cancel{3}}$. Now, as both the 3 and 7 are cancelled in the numerator, what remains? Is the numerator of the product 0? No. If we remember that cancellation is only *dividing* the terms of the fraction by the same number, we shall see that really there is a factor 1 to be *understood* when we cancel 7, and also when we cancel 3; so that the result would be $\frac{1 \times 1}{5 \times 1} = \frac{1}{5}$.

20–23. Perform the following: $\frac{2}{3} \times \frac{15}{16} \times \frac{3}{5} \times \frac{4}{9}$; $\frac{3}{7} \times \frac{21}{32} \times \frac{8}{9} \times 4$; $\frac{1}{5} \times \frac{1}{7} \times 35$; $\frac{12}{7} \times \frac{63}{5} \times 10 \times \frac{1}{2}$.

SECTION V.

DIVISION OF FRACTIONS.

To Divide a Fraction by an Integer.

144. Rule. — *Divide the numerator, or multiply the denominator, by the integer.*

This is the same as Principles III. and IV. (**122, 123**). If necessary, review those, and solve the following, giving the explanations as under those articles: —

1. $\frac{2}{7} \div 5 =$ what?	6. $\frac{128}{137} \div 7$.	11. $\frac{50}{7} \div 10$.
2. $\frac{11}{3} \div 4 =$ what?	7. $\frac{33}{73} \div 7$.	12. $\frac{39}{87} \div 13$.
3. $\frac{5}{6} \div 3 =$ what?	8. $\frac{343}{782} \div 151$.	13. $\frac{143}{782} \div 111$.
4. $\frac{4}{13} \div 2 =$ what?	9. $\frac{38}{163} \div 19$.	14. $\frac{279}{456} \div 93$.
5. $\frac{15}{13} \div 5 =$ what?	10. $\frac{1}{2} \div 5$.	15. $\frac{171}{500} \div 20$.

To Divide by a Fraction.

145. Principle. — *A* **Fraction** *inverted shows how many times that fraction is contained in 1.*

Ex. 1. — How many times is $\frac{2}{6}$ contained in 1?

Solution. — Since there are 6 sixths in 1, $\frac{1}{6}$ is contained 6 times in 1. And, as $\frac{2}{6}$ is 2 times $\frac{1}{6}$, $\frac{2}{6}$ is contained in 1 only $\frac{1}{2}$ as many times as $\frac{1}{6}$ is. Hence $\frac{2}{6}$ is contained $\frac{1}{2}$ of 6, or 3 times in 1.[1]

146. The **Reciprocal** of a number is 1 divided by that number: hence the *Reciprocal of a fraction* is the fraction inverted.

2–10. Find how many times each of the following is contained in 1, and give the solution in each case: $\frac{2}{9}$; $\frac{5}{7}$; $\frac{4}{13}$; $\frac{5}{4}$; $\frac{7}{2}$; $\frac{3}{2}$; $\frac{8}{3}$; $\frac{11}{13}$; $\frac{124}{237}$.

147. Rule. — *To divide by a fraction, invert the divisor, and multiply the result by the dividend.*

Demonstration. — The divisor inverted shows how many times the divisor is contained in 1. Then in 2 it will be contained 2 times as many times as in 1; in 3, 3 times as many times; in 4, 4 times as many times; in $\frac{2}{3}$, $\frac{2}{3}$ as many times; in $\frac{5}{7}$, $\frac{5}{7}$ as many times, etc.

Ex. 1. Divide 11 by $\frac{3}{7}$. | 2. $11 \div \frac{2}{3}$. | 3. $7 \div \frac{5}{6}$.

Solution. $\frac{3}{7}$ is contained $\frac{7}{3}$ times in 1, and in 11 it is contained 11 times as many times as in 1. Hence $11 \div \frac{3}{7} = 11$ times $\frac{7}{3} = \frac{77}{3} = 25\frac{2}{3}$.

4. Divide $\frac{5}{2}$ by $\frac{8}{11}$. | 5. $\frac{3}{7} \div \frac{5}{8}$. | 6. $\frac{4}{11} \div \frac{3}{5}$. | 7. $\frac{1}{2} \div \frac{3}{8}$.

Solution. $\frac{8}{11}$ is contained $\frac{11}{8}$ times in 1, and in $\frac{5}{2}$ it is contained $\frac{5}{2}$ times as many times as in 1. Hence $\frac{5}{2} \div \frac{8}{11} = \frac{5}{2}$ times $\frac{11}{8} = \frac{55}{16} = 3\frac{7}{16}$.

Perform the following, giving the solution in full: —

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[1] This is but dividing successively by the factors of the divisor according to (84). Thus, to divide 1 by $\frac{2}{3}$, we divide successively by $\frac{1}{3}$ and 2. $1 \div \frac{1}{3} = 3$, and $3 \div 2 = \frac{3}{2}$.

8. $8 \div \frac{2}{9} = 36.$	16. $12 \div \frac{5}{11}.$	24. $5 \div \frac{2}{3}.$
9. $\frac{4}{7} \div \frac{3}{11} = 2\frac{2}{21}.$	17. $\frac{15}{47} \div \frac{11}{13}.$	25. $\frac{1}{2} \div \frac{1}{3}.$
10. $\frac{5}{8} \div \frac{4}{7} =$	18. $\frac{2}{9} \div \frac{7}{5}.$	26. $\frac{1}{3} \div \frac{1}{5}.$
11. $11 \div \frac{2}{3} =$	19. $127 \div \frac{5}{19}.$	27. $\frac{1}{7} \div \frac{1}{8}.$
12. $17 \div \frac{8}{3} =$	20. $43 \div \frac{1}{2}.$	28. $\frac{6}{111} \div \frac{1}{41}.$
13. $\frac{11}{13} \div \frac{5}{9} =$	21. $71 \div \frac{4}{3}.$	29. $10 \div \frac{2}{5}.$
14. $\frac{7}{2} \div \frac{3}{13} =$	22. $\frac{8}{15} \div \frac{6}{21}.$	30. $100 \div \frac{1}{10}.$
15. $\frac{3}{2} \div \frac{5}{19} =$	23. $\frac{1}{10} \div \frac{3}{7}.$	31. $\frac{7}{103} \div \frac{302}{9}.$

32. Divide $4\frac{2}{3}$ by $\frac{7}{9}$.

SUGGESTION. $4\frac{2}{3} = \frac{14}{3}$. Now, $\frac{14}{3} \div \frac{7}{9} = \frac{\overset{2}{\cancel{14}}}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{7}} = 6.$

148. *When mixed numbers occur in dividend or divisor, reduce them to improper fractions, and then proceed as usual.*

Perform the following, reducing the mixed numbers to improper fractions, and then, putting the operation in the form of a problem in multiplication, cancel as much as possible: —

33. $10\frac{2}{3} \div 3\frac{1}{4}.$	37. $42\frac{3}{5} \div 4\frac{11}{15}.$	41. $25\frac{6}{7} \div 2\frac{1}{7}.$
34. $15\frac{2}{3} \div 6\frac{1}{2}.$	38. $\frac{2}{111} \div 4\frac{1}{7}.$	42. $122 \div 7\frac{5}{8}.$
35. $1\frac{3}{21} \div \frac{8}{15}.$	39. $1\frac{1}{5} \div 1\frac{2}{3}.$	43. $\frac{16}{9} \div \frac{4}{9}.$
36. $11\frac{3}{7} \div 7\frac{13}{21}.$	40. $121\frac{1}{2} \div 23\frac{1}{3}.$	44. $\frac{25}{3} \div \frac{5}{3}.$

Complex Fractions.

149. An expression in the form of a fraction having a fraction in its numerator or denominator, or in both, is called a **Complex Fraction**; as $\frac{\frac{2}{3}}{4}$, $\frac{\frac{5}{6}}{\frac{3}{7}}$, $\frac{4\frac{1}{2}}{3}$, etc.

150. A fraction with a whole number for a numerator and a whole number for a denominator is called a **Simple Fraction**; as $\frac{3}{8}$, $\frac{4}{5}$, $\frac{11}{3}$, etc.

To Reduce a Complex Fraction to a Simple Fraction.

151. Rule. — *Multiply both terms of the complex fraction by the least common multiple of all the denominators of the partial fractions.*

Demonstration. — This does not alter the value of the complex fraction, according to (**124**). It destroys the denominators of the partial fractions, because, to multiply a fraction by a number equal to its denominator, we simply drop the denominator (**139**); and, as the multiplier contains each denominator of a partial fraction as a factor, all the denominators will disappear in the process.

Ex. 1. — Reduce $\frac{\frac{2}{9}}{\frac{5}{12}}$ to a simple fraction.

Solution. — The least common multiple of 9 and 12 is 36. Hence we have $\frac{\frac{2}{9} \times 36}{\frac{5}{12} \times 36}$, or $\frac{\frac{2}{\cancel{9}} \times 4 \times \cancel{9}}{\frac{5}{\cancel{12}} \times \cancel{12} \times 3} = \frac{8}{15}$.

Reduce the following *Complex* fractions to simple fractions, whole or mixed numbers : —

2. $\frac{\frac{3}{4}}{\frac{1}{8}} = 6$.	8. $\frac{\frac{2}{11}}{5} = \frac{2}{55}$.	14. $\frac{121\frac{1}{2}}{327\frac{3}{4}}$.
3. $\frac{2}{\frac{5}{6}} = 2\frac{2}{5}$.	9. $\frac{10\frac{1}{2}}{5\frac{1}{4}}$.	15. $\frac{\frac{2}{3}}{111\frac{1}{2}}$.
4. $\frac{\frac{5}{7}}{\frac{2}{3}} =$	10. $\frac{\frac{3}{10}}{\frac{9}{5}}$.	16. $\frac{10\frac{1}{2}}{\frac{5}{6}}$.
5. $\frac{\frac{17}{45}}{\frac{34}{15}} =$	11. $\frac{5\frac{1}{4}}{7\frac{1}{3}}$.	17. $\frac{1114\frac{2}{3}}{56\frac{1}{2}}$.
6. $\frac{2\frac{5}{8}}{3\frac{1}{2}}$[1] $=$	12. $\frac{2}{3\frac{5}{7}}$.	18. $\frac{49\frac{1}{2}}{12\frac{3}{8}}$.
7. $\frac{2\frac{2}{3}}{5\frac{1}{7}} =$	13. $\frac{\frac{2}{7}}{7\frac{6}{7}}$.	19. $\frac{141\frac{2}{7}}{58\frac{1}{5}}$.

[1] It is not necessary nor best in such cases to reduce the mixed numbers to improper fractions. In this case multiply both terms by 8; thus, $\frac{2\frac{5}{8} \times 8}{3\frac{1}{2} \times 8} = \frac{21}{28} = \frac{3}{4}$.

Miscellaneous Exercises.

1. $\dfrac{\frac{7}{8} \text{ of } \frac{4}{7}}{\frac{4}{7} \text{ of } \frac{8}{14}} =$

2. $\dfrac{\frac{3}{5} \text{ of } 2\frac{1}{6}}{1\frac{5}{7}} =$

3. $\dfrac{\frac{7}{8} + \frac{4}{7}}{\frac{5}{7} + \frac{1}{6}} =$

4. $\dfrac{\frac{8}{9} - \frac{3}{10}}{\frac{5}{7} - \frac{2}{3}} =$

5. $\dfrac{\frac{1}{3}}{\frac{1}{6}} \times \dfrac{\frac{9}{16}}{\frac{5}{60}} =$

6. $\dfrac{\frac{1}{3}}{\frac{3}{7}} + \dfrac{4\frac{1}{5}}{12\frac{1}{2}}.$

7. $\dfrac{7\frac{3}{4}}{\frac{1}{5}} - \dfrac{7\frac{1}{2}}{\frac{5}{12}}.$

8. $\dfrac{4\frac{2}{3} + \dfrac{6\frac{3}{5}}{6\frac{1}{9}}}{12\frac{5}{8} - 3\frac{4}{5}}.$

9. $\dfrac{5\frac{1}{6} \times 2\frac{1}{3}}{4\frac{5}{6} \div \frac{1}{2}}.$

10. $\dfrac{3\frac{3}{4}}{7\frac{1}{2}} - \dfrac{\frac{5}{8}}{35}.$

11. $\left(\dfrac{\frac{2}{3}}{4\frac{1}{2}} \div \dfrac{\frac{3}{4}}{\frac{5}{4}}\right) \div \dfrac{5\frac{1}{2}}{3}.$

12. $\dfrac{5\frac{2}{5} \div \dfrac{2\frac{4}{7}}{9\frac{3}{4}}}{19\frac{4}{5} \div 8\frac{4}{15}}.$

13. $\dfrac{12\frac{3}{4} - 7\frac{1}{8}}{\frac{3}{8} \times \frac{2}{3}}.$

Practical Problems.

152. FOR MENTAL SOLUTION.

1. Three rolls of butter weigh severally $1\frac{1}{8}$ lb., $2\frac{1}{2}$ lb., and $1\frac{3}{4}$ lb. What is the total weight?

2. What is the sum of \$$11\frac{3}{8}$ and \$$1\frac{1}{8}$? Express the result in dollars and cents.

3. From a piece of sheeting containing $31\frac{1}{2}$ yd., I sold $8\frac{3}{4}$ yd. How much remained?

4. What part of any thing is $\frac{1}{3}$ and $\frac{1}{2}$ of $\frac{1}{3}$ of it? What is a third and a half a third of 10? Of 8? Of 9? Of 11?

5. What cost $1\frac{3}{4}$ lb. of steak, at $12\frac{1}{2}$¢ per pound?

6. What cost a dressed turkey weighing $11\frac{3}{4}$ lb. at 12¢ per lb.? At 8¢? 9¢? 10¢? $12\frac{1}{2}$¢?

7. How many cents in $\frac{1}{2}$ a dollar? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{8}$? $\frac{1}{10}$? $\frac{1}{20}$? $\frac{1}{25}$? $1\frac{1}{2}$? $1\frac{1}{4}$? $1\frac{3}{4}$? $2\frac{1}{4}$? $2\frac{1}{2}$? $2\frac{3}{4}$?

8. *One* is what part of 2? Of 3? 4? 5? 6? 7?

9. What part of 3 is 2?

SOLUTION. 1 is $\frac{1}{3}$ of 3; and 2, being 2 times as great as 1, is 2 times as great a part of 3 as 1 is. Hence 2 is 2 times $\frac{1}{3}$, or $\frac{2}{3}$ of 3.

10. What part of 7 is 5? 3? 6? 2? 4? Give the solution as above.

11. What part of 13 is 3? 2? 10? 9? 12?

153. ***We observe that all we have to do, to ascertain what part of one number another is, is to write the former as the denominator, and the latter as the numerator, of a fraction.***

Of course we should always put results, when fractions, in simple fractions, and these in their lowest terms.

12. What part of 16 is 12?

13. What part of 8 is $2\frac{2}{3}$? Ans., $\frac{2\frac{2}{3}}{8} = \frac{1}{3}$.

14. What part of 100 is 25? 10? 5? 16? $12\frac{1}{2}$? $66\frac{2}{3}$? $62\frac{1}{2}$? $37\frac{1}{2}$? $6\frac{1}{4}$? $16\frac{2}{3}$?

15. What part of 18 is 12? 6? 4? 2? 3? $5\frac{1}{3}$? $5\frac{1}{4}$?

16. What part of $\frac{4}{5}$ is $\frac{2}{3}$? Ans., $\frac{\frac{2}{3}}{\frac{4}{5}} = \frac{5}{6}$.

17. What part of $\frac{1}{2}$ is $\frac{1}{3}$?

18. What part of $\frac{1}{5}$ is $\frac{1}{3}$?

19. What part of $\frac{3}{7}$ is $\frac{1}{2}$?

20. How many times $\frac{2}{5}$ is $\frac{6}{7}$? Ans., $\frac{\frac{6}{7}}{\frac{2}{5}} = 2\frac{1}{7}$.

21. How many times $\frac{2}{3}$ is $\frac{11}{12}$?

22. How many times $\frac{3}{11}$ is $\frac{5}{7}$?

23. What part of a year (52 weeks) is 13 weeks? 26 weeks? 24 weeks? 12 weeks? 14 weeks? 6 weeks?

24. 320 rods make a mile. What part of a mile is 40 rods? 80 rods? 120 rods? 60 rods?

25. There are 3 feet in a yard, and 12 inches in a foot. What part of a yard is $4\frac{1}{2}$ inches? 9 inches? 18 inches? 27 inches? 4 inches? 8 inches? $2\frac{1}{4}$ inches? 6 inches?

26. In the common grocer's weight, 16 ounces make a pound. What part of a pound is 2 ounces? 4 ounces? 8? 12? 10? 5? 6? 11? 14?

27. At 20¢ per lb., what cost 5 lb. 8 oz.[1] of butter? What 7 lb. 4 oz.? 2 lb. 12 oz.?

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[1] *Oz.* is the abbreviation for "ounce" or "ounces."

28. What cost 3 lb. 4 oz. of sugar at 9¢ per pound? At 12¢? At $10\frac{1}{2}$¢? At $8\frac{1}{2}$¢?

29. What is two-fifths and one-half of two-fifths of 5? Of 12? Of 7? Of 20? Of 6? Of 35?

30. What is $\frac{1}{7}$ and $\frac{3}{4}$ of $\frac{1}{7}$ of 8? Of 10? Of 12? Of 11?

31. How much cheese, at $16\frac{2}{3}$¢ per pound, can I buy for $\frac{1}{2}$ a dollar? How much for 75¢?

32. How many pounds of $8\frac{1}{3}$¢ sugar can I buy for a dollar? How much for $12\frac{1}{2}$¢?

33. What part of a year is 3 mo.? 4 mo.? 6 mo.? 2 mo.? 8 mo.? 9 mo.? 10 mo.?

34. At $\frac{1}{2}$ a dollar a yard, how much cloth can be bought for \$$2\frac{1}{2}$? For \$$1\frac{3}{4}$?

35. What part of \$1 is $6\frac{1}{4}$¢? $12\frac{1}{2}$¢? 10¢? $16\frac{2}{3}$¢? 25¢? $33\frac{1}{3}$¢? $62\frac{1}{2}$¢?

36. At \$2 per yard, how much cloth can be bought for \$5? For \$$3\frac{1}{2}$? For \$$\frac{3}{4}$?

37. Which is the greater, $\frac{2}{3}$ or $\frac{3}{4}$? How much? $\frac{3}{5}$ or $\frac{1}{2}$? $\frac{2}{7}$ or $\frac{11}{35}$?

38. How many more months in $\frac{5}{6}$ of a year than in $\frac{1}{2}$ a year? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{2}{3}$? $\frac{3}{4}$?

39. A bushel of wheat weighs 60 lb. How many bushels in a bag which weighs 150 lb.? 100 lb.? 75 lb.? 200 lb.?

40. What part of a foot is 2 inches? 3 inches? 4 inches? 6 inches? 8 inches? 9 inches? 10 inches?

The character @ means "at," "per," or "by."

41. At $7\frac{3}{5}$¢ @ lb., what cost 6 lb. starch?

Give the answer to the nearest whole cent, as you would pay such a bill at the grocery. This would be 46¢.

42. Mr. A. has coffee at $12\frac{1}{2}$¢ @ lb., @ $16\frac{2}{3}$¢, and @ $33\frac{1}{3}$¢. How much of the first can I buy for \$1? Of the 2d? Of the 3d?

43. From a piece of broadcloth containing $15\frac{3}{4}$ yd., the merchant sold 1[illegible], $2\frac{1}{2}$, and $\frac{3}{4}$ yd. How much remained?

44. For casting interest, 30 days are called a month. What part of a month is 10 da.? 15 da.? 20 da.? 27 da.? 5 da.? 6 da.?

45. Mr. A. borrows $250 of me, and agrees to pay me for the use of it at the rate of $20 per year. What must he pay me for 3 mo.? 4 mo.? 2 mo.? 6 mo.? 7 mo.? 8 mo.? 9 mo.? 1 mo.?

Money paid for the use of borrowed money is called *Interest.*

46. I borrow $300, and agree to pay interest at the rate of $24 per year. How much do I pay per month? How much for 5 da.? For 6 da.? 10 da.? 15 da.? 20 da.? 1 da.? 27 da.?

47. 196 lb. of flour makes a barrel. When my grocer sells me 50 lb. for a quarter of a barrel, how much more than a quarter-barrel do I get?

48. How many 8ths in a quarter? How many 16ths? 24ths? 36ths? 20ths? 28ths? 48ths? 44ths? 12ths?

49. Change $\frac{2}{3}$ to 15ths; 27ths; 21sts; 30ths; 36ths; 33ds; 18ths; 24ths.

50. In the Western States a tract of land 1 mile square is called a section. It contains 640 acres. How many acres in a half-section? A quarter? What part of a section is 40 acres? 120 acres are how many quarter-sections? Mr. A. buys 3 quarter-sections: how many acres does he buy?

51. Capt. C. owns $\frac{2}{3}$ of $\frac{3}{5}$ of a vessel, and Mr. A. owns the remainder. How much does Mr. A. own?

52. If I own $\frac{4}{5}$ of a farm, and sell $\frac{1}{3}$ of my share, how much have I left?

53. If I own $\frac{2}{3}$ of a house, and then buy $\frac{1}{2}$ the remainder, how much do I then own?

54. Mr. Walter owned $\frac{2}{5}$ of a piece of land, Mr. Smith $\frac{3}{7}$, and Mr. Jones the remainder. Which owned the most, and which the least? How much did Mr. Jones own?

ANS., Mr. Smith owned the most, and Mr. Jones the least.

55. A sold to B $\frac{5}{7}$ of his farm, and then bought back $\frac{1}{3}$ of what he sold. Which then had the larger part of the farm? How much the larger part had he?

How do you compare two fractions in order to ascertain which is the greater? *Write a rule.*

56. A bought $\frac{2}{3}$ of $\frac{5}{6}$ of a piece of land, and B bought $\frac{7}{8}$ of the remainder. Which bought the most of it? How much the most? How much remained?

57. If 12 is $\frac{6}{7}$ of a number, what is $\frac{1}{7}$ of the number? What is $\frac{7}{7}$, or the whole of the number?

58. If 9 is $\frac{3}{4}$ of a number, what is the whole of the number?

SOLUTION.—Since $\frac{1}{4}$ is $\frac{1}{3}$ of $\frac{3}{4}$, if 9 is $\frac{3}{4}$ of a number, $\frac{1}{4}$ of that number is $\frac{1}{3}$ of 9, or 3; and, if 3 is $\frac{1}{4}$ of a number, $\frac{4}{4}$, or the whole of it, is 4 times 3, or 12. Hence 12 is the number of which 9 is $\frac{3}{4}$.

59. If 15 is $\frac{3}{7}$ of a number, what is that number?

60. Mr. Harris bought 27 acres of land of Mr. Whipple, which was $\frac{3}{10}$ of Mr. Whipple's farm. How much had Mr. Whipple at first? How much had he left?

61. A sold 56 sheep to B, which was $\frac{7}{8}$ of his flock. How many sheep had A left? How many had he at first?

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62. $\frac{2}{3}$ of 12 is $\frac{4}{5}$ of what number?

SOLUTION. $\frac{2}{3}$ of 12 is 8. Now, if 8 is $\frac{4}{5}$ of a number, $\frac{1}{5}$ of the number is $\frac{1}{4}$ of 8, or 2; and, if 2 is $\frac{1}{5}$, $\frac{5}{5}$, or the whole number, is 5 times 2, or 10. Hence $\frac{2}{3}$ of 12 is $\frac{4}{5}$ of 10.

63. $\frac{2}{7}$ of 21 is $\frac{3}{4}$ of what number?

64. $\frac{4}{5}$ of 15 is $\frac{3}{10}$ of what number?

65. $\frac{3}{10}$ of 40 is $\frac{4}{5}$ of what number?

66. $\frac{3}{9}$ of 27 is $\frac{3}{5}$ of what number?

67. $\frac{8}{9}$ of 27 is $\frac{8}{13}$ of what number?

68. $\frac{5}{9}$ of 81 is $\frac{9}{10}$ of what number?

69. A owned a farm of 260 acres, and sold $\frac{7}{13}$ of it, which was just equal to $\frac{5}{11}$ of B's farm? How many acres in B's farm?

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70. $\frac{3}{4}$ of 20 is $\frac{5}{9}$ of how many times 9?

SUGGESTION.—As above, we find that $\frac{3}{4}$ of 20 is $\frac{5}{9}$ of 27, which is 3 times 9.

71. $\frac{5}{8}$ of 24 is $\frac{3}{4}$ of how many times 10?

72. $\frac{4}{5}$ of 45 is $\frac{6}{11}$ of how many times 3?

73. $\frac{6}{7}$ of 35 is $\frac{5}{9}$ of how many times 2?

74. $\frac{5}{9}$ of 81 is $\frac{4}{5}$ of how many times 9?

75. $\frac{2}{3}$ of 15 is $\frac{5}{7}$ of how many times 7?

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76. If 5 oranges cost 20¢, what cost 8 oranges?

77. If I sell a load of 20 bushels of wheat for $25, what will a load of 30 bu. bring at the same rate?

78. Bought 3 yd. of cloth for $1. What will 11 yd. cost at the same rate?

79. If oats bring 65¢ for 3 bushels, how much shall I get for a load of 50 bu.? A load of 80 bu.?

80. When oats are selling at 75¢ for 3 bu., what will a load of 60 bu. bring? Of 40 bu.? Of 80 bu.? Of 50 bu.?

81. How much coffee at 6 lb. for $1 can I buy for 75¢? For $1.25? For $2? For $1.50? For $1.75? For 25¢?

82. If $\frac{2}{3}$ of a yard of cloth cost $\$\frac{3}{4}$, what will $\frac{2}{5}$ of a yard cost?

FULL ANALYSIS.—If 2-thirds of a yard cost $\frac{3}{4}$ of a dollar, 1-third costs $\frac{1}{2}$ of $\frac{3}{4}$, or $\frac{3}{8}$ of a dollar; and, if 1-third costs $\frac{3}{8}$ of a dollar, 3-thirds cost 3 times $\frac{3}{8}$, or $\frac{9}{8}$ of a dollar. Again: if 1 yard costs $\frac{9}{8}$ of a dollar, 1-fifth costs $\frac{1}{5}$ of $\frac{9}{8}$, or $\frac{9}{40}$ of a dollar, and 2-fifths cost 2 times $\frac{9}{40}$, or $\frac{9}{20}$ of a dollar. $\frac{9}{20}$ of a dollar is 45¢, since $\frac{1}{20}$ is 5¢.

83. If $\frac{5}{7}$ of a yard of silk cost $2\frac{1}{2}$ cents, what will $\frac{3}{8}$ of a yard cost?

84. If an 80-acre lot is $\frac{3}{4}$ of my farm, what is $\frac{2}{3}$ of it?

85. If a third of ten were three, what would a fourth of 40 be? ANS., 9.

86. If $\frac{1}{3}$ of a certain number is 3, what is $\frac{1}{4}$ of 4 times that number?

87. $2\frac{3}{4}$ bu. apples make a barrel. When apples are 60¢ per bu., what is that per bbl.?

88. If $\frac{3}{4}$ bu. apples cost $\frac{1}{2}$ a dollar, what is that per bbl.?

89. At \$$1\frac{1}{2}$ per bbl., what cost a bushel of apples?

90. John can do a certain piece of work in 5 days, and James in 4 days. Working together, how much will they do in a week, working time?

91. There are 3 pipes discharging water from a full vat. One would empty it in 3 hours, 1 in 2, and 1 in 4. How long will it take them, running together, to empty it?

92. I buy goods at \$$3\frac{1}{2}$ per yard, and sell at a profit equal to $\frac{1}{7}$ the cost. What do I get for $2\frac{1}{2}$ yd.?

93. At what price per lb. must I sell coffee which cost me \$27 per 100 lb., in order to make $\frac{1}{9}$ as much as the cost?

94. A case of one dozen pairs of boots cost \$27. How must I sell them per pair to make a profit equal to $\frac{1}{3}$ the cost?

95. Wishing to multiply 38 by 25, if I first multiply by 100 by annexing two 0's, how shall I get the required product from the result?

96. In the way suggested in the last example, multiply the following by 25: 48, 256, 37, 43, 154, 75, 864, 19.

97. What part of 100 is $12\frac{1}{2}$? How, then, can I multiply by $12\frac{1}{2}$ by first multiplying by 100?

98. Multiply the following numbers by $12\frac{1}{2}$, as suggested in the preceding example: 32, 46, 18, 27, 54, 73, 65, 124.

99. In a manner similar to the above, multiply the following by $33\frac{1}{3}$: 27, 15, 36, 48, 19, 22, 31, 61, 72.

100. To divide by 100, and then multiply by 4, is equivalent to dividing by what number?

101. Dividing by 100, and then multiplying by 3, is equivalent to dividing by what number? What if you multiply by 8?

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154. *Written Exercises.*

Required the cost of

1. $7\frac{1}{2}$ lb. veal @ $5\frac{3}{4}$¢ = \$0.43$\frac{1}{8}$.
2. $9\frac{7}{8}$ qt. berries @ $7\frac{2}{3}$¢ = \$0.75$\frac{17}{24}$.
3. $6\frac{7}{8}$ yd. ribbon @ $12\frac{1}{2}$¢ = \$0.85$\frac{15}{16}$.
4. $7\frac{2}{3}$ yd. muslin @ $9\frac{3}{4}$¢ = \$0.74$\frac{3}{4}$.
5. $12\frac{1}{2}$ bu. oats @ $62\frac{1}{2}$¢ = \$7.81$\frac{1}{4}$.
6. $6\frac{1}{2}$ bu. apples @ $74\frac{1}{2}$¢ = \$4.84$\frac{1}{4}$.
7. $5\frac{1}{2}$ qt. nuts @ $9\frac{3}{4}$¢ = \$0.53$\frac{5}{8}$.
8. 15 lb. cheese @ $9\frac{1}{2}$¢ = \$1.42$\frac{1}{2}$.
9. $2\frac{3}{5}$ yd. cloth @ $\frac{7}{8}$ of a dollar = \$2.27$\frac{1}{2}$.
10. $7\frac{1}{2}$ lb. rice @ $5\frac{3}{4}$¢.
11. $9\frac{7}{8}$ qt. beans @ $7\frac{2}{3}$¢.
12. $5\frac{7}{8}$ yd. ribbon @ $12\frac{1}{2}$¢.
13. $7\frac{2}{3}$ yd. lace @ $9\frac{3}{4}$¢.
14. $12\frac{1}{2}$ bu. oats @ $62\frac{1}{2}$¢.
15. $6\frac{1}{2}$ bu. apples @ $74\frac{1}{2}$¢.
16. $5\frac{1}{2}$ qt. nuts @ $9\frac{3}{4}$¢.
17. 15 lb. starch @ $9\frac{1}{2}$¢.
18. $2\frac{3}{5}$ yd. cloth @ \$$\frac{7}{8}$.
19. 15 lb. honey @ $26\frac{5}{9}$¢.
20. $7\frac{3}{4}$ lb. coffee @ \$$\frac{3}{5}$.
21. 42 bu. apples @ $63\frac{2}{7}$¢.
22. $7\frac{5}{8}$ yd. calico @ $12\frac{1}{2}$¢.
23. $13\frac{4}{5}$ bu. potatoes @ $37\frac{1}{2}$¢.
24. $10\frac{2}{5}$ lb. cheese @ $15\frac{2}{3}$¢.
25. $3\frac{4}{5}$ tons coal @ \$$12\frac{3}{5}$.
26. $12\frac{3}{4}$ bu. apples @ $37\frac{1}{2}$¢.
27. $13\frac{1}{2}$ lb. veal @ $9\frac{3}{4}$¢.
28. $23\frac{2}{3}$ pecks peaches @ $87\frac{1}{2}$¢.

29. 175 yd. flannel @ $39\frac{4}{5}$¢.

30. $5\frac{3}{4}$ bbl. apples @ \$$1\frac{3}{4}$.

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31. A vest cost \$$6\frac{2}{5}$, a hat \$$5\frac{1}{2}$, and a pair of boots \$$8\frac{2}{3}$. What is the cost of all?

32. Bought flour at \$$7\frac{3}{5}$ per barrel, and sold it at \$8. How much did I make on 50 barrels? Had I sold it at \$$7\frac{3}{4}$ per barrel, how much would I have made?

33. Bought 5 loads of potatoes, containing respectively $33\frac{1}{2}$, $27\frac{3}{4}$, $40\frac{1}{4}$, $35\frac{1}{2}$, and $29\frac{1}{4}$ bushels, and sold $12\frac{3}{4}$ bushels to each of 3 men, and $25\frac{1}{2}$ to each of 4 men. How many potatoes had I left?

34. Having on hand $19\frac{4}{5}$ tons of iron, if I sell $10\frac{1}{4}$ tons at \$45 per ton, and the remainder at \$43, what will I receive for it?

35. Mr. Jones owns $\frac{3}{5}$ of a tract of land, and Mr. Smith buys $\frac{2}{3}$ of Mr. Jones's share. How much has Mr. Jones left? How much does Mr. Smith buy if the tract is 160 acres?

36. Sold $5\frac{2}{7}$ yards of cloth at \$$3\frac{1}{2}$ per yard, and received in payment $32\frac{1}{2}$ pounds of butter at $15\frac{1}{2}$ cents per pound, $\frac{3}{4}$ of a ton of hay at \$15 per ton. What is the balance? and is it in my favor, or against me?

37. How many raisins can be bought for \$1, at $17\frac{1}{2}$ cents per pound?

38. Bought a box of soap containing 70 pounds. Keeping it all summer, it dried away $\frac{1}{3}$, when I sold it at $8\frac{2}{3}$ cents per pound. I gave 7 cents per pound. Did I make, or lose? How much?

39. Bought 10 cords of wood for \$$5\frac{1}{2}$ per cord. Paid \$$\frac{3}{5}$ per cord for sawing, \$$\frac{2}{5}$ for splitting, and \$$\frac{1}{2}$ per cord for wheeling it into the shed and piling it. How much did the wood cost me in all?

40. Owning 100 acres of land, I sold $27\frac{1}{2}$ acres to one ꟷn, and $\frac{1}{2}$ the remainder to another. How much had I ?

41. A man cuts $35\frac{2}{3}$ cords of wood in $2\frac{1}{2}$ weeks' working time. What is the average of each day's work? How much does he earn per day, at 50 cents per cord for cutting?

42. If $3\frac{1}{2}$ yards of cloth cost $\$4\frac{3}{8}$, and 20 bushels of wheat bring $\$26\frac{2}{3}$, and a farmer brings in a load of $36\frac{1}{2}$ bushels of wheat, and buys 40 yards of cloth, what is the balance, and to whom due?

43. A room is 5 yards wide, and $5\frac{1}{2}$ yards long. How much will it cost to carpet it with yard-wide carpeting at $1.10 per yard, allowing nothing for waste?

44. Two persons, 130 miles apart, start to travel toward each other, — one at the rate of 25 miles a day, and the other at the rate of 32 miles. How long before they will meet?

45. Two persons start from places $7\frac{1}{3}$ miles apart, and travel the same way, — one at the rate of $3\frac{2}{3}$ miles per hour, and the other at the rate of $4\frac{1}{2}$. How far apart will they be in 5 hours?

46. Two persons on the same east and west line, and 15 miles apart, start to travel towards each other. If they continue these directions, — one at the rate of $2\frac{3}{5}$ miles per hour, and the other at the rate of $3\frac{2}{3}$ miles per hour, — how far will they be apart at the end of 6 hours?

47. D bought of E $35\frac{1}{2}$ bushels of clover-seed for $142, and afterwards sold to F $\frac{1}{4}$ of his purchase at a profit of $\$1\frac{1}{4}$ per bushel. What did the part sold to F amount to?

48. At the rate of $\$61\frac{7}{8}$ for $11\frac{1}{4}$ cords of wood, what will be the cost of 6 loads of $\frac{7}{8}$ of a cord each, 4 loads of $\frac{3}{4}$ of a cord each, and 8 loads of $\frac{2}{3}$ of a cord each?

49. A purchased of B 40 yards of cloth for $260. He then sold to C $\frac{3}{5}$ of his purchase at a profit of $\$\frac{3}{8}$ per yard, and the remainder to D at a loss of $\$\frac{1}{8}$ per yard. What did A gain or lose by these several transactions?

50. How much more than $8\frac{3}{4}$ yards of ribbon, at 4¢ a yard, will $4\frac{1}{4}$ yards of calico cost at 11¢ a yard?

51. A grocer bought 100 barrels of flour at $\$6\frac{7}{8}$ per barrel. He sold 49 barrels at $\$7\frac{1}{2}$ per barrel, and the rest at $\$7\frac{1}{8}$ per barrel. How much did he gain?

52. A tree, whose length was 136 feet, was broken into two pieces by falling. $\frac{2}{3}$ of the length of the longer piece equalled $\frac{3}{4}$ of the shorter. What was the length of each piece?

53. A obtains from two fields 344 bushels of wheat. Provided the first field yielded $\frac{3}{5}$ as much as the second, required the yield of each field.

54. A owns $\frac{3}{17}$ of a ship's cargo, valued at $\$493000$; B owns $\frac{15}{29}$ of the remainder; C owns $\frac{3}{10}$ as much as A and B; and D owns the remainder. How much does each own?

55. Bought 3 crocks of butter weighing $25\frac{7}{16}$, $29\frac{3}{4}$, and $27\frac{1}{8}$ lbs. The empty crocks weigh $5\frac{3}{16}$, $5\frac{3}{4}$, and $5\frac{7}{8}$ lbs. What did the butter cost at $23\frac{3}{4}$ cents a pound?

Problems illustrating the Use of Cancellation.

56. If 5 yards of silk cost \$7, how much will 15 yards cost?

SUGGESTION. — In solving this, we first *find what 1 yard costs by dividing the cost of the given quantity by that quantity.* Then we multiply the cost of 1 (yard) by the number representing the required quantity (15 yards). The operations then are, —

$$\frac{\textit{Cost of quantity, } 7}{\textit{Quantity, } 5} \times 15 = \text{the required quantity.}$$

Hence $\frac{7}{\cancel{5}} \times \overset{8}{\cancel{15}} = 21$, the required quantity.

57. If $\frac{3}{4}$ of a yard of silk cost \$6, what will $2\frac{1}{2}$ yards cost?

OPERATION. $(6 \div \frac{3}{4}) \times 2\frac{1}{2} = \cancel{6} \times \frac{4}{\cancel{3}} \times \frac{5}{\cancel{2}} = 20.$

EXPLANATION. — If $\frac{3}{4}$ yd. cost \$6, 1 yd. cost $(6 \div \frac{3}{4})$ dollars, and $2\frac{1}{2}$ yd. cost $2\frac{1}{2}$ times $(6 \div \frac{3}{4})$ dollars. Hence we have $(6 \div \frac{3}{4}) \times 2\frac{1}{2} = 6 \times \frac{4}{3} \times \frac{5}{2} = 20$ dollars.

58. If $\frac{7}{8}$ of an acre of ground cost \$245, what will $3\frac{3}{8}$ acres cost?

59. If $2\frac{3}{4}$ yards of cloth cost $\$3\frac{1}{2}$, what cost $5\frac{1}{2}$ yards?

2 times $\$3\frac{1}{2}$ gives the answer. Why is it?

60. If $\frac{2}{7}$ of 6 yards of cloth cost $\$3\frac{3}{7}$, what will $\frac{3}{5}$ of 7 yards cost?

OPERATION.

$$3\frac{3}{7} \div \frac{2}{7} \text{ of } 6 \times \frac{3}{5} \text{ of } 7 = \frac{\overset{2}{\cancel{24}}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2} \times \cancel{6}} \times \frac{3 \times 7}{5} = \frac{42}{5} = 8\frac{2}{5}.$$

EXPLANTION. — If $\frac{2}{7}$ of 6 yd. cost $\$3\frac{3}{7}$, 1 yd. cost $3\frac{3}{7} \div (\frac{2}{7}$ of 6) dollars, and ($\frac{3}{5}$ of 7) yd. cost ($\frac{3}{5}$ of 7) times [$3\frac{3}{7} \div (\frac{2}{7}$ of 6)] dollars. Hence we have

$$3\frac{3}{7} \div \frac{2}{7} \text{ of } 6 \times 6 \times \frac{3}{5} \text{ of } 7 = \frac{24}{7} \times \frac{7}{2 \times 6} \times \frac{3 \times 7}{5} = 8\frac{2}{5} \text{ dollars.}$$

155. In order to use cancellation to abridge computation, we first examine the problem, and ascertain what numbers are to be multiplied together, and by what we are to divide. Then, representing these operations by signs, proceed to cancel all factors which appear both as multipliers and divisors.

Cancellation is serviceable only in cases in which there are multiplications and divisions to be performed. When the processes are chiefly addition and subtraction, it is of no use.

61. If $\frac{2}{7}$ of a barrel of flour cost $\$1\frac{3}{7}$, what cost $5\frac{2}{3}$ barrels?

62. If $\frac{2}{7}$ of 6 yards of cloth cost $\$2\frac{3}{7}$, what cost $\frac{3}{5}$ of 7 yards?

63. If $\frac{3}{5}$ of $\$2\frac{1}{4}$ is the cost of a yard of cloth, what is the cost of $\frac{1}{2}$ of a yard of the same? Of 5 yards?

64. If $\frac{3}{5}$ of $\frac{5}{6}$ of a barrel of flour cost $\$3\frac{1}{2}$, what will $\frac{1}{2}$ of $\frac{4}{7}$ of 21 barrels cost?

65. If $\frac{2}{3}$ of $\frac{3}{4}$ of $2\frac{1}{2}$ yards of cassimere cost $\frac{1}{2}$ of 5 dollars, what will 2 yards cost?

66. If $\frac{6}{7}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of 6 yards of satinet cost 84 cents, what will $\frac{4}{5}$ of $\frac{6}{9}$ of $\frac{5}{6}$ of 9 yards cost?

67. If $\frac{6}{8}$ of $\frac{4}{6}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of $3\frac{1}{3}$ yards of broadcloth cost $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{2}{3}$ of 4 dollars, what will $\frac{7}{12}$ of $\frac{1}{2}$ of $4\frac{4}{5}$ yards cost?

68. If $\frac{5}{8}$ of $\frac{3}{7}$ of $\frac{8}{12}$ of $\frac{7}{8}$ of 12 yards of petersham cost $\frac{7}{8}$ of $\frac{9}{10}$ of $\frac{8}{9}$ of 10 dollars, what will $\frac{3}{4}$ of $3\frac{1}{3}$ yards cost?

69. If $\frac{1}{7}$ of 7 yards of cloth cost 49 cents, what will $\frac{6}{7}$ of $\frac{8}{9}$ of $\frac{7}{8}$ of 8 yards cost?

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70. From Chicago to Detroit, by way of the Michigan Central Railroad, is 284 miles. If an express train leaves Chicago at 9 o'clock in the morning, and arrives at Detroit at 6 o'clock and 45 minutes ($6\frac{3}{4}$ hours after noon), making 2 stops of 20 minutes each, and 12 stops of 5 minutes each, on the way, what is the average rate per hour run by the train?

71. A man engages to do a certain piece of work in 100 days. What part ought he to do in 10 days. In 25 days? In $12\frac{1}{2}$ days? In $16\frac{2}{3}$ days? In $33\frac{1}{3}$ days? In $62\frac{1}{2}$ days? In $6\frac{1}{4}$ days? In 11 days?

72. If 20 men require $7\frac{1}{2}$ barrels of flour for their subsistence 5 months, how much will 30 men require for a year? 17 men for 6 months? A barrel of flour is 196 pounds. At the above rate, what is a man's daily allowance?

73. How long could 50 men subsist on a stock of provisions which would last 7 men 80 days? 10 men 100 days? 13 men 200 days? 5 men 8 days? 11 men 176 days?

74. A man bought 189 acres of land at $10 per acre, and $250\frac{1}{4}$ acres at $13 per acre. He sold $\frac{2}{3}$ of the former tract for $\$18\frac{1}{2}$ per acre, and $\frac{2}{3}$ of the latter at $19 per acre. What would he make on the whole by selling the remainder at $20 per acre?

75. Bought at one time 320 acres of land at $\$25\frac{1}{2}$ an acre, and at another 275 acres at $\$31\frac{1}{4}$ an acre. Sold $\frac{3}{7}$ of the whole at $20, and the remainder at $30, per acre. Did I gain, or lose? How much?

76. If $\frac{5}{8}$ of twice my age is 30 years, what is my age? If $\frac{3}{7}$ of 4 times of it is $22\frac{2}{7}$, what is my age? If $\frac{2}{5}$ of $\frac{1}{2}$ of it is 8, what is my age? If 3 times $\frac{2}{3}$ of it is 30, what is my age?

77. If $\frac{5}{12}$ of $\frac{3}{4}$ of the distance to a certain place is 48

miles, what is the distance? If $\frac{2}{3}$ of $\frac{3}{5}$ is 10 miles, what is the distance? If $\frac{1}{5}$ of $2\frac{2}{3}$ times the distance is $\frac{7}{8}$ of a mile, what is the distance?

78. If you multiply together two *proper* fractions, how does the product compare with the less? Illustrate by $\frac{3}{5} \times \frac{2}{7}$.

79. If you multiply together two improper fractions, each greater than 1, how does the product compare with the greater? Illustrate by $\frac{5}{4} \times \frac{7}{3}$.

80. How is the value of a proper fraction affected by *adding* the same number to both its terms? Try it by adding 3 to both terms of $\frac{5}{7}$. By adding 1 to both terms of $\frac{2}{3}$. Try other cases.

81. How is an improper fraction whose value is greater than 1 affected by adding the same number to both its terms? Try it by adding 2 to both terms of $\frac{5}{4}$; of $\frac{7}{3}$; of $\frac{11}{5}$. By adding 1 to both terms of $\frac{8}{3}$; of $\frac{13}{7}$. By adding 20 to both terms of $\frac{3}{2}$; of $\frac{5}{3}$.

82. A regular train starts from New York at 1 o'clock in the afternoon, and runs at the rate of 32 miles an hour. At 3 o'clock a special train is started in pursuit, and is run at 40 miles per hour. At what hour will the special overtake the regular train? How far from New York?

83. Hens' eggs vary so much in size, that in an ordinary lot you may select 7 which will weigh 1 pound by taking the largest, or 10 by taking the smallest. When the largest are worth 15 cents per dozen, what are the smallest worth? Taking the extremes as given, what is the average weight of eggs per dozen?

84. If 9 hens' eggs weigh 1 pound $5\frac{1}{2}$ ounces (16 ounces make 1 pound), how much are eggs per pound when they sell at 1 shilling ($12\frac{1}{2}$ cents) per dozen?

85. If eggs average $1\frac{1}{2}$ pounds to the dozen, at what per pound should they be sold as compared with the price per dozen?

86. What is the value of $\frac{1}{11}$ of $\frac{1}{12}$ of a vessel, if a person who owns $\frac{3}{11}$ of it sells $\frac{1}{9}$ of $\frac{7}{8}$ of his share for \$1750?

87. There are 60 seconds in a minute, and 60 minutes in an hour. At what rate per hour is a railroad-train running which goes $\frac{1}{4}$ of a mile in 18 seconds?

88. Two land-speculators went West to buy land. One bought $\frac{21}{23}$ of a section (640 acres), and the other $\frac{23}{25}$. Which bought the most land? How much?

89. If $\frac{3}{4}$ of a pound of silver dollars is worth \$10$\frac{2}{7}$, what is the weight of \$1 in ounces, 12 ounces making a pound?

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156. Principle. — *If to the sum of two numbers the difference be added, it gives twice the greater; but, if from their sum their difference be subtracted, it gives twice the less number.*

Since the difference is what the less lacks of being the greater, if we add to the sum this difference, the result is twice the greater. In like manner, as the greater is the less + the difference, if we subtract this difference from the sum the remainder is twice the less.

90. The sum of two numbers is 198, and their difference 52. What are the numbers? Sum 295, and difference 117. What are the numbers?

91. If the sum of two numbers is 70, and the difference 16, what are the numbers? Sum 222, difference 114? Sum 386, difference 214? Illustrate as above.

92. James and John together have 86 cents, and John has 15 more than James. How many have each?

93. Two men, A and B, agree to work for C for \$1$\frac{1}{2}$ per day's work. Together they work 38 days, and A works 5 days more than B. How much must each receive?

DECIMAL FRACTIONS

CHAPTER III.

SECTION I.

DEFINITIONS, AND READING AND WRITING DECIMAL FRACTIONS.

157. DIVIDING any whole, or a unit, first into 10 equal parts (tenths), and these again into 10 equal parts (hundredths), and these again into 10 equal parts (thousandths), etc., is called the *Decimal Division*.

158. We have learned in Common Fractions that one-tenth may be represented thus, $\frac{1}{10}$; 2-tenths thus, $\frac{2}{10}$; 3-tenths thus, $\frac{3}{10}$, etc. So, also, that 1-hundredth may be written $\frac{1}{100}$; 2-hundredths, $\frac{2}{100}$; 7-hundredths, $\frac{7}{100}$; 4-thousandths, $\frac{4}{1000}$, etc. But there is another way to represent such fractions as these; that is, those which arise from the *Decimal Division*. It is this: tenths are represented by putting a dot, called a **Decimal Point**, at the left of the figure representing tenths: thus .1 is one-tenth; .2 is 2-tenths; .3 is 3-tenths, etc. So, also, .01 is one-hundredth; .02 is 2-hundredths; .03 is 3-hundredths, etc.

159. Fractions which arise from the *Decimal Division*, and are represented by means of the *Decimal Point*, and without the denominator expressed, are called **Decimal Fractions.**

For brevity, Decimal Fractions are often called simply *Decimals;* although, strictly speaking, all numbers, whole or fractional, expressed in the *Decimal* or common notation, are Decimals.

160. Counting to the right from the *Decimal Point*, the *first* place is *Tenths* place, the second *Hundredths*, the third *Thousandths*, the fourth *Ten-thousandths*, etc. Thus the *orders* of decimal fractions succeed each other as we go from units order towards the right, just as the orders of whole numbers do as we go to the left, according to the following

161. DECIMAL NUMERATION TABLE.

Hundred Billions.	Ten Billions.	Billions.	Hundred Millions.	Ten Millions.	Millions.	Hundred Thousands.	Ten Thousands.	Thousands.	Hundreds.	Tens.	Units.	DECIMAL POINT.	Tenths.	Hundredths.	Thousandths.	Ten-Thousandths.	Hundred-Thousandths.	Millionths.	Ten-Millionths.	Hundred-Millionths.	Billionths.	Ten-Billionths.	Hundred-Billionths.
3	3	3	3	3	3	3	3	3	3	3	3	.	3	3	3	3	3	3	3	3	3	3	3

$\underbrace{\qquad\qquad\qquad\qquad}_{\text{Orders of Whole Numbers.}}$ $\underbrace{\qquad\qquad\qquad\qquad}_{\text{Orders of Decimal Fractions.}}$

Ex. 1.—Read the following, and tell what they mean: .2; .5; .04; .07; .006; .001; .0008; .00007; .000003.

To Read a Decimal Fraction.

162. Rule.—I. *Numerate the fraction; that is, begin at the decimal point and name the orders to the right, and bear in mind the name of the lowest or right-hand order.*

II. *Read the expression just as a whole number, and then pronounce the name of the lowest or right-hand order.*

We numerate to find what the denominator of the fraction is, since, as in whole numbers, the whole may be read as so many of the *lowest* denomination.

We then read the number as written, since it tells how many parts are represented (that is, it is the numerator); and, having read it, pronounce the name of the lowest order, since this tells the denomination or kind of parts (that is, it is the denominator).

This method of reading is just the same in principle as the method of reading a *common* fraction.

1. Read .23478. Of what order is the 8? What, then, is the denominator of the fraction? What is the numerator? How would you read $\frac{23478}{100000}$?

2. Read .00234. What is the denominator? What the numerator? What is the difference between .00234 and $\frac{234}{100000}$?

Read each of the following, also writing each in the form of a common fraction: —

3. .304.	7. .56.	11. .01.	15. .58437.
4. .0027.	8. .506.	12. .001.	16. .00236.
5. .00801.	9. .3004.	13. .782564.	17. .00023.
6. .0101.	10. .0501.	14. .586423.	18. .90801.

N.B. — Pupils *write* a rule for representing a decimal fraction as a common fraction.

19. Read 23.5.

This is the same as $23\frac{5}{10}$, and is read in the same way; that is, "23 and 5-tenths."

20–22. Read 7248.347; 46.8423; 348.57684.

In reading whole numbers and decimals as mixed numbers, it may promote clearness to omit *and* in each case, except before the fraction. True, it is a little harsh; but perhaps the perspicuity compensates for this. In this way 7248.347 will be read "7 thousand 2 hundred forty-eight, *and* 3 hundred forty-seven thousandths."

Read the following: —

23.	48.21.	37.	152.342.	51.	2.718281828.
24.	107.056.	38.	793.546.	52.	3.14159.
25.	5006.00205.	39.	100.001.	53.	127.0087602.
26.	5400.0064.	40.	888.999.	54.	1003.700101002.
27.	1000.0001.	41.	.4343.	55.	100801.7304008.
28.	7281.5436.	42.	3.1416.	56.	46.7021.
29.	54037.02501.	43.	2.71828.	57.	32.0012.
30.	437.543621.	44.	1.0071.	58.	107.0107.
31.	1002.000508.	45.	100.0705.	59.	.004043.
32.	7080.070936.	46.	3006.0642.	60.	5.0005.
33.	5006.000807.	47.	2024.008.	61.	300.003.
34.	52111.111111.	48.	480.706.	62.	.053053.
35.	5333.333333.	49.	.3007.	63.	.0020202.
36.	6420.022022.	50.	0.0008.	64.	.206020.

163. Principle. — *Annexing 0's to a decimal does not alter its value, since, according to our method of reading, we annex an equal number to the denominator, and hence multiply both terms of the fraction by the same number.*

1. Explain the following: —

$.5 = .50 = .500 = .5000 = .50000.$

$.25 = .250 = .2500 = .2500000.$

$.012 = .0120 = .012000 = .01200000.$

To Write Decimals.

164. Rule. — *Write the numerator as a whole number. Then, beginning at the* RIGHT, *apply the decimal numeration, calling the right-hand figure tenths,*[1] *the next at the left hun-*

[1] Teachers who are averse to this mechanical or tentative process can teach their pupils to repeat the decimal numeration *backwards*, — i.e., from right to left; and, when this is entirely familiar, the direction in the rule may read, "Beginning at the *right*, call this order by the name of the denominator required, and proceed to the left, filling vacant orders with 0's till tenths order is reached. At the left of this order write the decimal point."

dredths, etc., filling all vacant orders with 0's, till the name of the order designated by the denominator is reached. At the left of this write the decimal point.

Reasons for the Rule. — The numerator, being a whole number, is so written. The reason for beginning at the right to apply the decimal numeration is simply to get the right number of places after the decimal point. True, the orders are thus *falsely* named; but the expedient is convenient; and, if this *trial* numeration from right to left gives the right number of orders, the *true* numeration from *left to right* cannot fail to be right.

1. Write six hundred twenty-five hundred-thousandths.

Process. — The numerator is 625. Now, beginning at the 5, *call* it tenths, the 2 hundredths, the 6 thousandths, and, prefixing a 0, call it ten-thousandths, and then another 0, and call it hundred-thousandths. At the left of this last 0 place the decimal point thus, .00625.

2. Write fifteen hundredths. Nineteen thousandths. Six ten-thousandths. Twenty-four thousandths. Five hundred thousandths. Thirty-nine millionths. One hundred thousandths. Forty-nine hundredths. Ten ten-millionths. Fifty-two thousandths. Eight hundred-thousandths. Eight hundred thousandths. Seventy-one hundredths. Ninety-one millionths. Seventeen ten-thousandths. Two thousand eight hundred forty-five ten-thousandths. Three hundred sixteen thousandths.

3. Write sixty-nine, and nine hundred three thousandths.

Suggestion. — It will be well for beginners to write the whole number first, and then, if the decimal is not readily written, *write it in another place*, according to the rule given above, and then annex it to the whole number.

4. Write seven hundred three thousand, and two hundred seven ten-thousandths.

Suggestion. — The whole number is 703000. The decimal, written according to the rule, is .0207. Written together, they are 703000.0207.

Write the following: —

5. Five, and two hundred sixty-three millionths.

6. Nine hundred eighty, and four thousandths.

7. Two, and eighty-five billionths.

8. Two hundred, and seventy-four ten thousandths.

9. Eight thousand two hundred, and thirty-two thousandths.

10. Four hundred fifty two hundred-thousandths.

11. Sixty-five, and five hundred twenty-one thousandths.

12. Eighty-two, and sixty-five billionths.

13. 7 hundred 63 thousand twenty, and one hundred eight millionths.

14. Seven thousand five hundred 29 millionths.

15. Four hundred seventy-five hundred-thousandths.

16. Forty five, and three hundred seventy-five ten-thousandths.

17. Three billion seven hundred fifty-five million 2 hundred 26 thousand, and 5 hundred forty-three millionths.

18. Three, and one thousand four hundred sixteen ten-thousandths.

19. Nine hundred 27 million 3 hundred 64 thousand 5 hundred, and 2 thousand 5 hundred 68 ten-millionths.

20. 14, and five tenths.

21. Thirty-six, and three hundred forty-eight thousandths.

22. 35, and 347 ten-thousandths.

23. Four hundred 16, and five hundred-thousandths.

24. 437246, and seventy-nine thousandths.

25. Four hundred, and 64 thousandths.

26. 54 thousand three hundred, and 22 ten-thousandths.

27. 374 thousand 426, and two ten-thousandths.

28. 461372, and 461 ten-thousandths.

29. 2 thousand 2, and 2 thousand 2 ten-thousandths.

30. 4 thousand, and four hundred four ten thousandths.

31. 472 thousand, and 204 ten thousandths.

32. 437 thousand 349, and 467 hundred-thousandths.

33. 420 thousand, and two hundred hundred-thousandths.

34. 300 thousand, and 204 hundred-thousandths.

35. 500387, and five hundred and five thousandths.

36. 800 thousand, and 565 hundred-thousandths.

37. 4000000, and five thousand and six hundred-thousandths.

38. 343 thousand 43, and six thousand hundred-thousandths.

39. 241341, and four hundred fifteen hundred-thousandths.

40. 11, and eleven thousand one hundred-thousandths.

41. 5, and five thousand six millionths.

42. 303, and three thousand three millionths.

43. 463 thousand 42, and 42 thousand eight millionths.

44. 50 thousand, and 50 thousand fifty millionths.

45–60. Write the following as decimals: $405\frac{17}{100}$; $300\frac{5}{10}$; $1\frac{27}{1000}$; $57\frac{802}{1000000}$; $1002\frac{1804}{10000000}$; $7\frac{5}{10000}$; $6\frac{7}{10000}$; $105\frac{105}{100000}$; $\frac{406020}{1000000}$; $\frac{58264}{1000000}$; $2\frac{24008}{100000}$; $1\frac{5802}{10000}$; $3\frac{3006}{1000000}$; $10\frac{42005}{1000000}$; $\frac{50}{10000}$; $\frac{2000}{1000000}$.

SECTION II.

REDUCTIONS.

To Reduce Common Fractions to Decimals.

1. How many tenths in $\frac{3}{5}$?

SOLUTION. — Since $\frac{3}{5}$ is $\frac{3}{5}$ of a unit, and there are 10 times as many tenths in any number as there are units, there are 10 times $\frac{3}{5}$, or $\frac{30}{5}$ *tenths*, in $\frac{3}{5}$. $\frac{30}{5} = 6$: hence there are 6 tenths in $\frac{3}{5}$, or $\frac{3}{5} = .6$.

2. As above, reduce $\frac{1}{2}$, $\frac{4}{5}$, $\frac{1}{5}$, $\frac{2}{5}$, and $\frac{3}{6}$ to tenths, explaining the operation.

3. How many hundredths are $\frac{3}{4}$?

SOLUTION. — Since $\frac{3}{4}$ is $\frac{3}{4}$ of a unit, and there are 100 times as many 100ths as units in any number, there are 100 times $\frac{3}{4}$, or $\frac{300}{4}$ hundredths, in $\frac{3}{4}$. $\frac{300}{4} = 75$: hence there are 75 100ths in $\frac{3}{4}$, or $\frac{3}{4} = .75$.

4. Reduce $\frac{1}{4}$, $\frac{3}{25}$, $\frac{3}{20}$, $\frac{7}{25}$, and $\frac{9}{20}$ to hundredths.

5. Reduce the following to thousandths: $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{3}{40}$, $\frac{29}{500}$, and explain as above.

165. Rule. — *Annex 0's to the numerator, and divide the result by the denominator, continuing to annex 0's till the division is exact, or until as many decimal places are obtained as desired. Point off as many places of decimals in the quotient as there have been 0's annexed, prefixing 0's to the significant figures, if necessary.*

DEMONSTRATION. — Since a common fraction represents parts of a unit, and in any number of units there are 10 times as many 10ths, 100 times as many 100ths, etc., annexing 0's to the numerator reduces the fraction successively to 10ths, 100ths, etc. Then dividing by the denominator is but reducing the improper fraction to a whole number (of tenths, hundredths, etc.).

6–10. Reduce $\frac{1}{16}$ to a decimal; $\frac{7}{25}$; $\frac{5}{6}$; $\frac{1}{125}$; $\frac{7}{8}$.

OPERATION.

```
16 ) 100 ( .0625
      96
      --
      40
      32
      --
      80
      80
      --
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EXPLANATION. — In this case we have virtually annexed 4 0's, or multiplied $\frac{1}{16}$ by 10000, which reduces it to 10000ths. Thus $\frac{1}{16} = \frac{10000}{16}$ ten-thousandths, or 625 ten-thousandths. This, in the decimal notation, is written .0625.

ANOTHER EXPLANATION. — We can explain the process as we go along with our work of dividing thus: 16 is not contained in 1; but 1 is 10 tenths. Again: 16 is not contained in 10; so that there are no *tenths* in the quotient. Hence we write 0 in tenths place. Now, 10 tenths make 100 hundredths; and, as 16 is contained in 100 6 times, we have 6 hundredths in the quotient, with 4 hundredths remaining, etc. This is the explanation which was given in Division.

This process may also be explained as multiplying and dividing the given fraction by the same number. Thus $\frac{3}{5}$ multiplied by 10 is $\frac{30}{5} = 6$: this, divided by 10, is .6.

Reduce the following common fractions to decimals, carrying the decimal to *millionths* if the division is not exact: —

11. $\frac{15}{16} = .9375.$	20. $13\frac{7}{16}$.	29. $128\frac{3}{11}$.
12. $\frac{6}{25} = .24.$	21. $40\frac{15}{16}$.	30. $13\frac{5}{12}$.
13. $\frac{9}{20} =$	22. $\frac{2}{7}$.	31. $\frac{4}{9}$.
14. $\frac{21}{8} =$	23. $13\frac{5}{9}$.	32. $\frac{5}{9}$.
15. $5\frac{1}{4} =$	24. $\frac{21}{37}$.	33. $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}$.
16. $16\frac{1}{2} =$	25. $12\frac{2}{3}$.	34. $\frac{139}{225}$.
17. $103\frac{3}{8} = 103.375.$	26. $1\frac{5}{6}$.	35. $\frac{1}{99}, \frac{2}{99}, \frac{3}{99}$.
18. $\frac{1}{3} = .33333+.$[1]	27. $3\frac{1}{5}$.	36. $\frac{102}{13}$.
19. $\frac{12}{13} =$	28. $10\frac{1}{25}$.	37. $\frac{1256}{17}$.

38. Tell, *without writing*, the decimals equivalent to the following common fractions: $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{4}{5}$; $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$.

REPETENDS.

166. When we are annexing 0's in order to extend a decimal, if at any time we have a remainder which is the same as we have had before, the same figures will recur in the quotient. Such a decimal is called a **Repeating** or **Circulating Decimal,** or simply a *Repetend*. A repetend is indicated by writing a dot over the first and last repeating figures; thus $.34\dot{5}7\dot{6}$ means .34576576576, etc.; $\dot{5}\dot{7}$ means .575757, etc. $.34\dot{5}7\dot{6}$ is read "34 hundreds and repetend 576;" $\dot{5}\dot{7}$ is read "Repetend 57," etc.

Ex. 1. — Tell, *without* writing, what repetends the following give: $\frac{1}{9}$; $\frac{2}{9}$; $\frac{3}{9}$; $\frac{4}{9}$; $\frac{5}{9}$; $\frac{6}{9}$; $\frac{7}{9}$; $\frac{8}{9}$.

2. What repetend does $\frac{1}{3}$ give? $\frac{2}{3}$?

3. What is the repetend given by each of the following: $\frac{1}{7}$; $\frac{2}{7}$; $\frac{3}{7}$; $\frac{4}{7}$; $\frac{5}{7}$; $\frac{6}{7}$?

---

[1] The + sign, when thus used, signifies that the division does not terminate.

167. Principle. — *When reducing a common fraction to a decimal, if the decimal does not terminate, the farther we extend it the nearer we approach to the value of the common fraction.*

Thus it is easy to see that $\frac{1}{3} = .3$ nearly, but .33 more nearly, and .333 still more nearly, etc. When we wish to use only a few places of a repetend, if the first figure of the part we would drop is 5 or more, it is more accurate to increase the last figure we use by 1: thus, instead of using .27 for .276, it is more accurate to use .28; so, instead of 3.1415 for 3.14159+, 3.1416 is more accurate.

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168. In **United-States Currency**, since a dime is a tenth of a dollar, a cent a tenth of a dime, and a mill a tenth of a cent, these are respectively tenths, hundredths, and thousandths of a dollar, and are so written, as we have already seen. So also we have learned that it is customary to write and read dimes, or tenths of dollars, with cents; thus we read $4.75, not "4 dollars, 7 dimes, and 5 cents," but "4 dollars and 75 cents."

Thus we are enabled to treat fractions of a dollar, or cents and mills, as other decimals.

EXAMPLES. — Represent the following fractions in decimals; that is, in cents and mills: —

$\$5\frac{1}{2}$; $\$127\frac{1}{4}$; $\$1\frac{1}{8}$; $\$200\frac{3}{4}$; $\$1\frac{1}{3}$; $\$12\frac{1}{6}$; $\$12\frac{2}{7}$.

To Reduce Decimals to Common Fractions.

169. Rule. — *Suppress the decimal point, and underneath the numerator of the fraction as thus obtained write the denominator, which is 1 with as many 0's annexed as there were places in the decimal. Reduce the fraction thus obtained to its lowest terms.*

This is simply writing what the fraction means, and is a direct consequence of the method of writing decimals. In fact, we have already performed the operation, and only mention it here to refresh the memory, and give a little more practical exercise.

Express the following decimals as common fractions: —

1. .5.	7. 23.625.	13. 1.06.	19. 3.75.
2. .35.	8. 103.	14. 1.10.	20. 3.075.
3. .125.	9. 2.4.	15. 1.07.	21. 15.6.
4. .375.	10. .0025.	16. 1.08.	22. 15.60.
5. .25.	11. .0005.	17. 1.125.	23. 15.600.
6. .75.	12. 1.04.	18. .225.	24. 1.12½.

SECTION III.

ADDITION AND SUBTRACTION.

170. The rules for Addition and Subtraction of Decimal Fractions are exactly the same as for whole numbers, and need not be repeated.

1. Add 715.25, 3051.039, 4.8, and 25.3156.

SOLUTION. — Writing the numbers so that like orders shall fall in the same column, we add exactly as in whole numbers. Thus in the ten-thousandths column there are only 6; so we write it in the sum. In the thousandths column there are 5 and 9, which make 14 thousandths, which are 1 hundredth and 4 thousandths, since 10 thousandths make 1 hundredth. In like manner we proceed through the other orders.

$$\begin{array}{r} 715.25 \\ 3051.039 \\ 4.8 \\ 25.3156 \\ \hline 3796.4046 \end{array}$$

Perform the following additions: —

2.	3.	4.	5.
715.206	605.271	742	$ 86.45
34.73	342.156	85.6	$243.045
1280.008	83.805	207.006	$580.50
7.5643	1.27	84	$ 73.00
187.4	483.5	556.38	$100.40

6. From 782.19 take 325.536.

SOLUTION. — Writing so that the various orders of the subtrahend shall fall under like orders in the minuend, we subtract as in whole numbers. Thus, as there are no thousandths in the minuend from which to take the 6 thousandths of the subtrahend, we take one of the 9 hundredths, which makes 10 thousandths, and subtract the 6 thousandths, obtaining 4 thousandths for the remainder. 3 hundredths from 8 hundredths leaves 5 hundredths. As 5 tenths cannot be taken from 1 tenth, we take a unit from the 2, which makes 10 tenths. This, with the 1 tenth of the minuend, makes 11 tenths, from which taking the 5 tenths there remain 6 tenths. Thus we proceed through all the orders as in whole numbers.

782.19
325.536
456.654

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171. *Examples for Practice.*

1.	2.	3.	4.
408.207	$23.56	$100.00	1000
57.0583	$ 7.50	$ 47.23	325.008
351.1487	$16.06	$ 52.77	674.992

5.	6.	7.	8.
1765.004	700.584	48	1
843.02	23	.546	.003
921.984			

9. From 8.1 take 3.547.
10. From 123 take 78.256.
11. From 3.46 + 10 take 5.7.
12. From .46 take .0037.
13. From 71.86 + 4.313 take 18.5 + 20.007.
14. From 10 take one tenth.
15. From 1 take one thousandth.
16. From $1 subtract 15 cents.
17. From $5.68 take $1.17.
18. Add 7.52, 10.478, 600.5, .87, and 23.
19. From the sum of $347.63 and $20.59 take the sum of $100 and $57.13.
20. From the sum of 8.7, .32, 6.08, 25, and 2,45, take the sum of .75, 1.25, 13, and 3.5.

172. *Practical Problems.*

1. A man earned \$5.27 one week, \$6.52 another, and \$7.15 another. Out of this he bought a pair of boots for \$4.75, and a hat for \$3.38. How much remained?

ANS., \$10.81.

2. A fence-builder laid up 20.5 rods one day, 17.02 rods the next, 31.25 rods the next, and 27 rods the next. A wind blew down 52.37 rods, and so displaced 10.5 rods more as to make it necessary to lay it over. How much remained of his work?

3. My grocery bill for the several days of a week was \$3.73, \$1.10, \$4, \$2.05, \$1.88, and \$3. On this I paid at one time during the week \$2, and at another \$3.50. How much did I still owe?

4. Write the following in the decimal form, and then add them: $6\frac{1}{4}$, $12\frac{1}{2}$, $5\frac{3}{8}$, $6\frac{5}{8}$, $\frac{3}{5}$, $\frac{3}{4}$. SUM, 32.1.

5. From the sum of $\frac{5}{6}$, .25, 10, $\frac{2}{5}$, and $13\frac{1}{2}$, subtract the sum of $1\frac{1}{2}$, $\frac{4}{5}$, 8, and 2.56.

6. A steamer sailed in 6 successive days 305.24, 206.5, 182.402, 343.25, 287.13, and 313.008 miles; and a sail-vessel, starting from the same port, sailed after her, making 105 miles the 1st day, $204\frac{3}{4}$ the 2d, $98\frac{1}{2}$ the 3d, $110\frac{5}{8}$ the 4th, $212\frac{5}{6}$ the 5th, and $113\frac{3}{5}$ the 6th. How much in advance was the steamer at the end of the 6 days?

7. My meat bill for the 7 days of a week was, 1st day, \$1.37; 2d day, \$0.88; 3d day, \$1; 4th day, 75 cents; 5th day, \$1.25; 6th day, 97 cents; 7th day, \$1.10. At the beginning of the week, the marketman owed me \$5.00. How did the account stand at the close of the week?

8 During the week a grocer sold from a bin of potatoes, which contained $87\frac{1}{2}$ bushels at the first, $6\frac{3}{4}$, $\frac{1}{2}$, 17, $25\frac{1}{2}$, 42.25, 18.5, and 31 bushels; having in the mean time bought and put in 2 loads of $33\frac{1}{2}$ and 41.75 bushels respectively. How many potatoes remained in the bin?

9. How much less than a thousand is 156.75, $428\frac{5}{8}$, 243.125, and $89\frac{1}{2}$?

10. From a piece of cloth containing $41\frac{3}{4}$ yards I sold $\frac{1}{2}$ a yard, $1\frac{3}{4}$, 8, 5.25, and 21.125 yards. What was the remnant?

11. A man's property is worth \$7528; but he owes \$347.50 to one man, \$1000 to another, \$$75\frac{3}{4}$ to another, and \$$15\frac{1}{2}$ to another. How much would he be worth if his debts were paid?

SECTION IV.

MULTIPLICATION AND DIVISION.

Multiplication of Decimals.

173. The *process* of Multiplication of Decimals is precisely the same as that of whole numbers, the only thing needing further attention being the position of the *Decimal Point* in the product.

174. Principle. — *In Multiplication of Decimals, the number of decimals in the product must equal the number in both factors.*

DEMONSTRATION. — Any number wholly or in part decimal may be read as so many of the lowest order mentioned as are represented by the figures, regardless of the point. (For example, 4.5 is 45 *tenths*, 43.75 is 4375 hundredths, etc.) In reading in this way, the denominator of each factor becomes 1 with as many 0's annexed as there are decimal places in that factor and the numerators are whole numbers. Hence the product of these denominators, which will be the denominator of the product, will contain as many 0's as there are decimal places in both factors. Therefore the number of decimal places in the product equals the number in both the factors.

Ex. — Multiply 34.8 by 3.76, explaining the pointing off according to the last paragraph.

SOLUTION. $34.8 = \frac{348}{10}$, and $3.76 = \frac{376}{100}$. Hence $34.8 \times 3.76 = \frac{348}{10} \times \frac{376}{100}$. We therefore multiply the numerators together, disregarding the decimal points. This product is 130848. Now, as the denominator of the multiplicand is 10, and that of the multiplier 100, the product is thousandths. Therefore we point off three decimals, which is equivalent to writing 1000 as a denominator.

$$\begin{array}{r} 34.8 \\ \underline{3.76} \\ 2088 \\ 2436 \\ \underline{1044} \\ 130.848 \end{array}$$

Examples for Practice.

1. $327 \times .6 = 196.2$.
2. $327 \times .06 = 19.62$.
3. $327 \times .006 = 1.962$.
4. $\$375.6 \times .125 = \$46.95$.
5. 78.05×3.47.
6. $.543 \times .0027$.
7. $.00279 \times .008 = .00002232$.
8. $\$48.275 \times 6.421$.
9. $5832 \times .05$.
10. $\$70.01 \times .0001$.
11. $34 \times .000025$.
12. 46.702×81.54.
13. $1.764 \times .0547$.
14. $.0058 \times .073$.
15. 356.28×175.6.
16. $256 \times .053$.
17. 5.002×713.
18. $.003 \times 3.003$.
19. 47×3.5.
20. $\$73.6 \times .0002$.
21. $56400 \times .01$.
22. $150 \times .1$.
23. $\$79 \times .1$.
24. 34.76×3.1416.
25. $875 \times .4343$.
26. $\$17.58 \times 2.002$.
27. $.8 \times 1.1$.
28. 4.5×4.5.
29. 3.712×3.712.
30. 8246×3.14.
31. $.347 \times 28$.
32. $80.008 \times .0007$.
33. $5.2704 \times .0341$,
34. $.00702 \times .00702$.
35. $.1 \times .1$. $.01 \times .01$.
36. $.5 \times .5$. $.005 \times .005$.

Division of Decimals.

175. The *process* of Division of Decimals is precisely the same as that of Division of Whole Numbers, the only thing needing further attention being the position of the Decimal Point in the Quotient.

Ex. 1. — Divide 2190.89208 by 63.284.

[The following solution is given for the purpose of exhibiting a simple method of insuring accuracy in placing the decimal point, which is the only thing presenting any difficulty in division of decimals: —

EXPLANATION. — The first figure of the quotient being perceived to be 3, *we retain it in mind till after we have multiplied* and written the first partial subtrahend (189852), and then write this first quotient figure directly over the figure in this subtrahend which arises from multiplying the units of the divisor by this figure; viz., in this instance, over the 9. Then, proceeding with the division, we place the decimal point in the quotient directly over that in the dividend.

OPERATION.

```
              34.62
63.284 ) 2190.89208
         1898 52
          292 372
          253 136
           39 2360
           37 9704
            1 26568
            1 26568
```

176. Rule. — *Write the first figure in the quotient directly* OVER *the figure in the first subtrahend which arises from multiplying the* UNITS *of the divisor by this figure: the decimal point in the quotient will fall over the point in the dividend.*

The reason for this rule is, that, when units are multiplied by any order, the product is of the same name as this order; i.e., units by tens give tens, units by hundreds give hundreds, etc. Hence this arrangement brings like orders in quotient, dividend, and subtrahend in the same column. An example will make this clear.

Ex. 2. — Divide 1956671.95 by 74.257.

EXPLANATION. — The first quotient figure being perceived to be 2, *we retain it in mind till after we have multiplied*, and then write it over the 8 which arose from multiplying the units (4) of the divisor. Then, proceeding with the division, we place the decimal point in the quotient over that in the dividend; and, as in this instance the units order is vacant, we write 0 in it.

OPERATION.

```
            26350.
74.257 ) 1956671.95
         148514
          471531
          445542
           259899
           222771
            371285
            371285
```

Now observe, that, the 8 being the product of units by some other order, this other order must be of the same name as the 5 in the dividend under which

the 8 falls, in order that the 8 may be of this order: i.e., the 2 (ten-thousands) multiplied into the 4 units give 8 (ten-thousands), etc.

The only case in which there can be the least difficulty is when there are no integers in the quotient. We give a few such examples: —

3–6. Divide 78.642 by 3. By .3. By .03. By .003.

In Short Division it is equally well to write the quotient *under* the dividend. If the first subtrahend is needed in order to make the place of the units product more evident, write it *above*. The work in these examples stands thus: —

```
             0.6          0.06          0.006
3|78.642   .3|78.642   .03|78.642   .003|78.642
  26.214     262.14      2621.4       26214.
```

7–9. Divide 13.475 by .245. By .0245. By .00245.

```
    55.              550.              5500.
.245|13.475     .0245|13.475     .00245|13.475
     1.225            0.1225            0.01225
      1225              1225                1225
      1225              1225                1225
```

177. Another Method of Determining the pointing of the Quotient.

— *Make the number of decimal places in divisor and dividend the same by annexing 0's to that which has the least. Then drop the decimal points, and divide as in whole numbers.*

The reason for this is, that making the number of decimal places the same reduces the fractions to forms having a common denominator: whence the quotient is the numerator of the dividend divided by the numerator of the divisor.

10. Divide 1116.9 by .153.

Operation.

```
1116900|153
1071     7300.
  459
  459
   00
```

Explanation. — Reducing both to thousandths, we have 1116900 thousandths to divide by 153 thousandths, which is the same as 1116900 ÷ 153.

11. Solve in this manner Exs. 3–6.

The numbers are written for division thus:—

3000)78642   300)78642   30)78642   3)78642

The operations are,—

3|000)78|642	3|00)786|42	3|0)7864|2	3)78642
26.214	262.14	2621.4	26241

Examples for Practice.

1. $688.1875 \div 5.5 = 125.125$.
2. $.440946 \div .561 = .786$.
3. $242.451 \div 1.2245 = 198$.
4. $\$183.375 \div \$.489 =$
5. $.1728 \div 12 =$
6. $343 \div .007 =$
7. $\$67.8632 \div 32.8 =$
8. $\$983 \div 6.6 =$
9. $12 \div .002 =$
10. $147.828 \div 9.7 =$
11. $\$37.4 \div \$4.5 =$
12. $7.85 \div 3.43 =$
13. $.478 \div .58 =$
14. $.9009 \div .4051 =$
15. $2.25 \div 18 =$
16. $22.36 \div 4.3$.
17. $\$3.25 \div \$.3$.
18. $87.9 \div .3$.
19. $\$22.5 \div .005$.
20. $.001638 \div .07$.
21. $.08 \div 32$.
22. $\$.643 \div \$1.05$.
23. $.278 \div .07$.
24. $14 \div .7854$.
25. $3.1416 \div 4$.
26. $\$8.371 \div 500$.
27. $\$583.71 \div \$120$.
28. $4.737 \div 3000$.
29. $42 \div 600$.
30. $125 \div 25000$.

178. In case the divisor has 0's at its right, *neglect them* in the process of dividing, and obtain the quotient of the dividend divided by the significant figures. Then remove the decimal point to the left as many places as there have been 0's neglected. *This is the same as the corresponding case in division of whole numbers.* Thus, to divide 4.737 by 3000, we neglect the three 0's, and divide by 3, obtaining the quotient 1.579. Now, this quotient is to be divided by 1000, the other factor of 3000. *This is done* by removing the decimal point 3 places to the left.

3|000)4.737
   1.579 *Quot. of* $4.737 \div 3$
True quot., .001579

Perform the following: —

31. 7.82 ÷ 10.
32. \$540 ÷ 100.
33. .56 ÷ 10; by 100.
34. .072 ÷ 900.
35. 1800 ÷ 0006.
36. \$640 ÷ \$8000.
37. .05 ÷ 100.
38. 235 ÷ 50.
39. 473 ÷ 2300.
40. 5.276 ÷ 11200.
41. 30.03 ÷ 710.
42. 64 ÷ .64.

---

179. *Practical Problems.*

[NOTE. — In these problems, when *common* fractions occur, put them into the forms of *decimals* before performing the operation required.]

1. What is the cost of 3$\frac{3}{4}$ yards of cassimere at \$2$\frac{1}{4}$ per yard?

OPERATION.

```
  $2.25
   3.75
   ----
   1225
  1575
  675
  -------
  $8.4475, or $8 and 45 cents.
```

In business operations it is customary to drop from results all fractions of a cent less than $\frac{1}{2}$, or .5 of a cent, and to call all fractions of a cent equal to or greater than .5 an additional cent.

2. What cost 14$\frac{1}{2}$ cords of wood at \$6$\frac{3}{4}$ per cord?

3. Bought of the groceryman 17$\frac{3}{8}$ pounds of sugar at 11$\frac{5}{8}$ cents per pound, and handed him in payment a \$5 bill. How much change should I receive?

4. How many dozen hats, at \$5$\frac{3}{4}$ each, can be bought for \$103$\frac{1}{2}$?

5. Bought a case of 1 dozen pairs of boots for \$112.50. What was that per pair?

6. If a railroad-train averages 23$\frac{3}{5}$ miles an hour, including stops, how long will it be in running 288 miles?

7. If a railroad-train runs 350 miles in $19\frac{1}{2}$ hours, but stops 3 times 20 minutes each, and 10 other times 6 minutes each, what is its average rate per hour while running?

8. How much will it cost to fence my house-lot, which is 4 rods by 8, on two adjacent sides, if I pay $\$1\frac{3}{8}$ per foot for the fence, a rod being $16\frac{1}{2}$ feet?

9. If a ship sails at the rate of 130.75 miles per day, in what time will she make a trip of $69\frac{1}{3}$ miles?

10. If a man can dig $\frac{1}{4}$ of a rod of a certain ditch in $\frac{1}{2}$ an hour, how much can 3 men dig in $3\frac{1}{5}$ days, working $8\frac{1}{2}$ hours per day?

Bills of Goods.

180. **A Bill of Goods** is a written statement in proper form, which the seller gives to the buyer, specifying the amount and price of each article, and the aggregate value of the whole.

The person who buys is called *Debtor*, and the person who sells is called *Creditor*.

The letter @ made in this way is used in such bills for "at," and is followed by the price of a unit; as 1 yard, 1 pound, 1 gallon, etc.

Find the amount of each of the following Bills of Goods: —

1. ANN ARBOR, Dec. 1, 1874.

MR. JAMES SMITH

BOUGHT OF PHILIP BACH:

$16\frac{1}{4}$ yd. sheeting, @ 22¢	$3.58
$7\frac{3}{4}$ yd. flannel, @ $62\frac{1}{2}$¢	4.84
$\frac{1}{2}$ doz. hdkfs.,[1] @ $37\frac{1}{2}$¢	2.25
$2\frac{3}{4}$ yd. drilling, @ $15\frac{3}{4}$¢	.43
Received Payment,	$11.10

PHILIP BACH.

---

[1] Hdkfs. means handkerchiefs.

2. DETROIT, NOV. 25, 1874.

MR. J. B. ANGELL

BOUGHT OF S. C. JOHNSON:

5	cans oysters,	@ $37\frac{1}{2}$¢	. . . . . .
$19\frac{1}{4}$	lb. turkey,	@ $12\frac{1}{2}$¢	. . . . . .
20	lb. raisins,	@ $17\frac{3}{4}$¢	. . . . . . ____

Received Payment,

S. C. JOHNSON.

3. ANN ARBOR, Oct. 7, 1874.

MRS. A. E. PARKS

TO COLE & TREMAIN, *Dr.*[1]

For 10	lb. granulated sugar,	@ $11\frac{1}{2}$¢	. . .	
" 5	heads celery,	@ $7\frac{1}{2}$¢	. . .	
" $6\frac{3}{4}$	lb. butter,	@ $31\frac{1}{4}$¢	. . .	
" 1	lb. tea,	. . .	. . .	$1.40
" $2\frac{1}{2}$	lb. Mocha coffee,	@ $42\frac{3}{4}$¢	. . .	
" $10\frac{1}{4}$	lb. codfish,	@ $10\frac{1}{2}$¢	. . .	____
	Charged in Account		. . . .	

When an account at a store has been running some time, the merchant often makes a copy of it for the creditor upon settlement. The following is such a transcript. What was due on the account?

4. MR. AMOS WHITE

IN ACCOUNT WITH C. H. MILLEN & CO.

1874.	*Dr.*			
May 1.	To 10 yd. calico,	@ $8\frac{3}{4}$¢	. .	
	" 6 spools thread,	@ $6\frac{1}{4}$¢	. .	
	" $2\frac{1}{4}$ yd. sheeting,	@ $17\frac{3}{4}$¢	. .	
June 15.	To 3 yd. cassimere,	@ $\$2\frac{1}{4}$	. .	
	" trimmings, $1.25	. .	. .	
Aug. 21.	To 4 table-cloths,	@ $1.75	. .	
	" 3 prs. cotton hose,	@ 28¢	. .	____
	Cr.			
Sept. 1.	By cash .	. . . .	. . .	$2.00
	Due	. . . .		$

ANN ARBOR, Dec. 31, 1874.

[1] This form of bill-head means just the same as the preceding. Mrs. Parks is the *Debtor*, and Cole & Tremain are the *Creditor*.

181. ***Goods sold by the 100, or 1000, or the Ton.***

1. What is the amount of a bill of 5728 ft. of lumber at $37½ per thousand feet?

EXPLANATION. — If the lumber had cost $37½ per foot, the cost would have been $37½ × 5728, or $214700. At $37½ for 1000 feet the cost is 1000th part as much. Hence we divide $214700 by 1000, and have $214.70.

OPERATION.
```
    5728
     37½
    ----
    2864
   40096
  17184
--------
$214.700
```

182. Rule. — I. *To find the cost of articles sold by the 1000, multiply together the price per thousand and the number, — using the number or the price for multiplier, as is most convenient, — and then cut off 3 decimals from the product, or remove the decimal point 3 places to the left.*

II. *If the articles are sold by the 100, proceed in the same way; except that 2 instead of 3 figures are to be cut off as decimals, or the point removed 2 places to the left.*

III. *If the price is given per ton, and the quantity in pounds, proceed as if the price were per 1000, and divide the result by 2, since a ton is 2000 pounds.*

2. What is the cost of 72568 brick at $6.75 per M?[1]

3. What is the cost of 85670 lath at 23¢ per C?[1]

4. It is estimated that a certain public building will require 2 million bricks in its construction. What will these cost at $6½ per M?

5. I wish to put a board fence around a lot which is 10 rods by 20 (16½ feet make a rod). I find that for 1 panel (length) of fence (16 feet) it will take 1 board 1 foot wide, which measures 16 square feet; one 8 inches wide, which measures 10⅔ square feet; 4 boards each 5 inches wide, which together measure 26⅔ square feet. How much will the boards for the whole cost at $17¼ per M? Observe that I shall have to pay for boards for *whole* panels. They will not cut the boards for me at the lumber-yard.

[1] M stands for 1000, and C for 100.

6. If it takes 8750 shingles for the roof of my house, how much will they cost at $4.75 per M?

7. What is the cost of 117800 lath at $2.50 per M?

8. What is the cost of a load of hay weighing 1875 lb. at $12.50 per ton (2000 lb.)?

OPERATION.

```
      1875
       12½
     -----
     937.5
    22500
   -------
2 ) 23.437.5
   $11.72
```

9. A farmer brought me 5 loads of hay, which he had weighed at the public scales. They weighed 2785½ lb., 3056¾ lb., 2907¼ lb., 3000 lb., and 3172½ lb. each, including the wagon, which weighed 950 lb. What is the value of the hay at $15½ per ton?

10. What will a load of coal weighing 1725 lb. cost at $11½ per ton?

11. I paid a coal-dealer $9.92 for a load of coal weighing 1725 pounds. How much was that per ton?

12. I paid a coal-dealer $9.92 for a load of coal at $11½ per ton. How many pounds of coal should there have been?

13. What is the cost of 15500 lb. of plaster at $7.50 per ton?

14. A man paid $58.12½ for a car-load of plaster at $7½ per ton. How many pounds should there have been?

15. If 31000 lb. of plaster cost $116.25, what is that per ton?

16. If freight from New York to Detroit is 35½¢ per hundred, how much will be the transportation on 6 boxes of goods weighing severally 347⅜, 158¼, 527⅝, 250, 348⅛, and 629⅞ lb.?

17. If railroad freight on wheat from Detroit to New York is 37½ cents per 100 pounds, what is that per bushel of 60 pounds?

18. Nov. 25, 1874, freights by water from Milwaukee to Buffalo are quoted at 5½¢ (i.e., 5½¢ per bushel). What is this per 100 lb.?

19. Which would be better, to charter a vessel which would carry 1200 tons from Milwaukee to Buffalo for \$2500, or to pay freight at the rate of $5\frac{1}{2}$¢ per bushel of wheat?

20. What is the freight from Detroit to New York on a car-load of wheat (375 bushels) at $37\frac{1}{2}$¢ per 100 lb.?

21. Expressage from Ann Arbor to East Saginaw is now \$$1\frac{1}{4}$ per hundred. What will this add to the cost of butter per pound if I ship 6 crocks weighing in gross (i.e., crocks and all) $35\frac{1}{2}$ lb. each, the crocks weighing 4 lb. each, and the return expressage to be paid on the crocks?

22. This fall (1874), freight on wheat by vessel from Toledo, O., to Oswego, N.Y., is quoted at 14¢ (14¢ per bu.). What is the freight on a schooner's cargo of 1200 tons?

23. Which will cost the more, to ship 1000 bushels of wheat by water at $15\frac{1}{2}$ cents per bushel, or by railroad at $31\frac{1}{4}$¢ per 100 lb.? How much?

24. What ought eggs to be per pound when they are selling at $18\frac{3}{4}$¢ per doz, if they average $9\frac{1}{2}$ eggs to a pound?

25. If a compositor (one who sets type) receives 40¢ per 1000 ems for setting, how much will he receive for setting a book of 296 pages, which measures 1576 ems to a page?

26. What is the amount of a bill of lumber for 25371 ft. boards @ \$$37\frac{1}{2}$ per M, 1483 ft. scantling @ \$$18\frac{3}{4}$ per M, 21000 lath @ 21¢ per C, and 7342 shingles @ \$$6\frac{1}{2}$ per M?

27. Which is the higher freight on wheat, $37\frac{1}{2}$ cents per hundred, or $22\frac{1}{2}$ cents per bushel?

28. Four persons bought 640 acres of land, the first having .2 of it, the second .35, the third .15, and the fourth the remainder. How many acres did the fourth have?

29. At $62\frac{1}{2}$ cents per cord for cutting, how many months (26 days) will it take a chopper to earn \$100, if he cut $2\frac{1}{2}$ cords per day?

Denominate Numbers.

CHAPTER IV.

SECTION I.

DEFINITIONS AND TABLES.

183. An Abstract Number is a *mere* number; that is, a number not applied to any specified things. Thus *ten*, *seven*, 146, $\frac{2}{3}$, $4\frac{3}{4}$, are abstract numbers.

184. A Concrete Number is a number applied to some specified thing; as *ten* men, *seven* trees, 146 feet, $\frac{2}{3}$ of an acre, $4\frac{3}{4}$ pounds, etc.

185. Denominate Numbers means, literally, *Named Numbers;* but the term is applied only to concrete numbers which represent *money*, *weight*, or *measure*. Thus $5, 10 *lb.*, 3 *gal.*, are denominate numbers; but 5 *men*, 10 *trees*, 3 *stones*, are not so called.

186. In denominate numbers, the different *Orders*, as of money, weight, or measure, are called **Denominations.** Thus *dollars*, *dimes*, and *cents* are denominations of money; and *rods*, *feet*, and *inches* are denominations of measure.

187. A Compound Number consists of several related denominations written together, and to be read as one number. Thus 4 *gal.* 2 *qt.* 1 *pt.* is a compound number; so also is 6 *mi.* 25 *rd.* 10 *ft.*

MEASURES OF VALUE.

188. **Money** is the measure of value used for purposes of buying and selling.

189. **Coins** are pieces of metal bearing the government stamp, and having a value fixed by law. Coin is also called *Specie.* (See Appendix III. for cuts of coins in most common use.)

190. **Currency,** as a business term, means notes or bills issued by the government or by banks to be used as money.

FEDERAL OR UNITED-STATES MONEY.

191. The **Denominations** of United-States money are eagles, dollars, dimes, cents, and mills; the abbreviations indicating each being, respectively, E., \$, d., ¢ or ct., and m. For ordinary purposes, amounts are mentioned in dollars and cents.

192. **Table.** 10*m.* = 1¢, 10¢ = 1d., 10d. = \$1, \$10 = 1 eagle. Hence we see that United-States money is decimal.

193. The coins of the United States at present struck (1879) are, of *Gold*, \$1, \$2½, \$3, \$5, \$10, and \$20 pieces; of *Silver*, dime, quarter-dollar, half-dollar, and dollar pieces; of *Nickel*, 5-cent and 3-cent pieces; of *Bronze*, a 1-cent piece.

194. The *Trade Dollar*, 20-cent, 5-cent, and 3-cent pieces, of silver, and the copper 2-cent and 1-cent pieces, though found in circulation, are not now coined. The mill was never coined.

195. None of the present United-States coinage is pure metal. The (so-called) *gold* is .9 gold and .1 copper and silver; the *silver* is .9 silver and .1 copper; the *nickel* is $\frac{3}{4}$ *copper and* $\frac{1}{4}$ nickel; the *bronze* is .95 bronze and .05 tin *and zinc.*

196. The **Legal Weights** are, the *gold dollar* 25.8 grains, and the other gold coins in proportion. The legal *silver dollar* weighs 412½ grains, and the *trade dollar* 420 grains. The subsidiary silver coins are on the basis of 385.8 grains to a dollar. The *nickel* 5-cent piece weighs 5 grams (metric weight) or 77.16 grains, and the 3-cent piece 30 grains. The *bronze* cent weighs 48 grains. When gold coin comes into the United-States Treasury reduced in weight more than .005, it is recoined.

MONEY OF FOREIGN COUNTRIES.

197. The Denominations and Values of **Canada Coins** are similar to those of the United States; and Canada currency is reckoned in dollars and cents, though reckoning in English currency is still common.

198. The Denominations of **English Money** are pounds, shillings, pence, and farthings, represented respectively by £, *s.*, *d.*, and *far.*

The *gold coin,* whose value is £1, is called a *sovereign* (*sov.*). Its value in United-States gold is $4.8665. From this value and the following table the pupil will be able to calculate the value of any English coin.

199. Table. 4 *far.* = 1*d.*, 12*d.* = 1*s.*, 20*s.* = £1.

A *guinea* is a gold coin equal to 21*s.* A *crown* is a silver coin equal to 5*s.* A *florin* is a silver 2-shilling piece.

200. The **French Coins** most frequently mentioned in this country are the *franc* (*frank*), the *Napoleon*, and the *centime* (*centeem′*).

The *centime* is $\frac{1}{100}$ of a franc, just as our cent is $\frac{1}{100}$ of a dollar. A *Napoleon* is a 20-franc gold piece. The franc is silver, and the centime bronze. The abbreviation for francs is ^{fr}, and centimes are written as decimals; thus 5^{fr} .17 is 5 francs and 17 centimes. A *franc* = 19.3¢ United-States gold.

201. The standard Denominations of **German Coins** are *the mark* (23.8 *cents*), and *the pfennig* $= \frac{1}{100}$ of a mark.

The Prussian *silver thaler* (74.6¢) and the *silver groschen* ($2\frac{1}{2}$¢) are coins frequently referred to in this country, the mark and pfennig having been made the standards in 1872.

Examples.

1. What is the value in our coin of an English shilling? Of a penny? Of a farthing? Of a crown? A guinea?

2. What is £5 12*s*. in dollars and cents?

3. What English coin is very nearly a half-dollar? What is very nearly a half-eagle?

4. What is 10*s*. 6*d*. in our coin? What 5*s*. 10*d*.?

5. What is the value in our coin of 1 centime? Of a Napoleon?

6. How near is a 5-franc piece to $1?

7. The French *gold* coins are 100, 40, 20, 10, and 5 franc pieces. How much is each in our coin?

8. The French *silver* coins are 5, 2, and 1 franc pieces, and 50 and 25 centime (i.e., half and quarter francs) pieces. How much is each in our coin?

9. The *bronze* French coins are 10, 5, 2, and 1 centime pieces. How much is each in our coin?

10. 1000000^{fr} are how many dollars and cents? $25^{fr}.37$? $15^{fr}.75$? $258^{fr}.60$? 100^{Nap}? $550 = how many francs?

11. The German *gold* coins are 5-mark, 10-mark, and 20-mark pieces. What is the value of each in our coin?

12. The German *silver* coins are 20-pfennig, 1-mark, 2-mark, and 5-mark pieces. What is the value of each in our coin?

13. The German *copper* coins are 5-pfennig and 10-pfennig pieces. What is the value of each in our coin? *How many* pfennigs make a cent?

14. \$100 gold = how many marks?

15. How many pfennigs make $\frac{1}{4}$ of a dollar?

16. How many marks make \$1 gold?

202. Chicago **Quotations of Foreign Exchange** are to-day: "London $4.88\frac{1}{2}$, Paris $5.15\frac{5}{8}$, Hamburg $95\frac{3}{4}$." This means that you can buy a *Draft* (i.e., an order) on a bank in either of these foreign cities at these rates. The "London $4.88\frac{1}{2}$" means that you will have to pay \$$4.88\frac{1}{2}$ for £1. The "Paris $5.15\frac{5}{8}$" means that you will have to pay at the rate of \$1 for $5^{fr}.15\frac{5}{8}$. The "Hamburg $95\frac{3}{4}$" means that you will have to pay at the rate of $95\frac{3}{4}$¢ for 4 marks.

17. According to the above quotations, how much will a draft on Hamburg cost me which will pay for a book which is offered me there at 175 *M.*?

18. At the above rates, what will a draft on London cost which will pay a bill there of £125 12*s.*? On Paris, to pay a bill of 1000^{fr}?

MEASURES OF EXTENSION.

203. A Point is a place without size.

Points are designated by letters; as D, C, M, N, etc.

204. A Line is the path of a point in motion.

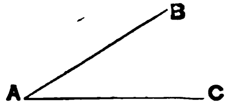
A line is designated by letters placed at its ends; as the line DC, the line MN, etc.

205. A Straight Line is the path of a point moving all the time in the same direction. Generally, when we say "Line," we mean a "straight line."

A *Line*, or *Distance*, is *measured* by another line or distance.

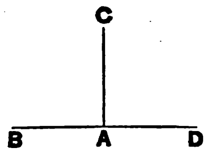
The lines, or distances, commonly used in measuring length, or distance, are an *Inch*, a *Foot*, a *Yard*, a *Rod*, and a *Mile*.

206. An Angle is the opening between two lines which meet. The point where the lines meet is called the **Vertex** of the angle.

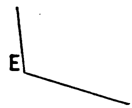


In common language we call an angle a *Corner*. Thus the corner or opening between the two lines BA and CA is the angle BAC. In designating an angle we use a letter placed at the vertex, or three letters, one on each of the lines, with the letter which stands at the vertex named between them. Thus the angle above may be spoken of as the "angle A," or the "angle BAC."

207. When one straight line, as CA, meets another, as BD, so as to make the angles CAB and CAD equal (i.e., just alike), the angles are called **Right Angles.** A right angle is a *square corner*.



208. An Acute Angle is an angle which is less than a right angle, and an **Obtuse Angle** is one which is greater than a right angle.



For the method of measuring angles, see *Circular Measure*.

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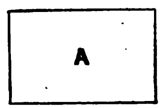
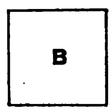
209. A space which has length and breadth, but not thickness, is called a **Surface.**

210. A Rectangle is a plane (flat) surface or figure bounded by four straight lines, and having all its angles right angles. A is a rectangle.



211. A Square is a rectangle having all its sides equal each to each. B is a square.



A Surface is measured by another surface, — usually by a square.

The **Area** of a surface is the number of times it contains the measure.

The surfaces commonly used for measuring surface are the *Square Inch, Square Foot, Square Yard, Square Rod, Square Mile,* and the *Acre.*

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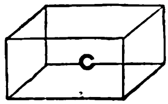
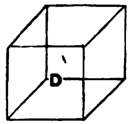
212. A Solid, or Body, is a space having length, breadth, and thickness.



213. A Right Parallelopiped is a solid, or space, bounded by six rectangles, and having all its angles right angles. C represents a parallelopiped.



214. A Cube is a parallelopiped having all its faces (sides) squares. D represents a cube. A cube, each of whose edges is 1 inch, is a *Cubic Inch;* one whose edges are each 1 foot is a *Cubic Foot;* one whose edges are each 1 yard, a *Cubic Yard,* etc.

A *Solid is measured* by another solid, — usually by a cube. The **Volume** or **Contents** of a solid is the number of times it contains the measure.

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215. Principle. — *The* **Area** *of a* **Rectangle** *is the product of its length by its width.*

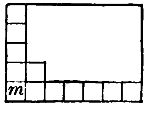

ILLUSTRATION. — If surface A is 7 long and 5 wide, and m, the measure, is 1 on a side, we can apply 7 of the measures along one side of the surface, as from B to C, making a row of 7 measures. Now, we can apply the measure so as to make as many such rows as there are units in the width, — in this case 5; and 5 times 7 measures (m) make 35 measures. Hence the area of A is 35, m being the measure.

1. A square foot is 12 inches on a side. How many square inches in a square foot?

2. A square yard is 3 feet on a side. How many square feet in a square yard?

3. Show that there are $30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet, in a square rod.

4. Show that there are 102400 square rods in a square mile. 160 square rods make an acre. How many acres in a square mile?

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216. Principle. — *The* **Volume** *of a* **Right Parallelopiped** *is the product of its three dimensions; that is, of its length, breadth, and thickness.*

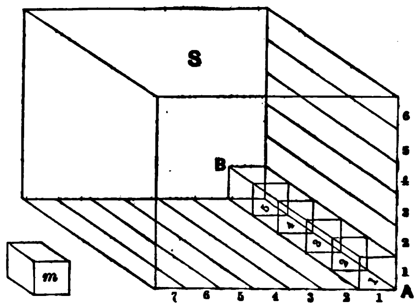

ILLUSTRATION. — If solid **S** is 7 long, 5 wide, and 6 high, and *m*, the measure, is a cube 1 on each edge, we can place 5 of the measures along one side of the bottom (base) of the parallelopiped **S**, as from **A** to **B**, and 7 such rows will cover the base. Thus it takes 7 × 5 of the measures to cover the base 1 deep. Then, as the parallelopiped is 6 high, it will take 6 such layers, or 7 × 5 × 6, to fill the whole space. Hence the volume of the parallelopiped **S** is 7 × 5 × 6, or 210, *m* being the measure.

1. How many cubic feet in a cubic yard?

2. How many cubic feet in a parallelopiped 8 feet long, 4 feet wide, and 4 feet high? This is called a *cord* of wood.

3. When 4 ft. wood is piled 4 ft. high, 1 ft. in length of pile is called a *cord foot*. How many cubic feet in a cord foot? How many cord feet in a cord?

4. How many cubic inches in a cubic foot?

217. Abbreviations. — *in.* stands for inch or inches, *ft.* for foot or feet, *yd.* for yard or yards, *rd.* for rod or rods,

mi. for mile or miles, *sq.* for square, *cu.* for cubic, *A.* for acre, *cd.* for cord.

218. ***Tables of Measures of Extension.***

Long Measure.	Square Measure.	Cubic Measure.
12 in. = 1 ft. ∴	144 sq. in. = 1 sq. ft., and	1728 cu. in. = 1 cu. ft.
3 ft. = 1 yd. ∴	9 sq. ft. = 1 sq. yd., and	27 cu. ft. = 1 cu. yd.
5½ yd. = 1 rd. ∴	30¼ sq. yd. = 1 sq. rd.	16 cu. ft. = 1 cd. ft.
320 rd. = 1 mi.	160 sq. rd. = 1 A.	8 cd. ft. = 1 cd.
	640 A. = 1 sq. mi.	128 cu. ft. = 1 cd.

219. On the common carpenter's square the inch is usually divided into *halves, quarters, eighths*, and *sixteenths*, or into *twelfths*. On other scales it is often divided into *tenths*. A *line* is $\frac{1}{12}$ of an inch.

A *size*, as used by shoemakers, is ⅓ of an inch, sometimes called a *barleycorn*. *Children's sizes* run from *size* 1, 4⅝ in. long, to *size* 13, 8⅝ in. long. *Youth's, women's*, and *men's sizes* run from *size* 1, 8⅞ in., to *size* 15, 13⅝ in.

The ancient *Roman mile* (*mille passum*, 1000 paces) was about 1618 yd., and hence a little shorter than ours. The modern *Roman mile* = .925 *Eng. mi.* The *Irish mile* = 1.273 *Eng. mi.* The *French mile* (*mille marin*) is the same as our marine or geographic mile. The *German short mile* (*meile*) = 3.897 *Eng. mi.*; the *long mile* = 5.753 *Eng. mi.*; the *Prussian mile* = 4.68 *Eng. mi.*

A *geographic, nautical, or marine mile* is 1′ of the equator or of a meridian, and hence is 1.1527 *Eng. mi.* very nearly; a degree being 69.164+ *Eng.* or *statute miles.* A *league* is 3 marine miles. A *furlong* is ⅛ mi. See table of *Circular Measure.*

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220. Cloth, ribbons, laces, etc., are sold by the yard in length, irrespective of the width; the differing widths being considered in fixing the price. For such measurements the yard is divided into halves, quarters, and eighths.

Ex. How many inches in half a yard? How many in a quarter? An eighth? A sixteenth?

221. Sea-depths are measured in fathoms. A **Fathom** is 6 feet.

1. How many feet in a mile? How many fathoms in a mile? In 3 miles?

2. At a certain place the sea was reported as 900 fathoms deep. How much more than a mile deep was it?

3. How many miles in depth is that place in the Atlantic Ocean which is reported as 2640 fathoms deep?

4. In our Western forests many of the trees are 100 feet high. How many fathoms deep would a lake be which would submerge these standing forests?

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222. A **Hand** is 4 in., used in measuring the height of horses.

1. How many feet high is a horse which measures $15\frac{1}{2}$ hands? 17 hands? 18? 12?

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223. A **Pace** is reckoned at 3 ft., although in pacing long distances 5 paces are reckoned a rod.

Ex. How many paces in a mile?

224. For measuring land, surveyors use a *Chain* 4 rods long, and made of 100 links of equal length.

1. How many feet in 4 rods? How many inches in 66 feet? Then how long is 1 link of the surveyor's chain?

2. How many rods in a mile? Then how many chains in a mile?

225. The *Public Lands* of the United States which have been surveyed during the present century have been laid out in **Townships**, which are squares 6 miles on a side. These are divided in what are called **Sections**. A section is a square mile.

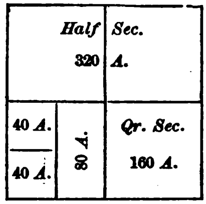
3. How many sections in a township?

4. How many acres in a section? In a half-section? A *quarter*-section? An eighth of a section?

5. What part of a section is 320 acres? 240? 160? 120? 80? 40?

6. How many 80-acre lots in a section? A half-section? A quarter-section?

7. This figure represents the way in which a section is usually divided. How long and how wide is a quarter-section, or 160 A.? 80 A.? 40 A.?



[*See Appendix* for the method of our Public Land Surveys; also the section on *Mensuration* for applications to the problems of measuring surfaces and solids; as boards, wood, etc.]

MEASURES OF CAPACITY.

226. The Capacity of a vessel is the amount which it contains, — its *contents.*

There are two varieties of measures of capacity in common use; viz., *Liquid Measure* and *Dry Measure.*

227. Liquid Measure is used in measuring liquids, or in estimating the capacity of vessels designed to contain liquids, as water, milk, oil, molasses, alcohol, etc. The *Denominations*, or measures, are gills (*gi.*), pints (*pt.*), quarts (*qt.*), gallons (*gal.*), and barrels (*bbl.*).

228. Dry Measure is used in measuring grain, seeds, fruit, etc., or in estimating the capacity of vessels designed to contain such articles. The denominations are pint (*pt.*), quart (*qt.*), peck (*pk.*), and bushel (*bu.*).

229. The denominations of like name in these two measures do not represent the same amounts. The *Dry pint* and *quart* are about $\frac{1}{7}$ larger than the *Liquid pint* and *quart;* or, more exactly, the *Dry quart* contains 67.2 cubic inches, and the *Liquid quart* 57.75 cubic inches.

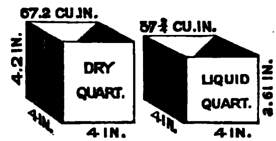


ILLUSTRATION. — These two figures represent boxes, each of which is 4 inches square on the bottom; but the *Dry quart* is 4.2 inches deep, while the *Liquid quart* is but 3.61 inches deep.

1. 4 quarts make a gallon. How many cubic inches make a gallon?

2. 8 quarts make a peck, and 4 pecks make a bushel. How many cubic inches in a bushel?

3. Which is the most, $1\frac{1}{4}$ cu. ft., or 1 bu.?

ANS., The difference is less than $\frac{1}{3}$ pt. ($\frac{2}{7}$).

230. *Tables of Measures of Capacity.*

LIQUID MEASURE.	DRY MEASURE.
4 gi. = 1 pt.	
2 pt. = 1 qt.	2 pt. = 1 qt.
4 qt. = 1 gal.	8 qt. = 1 pk., or 2 gal.
$31\frac{1}{2}$ gal. = 1 bbl.	4 pk. = 1 bu.

Barrels are made of various sizes, from 30 to 40 or even 56 gallons; but in estimating the capacity of cisterns, vats, etc., $31\frac{1}{2}$ gal. is usually considered a barrel. There is no definite measure in use called a hogshead. Any large cask is freqently so called.

231. Physicians and apothecaries use a kind of liquid measure, of which the denominations are *Minims* (♏), *Fluid Drachms* (f ʒ), *Fluid Ounces* (f ℥), *Pints*, and *Gallons*. The pint and gallon are the same as the common *Liquid Pint* and *Gallon*, but are designated by the abbreviations (O.) (Latin *octarius*, pint), and *Cong.* (Latin *congius*, gallon). 60 ♏ = 1 f ʒ, 8 f ʒ = 1 f ℥, and 16 f ℥ = 1 (O.).

Physicians in making prescriptions frequently call a minim a *drop*, a fluid drachm a *teaspoonful*, 4 fluid drachms a *tablespoonful*, a fluid ounce 2 *tablespoonfuls*, 4 fluid ounces a *teacupful*, and a pint 4 *teacupfuls*.

These measures are very indefinite, and in fact are much in excess of what they are called. Thus a drop of most liquids is much more than a *minim*. A common teaspoon holds nearer 90 than 60 drops of water, and we more frequently find teacups that hold $\frac{1}{2}$ a pint than a *gill*.

℞ is an abbreviation for *recipe*, or take; ā, aa, for equal quantities; *ss.*, for *semi*, or half; gr., for grain; *gtt.*, for drop; P., for *particula*, or little part; P. æq., for equal parts; q. p., as much as you please.

1. How many pints in a gallon? In 5 gallons? 7 gallons? A barrel?

2. How many quarts in a barrel? In 5 bbl.? In 4 bbl.?

3. A box which would hold a cord of wood would hold how many bushels of wheat? How many barrels of water?

4. How many bushels in a barrel of 31½ gallons?

5. It is customary to heap the measure in measuring apples, potatoes, corn in the ear, ashes, and some other substances; so that about 5 pecks are sold as a bushel. According to this method of measuring, how many bushels does a barrel of 31½ gal. contain?

6. How many drops make a teaspoonful? How many teaspoonfuls make a teacupful?

7. A man sold me 32 quarts, which he called a bushel of berries; but he measured them in a liquid-quart measure, instead of a dry-quart measure. What part of a bushel did I get? How many liquid quarts should he have given me?

[For further practical applications, see *Mensuration.*]

MEASURES OF WEIGHT.

232. There are *Three Varieties* of measures of weight; viz., *Avoirdupois*,[1] *Troy*,[2] and *Apothecaries'*.

233. Avoirdupois Weight is the common weight used by grocers, and for most ordinary purposes. The denominations are *Ounces* (*oz.*), *Pounds* (*lb.*), *Hundred Weight* (*cwt.*), and *Tons* (*T.*).

234. Troy Weight is the weight used for weighing gold,

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[1] From three French words, *avoir du poids*, meaning "to have weight."

[2] From *Troyes* in France, whence this weight came into use.

silver, and precious stones, and in philosophical experiments. The denominations are *Grains* (*gr.*), *Pennyweights* (*pwt.*), *Ounces* (*oz.*), and *Pounds* (*lb.*).

235. Apothecaries' Weight is used in medical prescriptions. The denominations are *Grains* (*gr.*), *Scruples* (℈), *Drams* (ʒ), *Ounces* (℥), and *Pounds* (*lb.*). Drugs and medicines are bought and sold in the quantity by Avoirdupois Weight.

236. ***Tables of Measures of Weight.***

AVOIRDUPOIS WEIGHT.

16 oz.	= 1 lb.
100 lb.	= 1 cwt.
20 cwt.	= 1 T.

TROY WEIGHT.

24 gr.	= 1 pwt.
20 pwt.	= 1 oz.

APOTHECARIES' WEIGHT.

20 gr.	= 1 sc., or ℈.
3 ℈	= 1 dr., or ʒ.
8 ʒ	= 1 oz., or ℥.

12 oz. = 1 lb., or ℔.

237. The denominations of like name in troy and apothecaries' weights are the same; but the avoirdupois pound is heavier, whereas the ounce is lighter, than the pound and ounce of the other weights. The troy pound is 22.79 cu. in. of water; and the avoirdupois pound is $\frac{175}{144}$ times as much, or 27.69 cu. in. of water. A pint of water ($28\frac{7}{8}$ cu. in.) is a little more than 1 lb. avoirdupois.

The only difference between the troy and the apothecaries' tables is in the subdivision of the ounce. In troy weight there are two subdivisions, — pennyweights and grains; whereas in apothecaries' there are three, — drams, scruples, and grains: but, in each, 480 grains make an ounce. 7000 troy grains are equal to a pound avoirdupois.

238. Physicians in writing prescriptions use the Roman notation to designate the number of grains, scruples, etc., and write them *after* the symbol. In writing thus, a final i is written j; thus gr. vj is 6 *grains*, ℈ ij is 2 scruples, ʒ iij is 3 drams, ℥ iv is 4 ounces, etc.

239. In wholesale transactions in coal and iron, and in the United-States custom-houses, 112 lb. are called a cwt.: hence 28 lb. is a *Quarter* (of a cwt.), and 2240 lb. make a *Ton*. This is sometimes called the *Long Ton*.

240. 196 lb. *flour* make a barrel: 200 lb. *pork* or *beef* make a barrel. 56 lb. *butter* = 1 firkin. 100 lb. *grain* or *flour* are called a *Cental*, of *dried fish* a *Quintal*, and of *nails* a *Keg*. 280 lb. *salt* = 1 barrel at New-York Salt-Works. A *bushel* of coal is 80 lbs.

241. ***Table showing the Weight of a Bushel of the Principal Grains and Seeds, as established by Law in the Several States.***

Barley.	Ill., Ind., Io., Ky., Mich., Minn., Mo., N. C., N. J., Ohio, Wis., **48** lb. Mass., Ore., Vt., 46 lb.; W. T., 45 lb.; La., 32 lb.; Penn., 47 lb.; Cal., 50 lb.
Buck-wheat.	Mich., Minn., Ore., Wis., 42 lb.; Io., Ill., Ky., Mo., 52 lb.; Ind., N. C., N. J., 50 lb. Cal., 40 lb.; Mass., Vt., 46 lb.; N. Y., Penn., 48 lb.; Conn., 45 lb.
Clover Seed.	Ill., Ind., Io., Ky., Mich., Minn., Mo., N. Y., Ohio, Ore., W. T., Wis., **60** lb. N. J., 64 lb.
Indian Corn.	Conn., Del., Ind., Io., Ill., Ky., La., Mass., Mich., Minn., N. J., Ohio, Ore., Penn., Vt., W. T., Wis., **56** lb. Cal., Mo., 52 lb.; N. C., 54 lb.; N. Y., 58 lb.
Oats.	Cal., Ill., Ind., La., Mich., Minn., N. Y., Ohio, Penn., Vt., Wis., **32** lb. Me., Mass., N. C., N. H., N. J., 30 lb.; Io., Mo., 35 lb.; W. T., 36 lb.; Conn., 28 lb.; Ky., 100 lb. to 3 bu.
Rye.	Conn., Ind., Io., Ill., Ky., Mass., Mich., Minn., Mo., N. J., N. Y., Ohio, Ore., Penn., Vt., W. T., Wis., **56** lb. Cal., 54 lb.; La., 52 lb.
Timothy Seed.	Ill., Ind., Io., Ky., Mo., **45** lb. N. Y., 44 lb.; Wis., 46 lb.
Wheat.	**60** lb. in all except Conn. In Conn., 56 lb.

Peas, *Beans*, and *Potatoes* are usually weighed at 60 lb. to the bushel.

1. On the common grocers' scales, what part of a pound is 8 oz.? 4 oz.? 12 oz.?

2. What is the difference in weight between 3 oz. of butter and 3 oz. of gold, each being weighed by its proper weight?

3. How many bushels in a ton of wheat, Michigan standard?

4. On a road when 1 T. would be a fair load for a span of horses, how many barrels of salt ought a man to put on as a load? How many of flour?

5. How much does a barrel of water weigh, if 1 pt. weighs 1 lb.? How much a gallon?

6. In a certain hay-field there were 1250 heaps, which would average 85 lb. each. How many tons in the field?

7. How many barrels of pork will be cut from 120 hogs which average 175 lb. each?

8. How many barrels of flour will be made from 2160 bu. of wheat, if it yield 40 lb. to the bushel?

9. How many scruples in 10 drams? How many grains in ℈ vij? In ʒ v? In ℥ iij?

10. A man ordered a 3-oz. gold watch-case; but, when it came, it weighed only 57 pwt. How much did it fall short of the required weight?

11. How much will a gold watch-case weighing $2\frac{1}{2}$ oz. cost, at $0.90 per pwt. and $20 for making?

12. How many bushels of coal in a ton as weighed at the mines?

13. At $6.00 per ton, what is coal per bushel?

14. The specific gravity of ice being .93 (i.e., it being .93 times as heavy as water), how many cubic feet in a ton of ice?

MEASURES OF TIME.

242. The denominations of time are *Seconds* (*sec.*), *Minutes* (*min.*), *Hours* (*hr.*), *Days* (*da.*), *Weeks* (*wk.*), *Months* (*mo.*), *Years* (*yr.*), and *Centuries* (*cen.*).

243. *Table.*

60 sec. = 1 min.	365 da. = 1 common yr.
60 min. = 1 hr.	366 da. = 1 leap yr.
24 hr. = 1 da.	100 yr. = 1 cen.
7 da. = 1 wk.	

In *Computing Interest,* 30 da. = 1 mo. For many purposes, 4 wk. *are called* a month.

Of the 12 calendar months which make up the year, September, April, June, and November have 30 da. each. All the others, except February, have 31 da. each. In common years February has 28 da., in leap year 29 da. What two exceptions are there to the law that each alternate month has 31 days?

244. Every year whose number is divisible by 4, except the centennial years which are not divisible by 400, is a **Leap Year.**

Thus 1840, 1844, 1880, 1876, 1600, 1200, 2000, are leap years. 1551, 1842, 1883, 1500, 1100, 1900, are not leap years.

The reason for *Leap Year* is this: A year is the time it takes the earth to go around the sun. But this is a little more than 365 days. Instead of reckoning this part of a day, it is neglected, and a *whole* day is added to the year every 4th year (in general); but, as this is a little too much, the centennial years (in general), although they are the 4th years, are reckoned as common years (365 da.). But this again is rejecting too many leap years: so that every centennial year which is divisible by 400 is made a leap year. With this correction the error does not amount to a day in 100000 years.

1. How many seconds in $\frac{1}{2}$ a minute? In $\frac{1}{4}$ of a minute? In 2 min.? In 10 min.? In $2\frac{1}{2}$ min.?

2. How many minutes in $\frac{1}{2}$ an hour? In $\frac{1}{4}$ an hour? In $\frac{3}{4}$ an hour? In $1\frac{1}{2}$ hr.? In 3 hr.? In $5\frac{1}{4}$ hr.?

3. What part of a day is 12 hr.? 6 hr.? 1 hr.? 5 hr.?

4. How many days in 48 hr.? In 36 hr.? In 80 hr.?

5. Point out the leap years in the following: 1540, 1320, 1000, 800, 560, 2100, 2290, 2400, 1875, 1876, 1877, 1878, 1879, 1880, 1881, 1882, 1883, 1890.

Important Facts concerning Dates.

245. Let the pupil verify the following: —

(*a*) A *Common Year* has 52 weeks and 1 day.

A *Leap Year* has 52 weeks and 2 days.

(*b*) *Any Particular Date*, as July 4, falls one day later in the week in a year succeeding a *Common Year*, and 2 days later in a year succeeding *Leap Year*.

(*c*) *To find from the present date the day of the week of the same day in any other month of the same year.* — Take the sum of the excesses over 28 da. of the days in each month ending in the interval. Divide this sum by 7, and, if the date be future, *advance* the day of the week by this remainder; if past, move the day of the week *back*.

(*d*) *To find from the present date the day of the week of any preceding or succeeding date.* — Reckon a day for each year closing in the interval, adding 1 da. more for each leap year. Divide the sum by 7, and, if the date be *future*, *advance* the day of the week by the remainder; if past, move the day of the week *back* by the same number. This will give the day of the week of the present date in the required year. Then pass to the required date in that year, as in the preceding.

(*e*) *To find the number of leap years that have ended in any given period.* — Divide by 4 the number of years that have ended in the time: *in general*, this quotient will be the number of leap years. But if the 1st year ending in the period is leap year, and the remainder is 1, 2, or 3; or if the 2d year is leap year, and the remainder is 2 or 3; or if the 3d year is leap year, and the remainder is 3, — add 1 to the quotient.

(*f*) It of course amounts to the same thing to advance the date of a particular day of the week as to keep the same date and throw the day of the week back, etc. This is often more convenient in practice.

Ex. 1. — To-day (April 16, 1879) is Wednesday. What day of the week was July 24, 1827?

Between dates 52 years have ended, 13 of which were leap years. $(52+13)\div 7=9$ and 2 remainder. Hence April 16, 1827, was Monday. Now, the excesses of 28 in the months ending between this and July 24 is $2+3+2=7$. Hence the days of the week fall on the same days of the month in July as in April. July 16, 1827, was therefore Monday, and July 24 Tuesday.

2. Find from your present date that July 4, 1776, was Thursday.

3. On what day of the week were you born?

4. On what day of the week will next Christmas fall? The 4th of July in the year 2000? (Find these from your present date.)

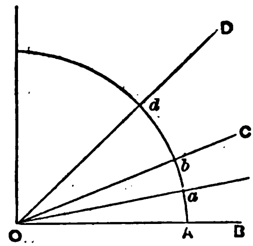
5. Show that the days of the week fall on the same days

of the month in January as in October, in March as in November, in April as in July, in September as in December.

6. If May, June, or Aug. 1, of a common year, falls on Sunday, show that no other month of that year begins on Sunday. How is this in leap year?

CIRCULAR OR ANGULAR MEASURE.

246. A Degree is $\frac{1}{360}$ part of the circumference of a circle, or an angle measured by this part of a circumference, and is designated thus, 1°.

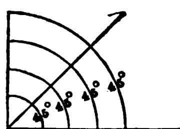


In the figure from **A** to *a* is 10 degrees, written 10°. From **A** to *b* is 20°, and the angle **BOC** is an angle of 20°. From **A** to *d* is 45°, and the angle **BOD** is an angle of 45°. A quarter of a circumference, or a right angle, is 90°.

247. A Minute (as circular or angular measure) is $\frac{1}{60}$ part of a degree, and is designated thus, 1′. 5 minutes of this measure is written 5′.

248. A Second (as circular or angular measure) is $\frac{1}{60}$ part of a minute, and is written 1″. 12 seconds is written 12″.

It will be observed that a degree, *circular* measure, is not a definite length: thus a degree on the circumference of the earth is about $69\frac{1}{6}$ miles, while a degree on such a circle as the one in the last figure is so small that we could not see it. But a degree as the measure of an angle is a definite thing. Thus, if we draw an angle equal to $\frac{1}{2}$ a right angle, and then strike a circumference with the vertex of the angle as a centre, there will be 45° of the circumference included between its sides, whether the circle be large or small.



Angle of 45°

249. ***Table of Circular or Angular Measure.***

$60'' = 1'$, $60' = 1°$, $90° = 1$ Quadrant or Right Angle.

250. A *Geographic* or *Nautical* (marine) mile is $1'$ of the equator, or about $1\frac{1}{6}$ common miles.

251. Degrees of Latitude being distances north or south of the equator, and measured on equal circles (great circles which pass through the poles), are nearly equal;[1] but degrees of Longitude vary from $69\frac{1}{6}$ miles at the equator to 0 at the poles. In the latitude of Ann Arbor, Mich., a degree of longitude is 51.1 miles. At 60° latitude, a degree of longitude is 34.53 miles.

1. How many minutes in 2°? In $1\frac{1}{3}$°? In 10°? In .5 of a degree? In .3?

2. How many degrees in 240′? In 580′? In 200′? In 1260′ how many degrees and decimals of a degree?

3. How many seconds in $\frac{1}{2}$ a minute? In .75 of a minute? In .25 of a minute?

4. On a circumference which is 1800 ft. in length (around), how long is a degree? A minute? A second?

5. The circumference of a circle is 3.1416 times its diameter (nearly). Show that a degree on the circumference of the earth is about $69\frac{1}{6}$ miles, the radius of the earth being 3962 miles.

Miscellaneous.

252. Of Paper 24 *Sheets* are called a *Quire*.

24 *Sheets*	are	called	a *Quire*.
20 *Quires*	"	"	a *Ream*.
2 *Reams*	"	"	a *Bundle*.
5 *Bundles*	"	"	a *Bale*.

[1] In consequence of the flattening of the earth at the poles, degrees of latitude are not absolutely equal. A degree of latitude at any place is $\frac{1}{360}$ part of the circumference of a circle which has the same curvature as the meridian at that place.

253. In Counting, 12 *Things* are called a *Dozen.*

12 *Dozens*	"	"	a *Gross.*
12 *Gross*	"	"	a *Great Gross.*
20 *Things*	"	"	a *Score.*

254. Two things are often called a *pair*, and six things a *set;* as a *pair* of birds, a *set* of spoons.

255. The terms *folio, quarto, octavo,* applied to books, indicate the number of leaves into which a sheet of paper is folded. Thus, when a sheet of paper is folded into 2, 4, 8, 12, 16, 18, or 24 leaves, the forms are called respectively, folio, 4to or quarto, 8vo or octavo, 12mo, 16mo, 18mo, and 24mo.

LONGITUDE AND TIME.

256. As the sun appears to pass around the earth from east to west, and as the hours of the day are determined by its apparent position, any given hour comes to a place at the east *before* it does to a place at the west: thus it is noon at New York *before* it is noon at Detroit, Mich. Hence, when it is noon at Detroit, it is *after* noon at New York. In like manner, when it is noon at Detroit, it is *before* noon at Omaha, Neb.

257. Principle. — *When it is any given hour of the day at any given place, it is Later at places Eastward, and Earlier at places Westward.*

258. To ascertain just how great this *Difference in Time* is, we have only to consider, that, as the sun appears to go around the earth — that is, to pass over 360° of longitude — in 24 hours, in 1 hr. it passes over $360 \div 24$, or 15° of longitude. Then 1° of longitude makes a difference of $\frac{1}{15}$ of 1 hr. (60 min.), or 4 min. in time; and 1′ of longitude makes a difference of $\frac{1}{60}$ of 4 min. (240 sec.) or 4 sec. in time.

259. ***Table of Longitude and Time.***

15° longitude make 1 hr. diff. in time.
1° longitude makes 4 min. diff. in time.
1′ longitude makes 4 sec. diff. in time.

260. Rule. — I. ***To change longitude to time, multiply the degrees, minutes, and seconds by 4, calling the product minutes, seconds, and 60ths of a second.***

II. ***To change time to longitude, express the time in minutes and seconds, and, dividing by 4, call the quotient degrees and minutes.***

The reason for this is apparent; since 1° corresponds to 4 min. of time, 1′ to 4 sec. of time, and 1″ is $\frac{1}{60}$ of 1′.

1. Adrian, Mich., is in 84° west longitude; and Fort Kearney, Neb., is in 99° west longitude. When it is 9 o'clock A.M. at Adrian, what time is it at Fort Kearney? When it is noon at Fort Kearney, what time is it at Adrian?

How many degrees *west* of Adrian is Fort Kearney?

2. New-York City is in longitude 74° west, and San Francisco in 122½° west. What is the difference in time? When it is 6 A.M. at New York, what time is it at San Francisco? When it is 4 P.M. at San Francisco, what time is it at New York?

3. In the latitude of Ann Arbor, Mich., 51.1 miles make a degree of longitude. Detroit is 38 mi. east of Ann Arbor. What is the difference in time?

4. What is the difference in longitude between two places whose difference in time is ½ an hour?

5. What is the difference in longitude between two places whose difference in time is 3½ hr.? 2½ min.? 2 hr. 15 min. 20 sec.?

6. New York being in longitude 74° west, and New Or-

leans in 90° west, when it is 2 P.M. at New York, what time is it at New Orleans? When it is 11 A.M. at New Orleans, what time is it at New York?

7. What is the difference in time between Chicago, 87° 37′ 37″ W. long., and Cincinnati, 84° 27′ W. long.? When it is 5 P.M. at Chicago, what is the time at Cincinnati? When it is 8 A.M. at Cincinnati, what time is it at Chicago?

8. The difference in time between two places being 1 hr. 22 min. and 32 sec., what is the difference in longitude?

9. A telegraphic signal given from Washington, D.C., at 3.20 P.M., was received at St. Louis, Mo., at 2.27 P.M. The longitude of Washington being 77° 0′ 15″, what is that of St. Louis?

10. The difference in time between Buffalo and Cleveland being 11 min., what is the difference in longitude?

11. Boston, Mass., and Ann Arbor, Mich., are in about the same latitude; and the difference in time is 51 min. At this latitude 51.1 mi. make a degree of longitude. What is the distance in a direct line from Ann Arbor to Boston?

12. Berlin, Prussia, is in longitude 13° 23′ 53″ E., and Boston is in longitude 71° 4′ 9″ W. When it is 4 A.M. at Berlin, what time is it at Boston?

13. Annapolis, Md., and Cincinnati, O., are both in latitude 39° N. (nearly). On this parallel a degree of longitude is 53.47 mi., and the distance between the places is 422.413 mi. What is the difference in time?

14. The difference in time between Logansport, Ind., and Omaha, Neb., is 39 min.; and the distance between them is 509 mi. How many miles make a degree of longitude at this latitude?

15. In the latitude of Milwaukee, Wis. (43° 2′), $50\frac{1}{2}$ mi. make a degree of longitude. How far east or west of Milwaukee do you have to go to make a difference in time of 15 min.? Of 20 min.? Of $\frac{1}{2}$ an hour? Prairie du Chien has

the same latitude. A man's watch being set to Milwaukee time, he found it $13\frac{1}{2}$ min. fast at Prairie du Chien. How far is it in a direct line from one place to the other?

16. The difference in time between Detroit and Chicago is 19 min. If my watch is set to Chicago time, how will it compare with Detroit time? The longitude of Detroit is $82° \ 58'$ west. What is the longitude of Chicago?

17. Allowing no time for the passage of the current, at what time would a telegraphic signal sent from Ann Arbor, $83° \ 50' \ 48''.3$ W., at $3.45\frac{1}{2}$ P.M., reach Cambridge, Mass., $71° \ 7' \ 24''.9$ W.?

18. Pekin, China, is $116° \ 27'$ east longitude, and Washington is $77°$ west longitude. When it is noon on Jan. 1 at Washington, what time is it at Pekin?

19. Suppose the explosion of a meteor to have been observed at Ann Arbor at 2 o'clock 35 min. 27 sec. A.M., and at another point in the same parallel at 3 o'clock 4 min. 10 sec. A.M.: how many miles are the places apart? Is the second east, or west, of Ann Arbor? What is its longitude? See Exs. 17 and 11.

20. A and B sailed together from San Francisco. A kept his watch by San Francisco time, and B set his by the sun every day. After 10 days, A's watch was 4 hours 39 minutes faster than B's. In what longitude were they then, the longitude of San Francisco being $122° \ 26' \ 15''$ west?

21. At which point does the sun rise first, — at Philadelphia, $75° \ 9' \ 23''.4$ W., or St. Louis, $90° \ 15' \ 10''$ W.? How much?

SECTION II.

REDUCTION.

261. Reduction of Denominate Numbers is the process of changing the denomination in which a number is expressed without altering the value represented by it. If the change is from higher denominations to lower, the reduction is said to be *Descending;* if from lower to higher, *Ascending.*

1. In 4 yd. 2 ft. 9 in. how many inches?

OPERATION.

4 yd. 2 ft. 9 in.
3
14 ft.
12
177 in.

EXPLANATION.

Since 3 ft. make a yard, in any number of yards there are 3 times as many feet as yards. Hence in 4 yd. there are 3 times 4, or 12 ft.; which, with the 2 ft., make 14 ft.

Since 12 in. make a foot, in any number of feet there are 12 times as many inches as feet. Hence in 14 ft. there are 12 times 14, or 168 in.; which, with the 9 in., make 177 in. Hence 4 yd. 2 ft. 9 in. are 177 in.

Is this reduction descending, or reduction ascending? Why?

2. In 5127 pt. of water, how many barrels, gallons, quarts, and pints? Or reduce 5127 pt. liquid measure to the higher denominations of that measure.

OPERATION.

2) 5127 pt.
4) 2563 qt. 1 pt.
640 gal. 3 qt.
31.5) 640.0 (20 bbl.
630
10.0 gal.

EXPLANATION.

Since every 2 pt. make a quart, in 5127 pt. there are as many quarts as 2 is contained times in 5127; i.e., 2563 qt. and 1 pt.

[Let the pupil fill out the explanation.]

∴ 5127 pt. = 20 bbl. 10 gal. 3 qt. 1 pt.

Is this reduction descending, or reduction ascending? Why?

To Reduce Denominate Numbers.

262. Rule. — I. **Reduction Descending** *is performed by beginning with the highest denomination given, multiplying this by the number which it takes of the next lower to make one of this higher, and adding to this product the given number of this next lower denomination. Reduce this sum to the next lower denomination in a like manner, and add to the result the given number of this lower. Proceed in this manner till the denomination required is reached.*

II. **Reduction Ascending** *is performed by dividing the given number by the number of this denomination which it takes to make one of the next higher denomination, and treating the quotient thus arising in like manner, proceeding thus till the desired denomination is reached.*[1]

Examples for Practice.

Each of the following will afford an exercise for both forms of reduction: —

1. 3 da. 5 hr. 17 min. = 278220 sec.[2]
2. 442 pt. = 6 bu. 3 pk. 5 qt.
3. 6 gal. 3 qt. 1 pt. = 55 pt.
4. £32 12*s.* 10*d.* = 7834*d.*
5. 8 lb. 6 oz. 12 pwt. 16 gr. = 49264 gr.
6. ℥ vj ʒ iij ℈ j = 154 sc.

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[1] It is scarcely expedient to give an abstract demonstration of a rule so mechanical as this. In fact, the author has doubts about the expediency of giving the *rule* at all. Be careful, however, that the *rationale* of the process is understood, and can be given with ease and elegance.

[2] These examples should be assigned in both ways, each making two examples, — one in reduction descending, and one in reduction ascending. Thus this one will be given, "Express 3 da. 5 hr. 17 min. in seconds;" and again, "Express 278220 sec. in days, hours, and minutes." In assigning them for work on the blackboard the two examples may be assigned to two students standing together, and thus one will "*prove*" the other's work.

7. 78692 gr. = 13 lb. 7 oz. 18 pwt. 20 gr.
8. 5382 oz. (avoirdupois) = 3 cwt. 36 lb. 6 oz.
9. 78562 cu. ft. = 613 cd. 98 cu. ft.
10. 4268 sq. rd. = 26.675 A.
11. $\frac{3}{4}$ of a bbl. = 189 pt.

SUGGESTION. — This should be solved thus: —

$$\frac{3}{\cancel{4}} \times \frac{63}{\cancel{2}} \times \cancel{4} \times \cancel{2} = 189.$$

Why is $\frac{3}{4}$ multiplied by $\frac{63}{2}$? Is this quite in accordance with the rule (**262**)?

The converse process is thus performed: —

$$\frac{189}{2 \times 4} \div 31\frac{1}{2} \text{ is } \frac{189}{4 \times 2} \times \frac{2}{63} = \frac{3}{4}.$$

Does this process differ from that given in the rule (**262**)?[1]

12. $4\frac{3}{4}$ cords = 608 cu. ft.
13. $\frac{2}{3}$ gal. = $5\frac{1}{3}$ pt. $\frac{1}{2}$ pt. = $\frac{1}{16}$ gal.
14. 2 A. 110 sq. rd. = 430 sq. rd.
15. 6875988 cu. in. = 31 cd. 11 cu. ft. 276 cu. in.
16. 12 bbl. 19 gal. 2 qt. = 3180 pt.
17. 368 pt. = 5 bu. 3 pk.
18. 30630 min. = 21 da. 6 hr. 30 min.
19. $172800'' = 48°$.
20. 17 sq. ft. 27 sq. in. = 2475 sq. in.
21. 75288 gr. = 13 lb. 17 pwt.
22. 3.25 gal. = 26 pt.

Does the process by which we perform this constitute an exception to the general rule (**262**)?

23. 12 ft. 9 in. = 12.75 ft.

How are inches reduced to feet? $9 \div 12 =$ what decimal? Does this process constitute an exception to the rule (**262**)?

24. 3 in. = $.083\frac{1}{3}$ yd.

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[1] It should be the aim of the teacher to show that we need no *special* rules for reduction of denominate fractions.

How are inches reduced to feet? How are feet reduced to yards? $3 \div 12 = .25$. $.25 \div 3 = .083\frac{1}{3}$.

25. £4.67 = £4 13*s.* 4*d.* 3.2 *far.*

How is the £ .67 reduced to shillings? How is .4*s.* reduced to pence?

26. $\frac{7}{30}$ wk. = 1 da. 15 hr. 12 min.

$\frac{7}{30} \times 7 = 1\frac{19}{30}$. $\frac{19}{\underset{5}{\cancel{30}}} \times \overset{4}{\cancel{24}} = 15\frac{1}{5}$. $\frac{1}{5} \times 60 = 12$.

$\therefore \frac{7}{30}$ wk. = 1 da. 15 hr. 12 min.

$12 \div 60 = \frac{1}{5}$. $15\frac{1}{5} \div 24 = \frac{\overset{19}{\cancel{76}}}{5} \div \overset{6}{\cancel{24}} = \frac{19}{30}$. $1\frac{19}{30} \div 7 = \frac{49}{30} \div 7 = \frac{7}{30}$.

N.B. — Let the pupil trace this work till he sees clearly that the processes form no exceptions to the general rules for reduction.

27. $.475° = 28' \ 30''$.
28. ʒ iv ℈ ij = $\frac{7}{12}$ oz.
29. .345 of a bbl. of flour = 67 lb. $9\frac{23}{25}$ oz.
30. 16 lb. 8 oz. = .275 bu. of wheat, or $\frac{11}{40}$ bu.
31. 43 rd. 11.7 in. = .13456 mi.+.
32. $\frac{5}{6}$ oz. = 16 pwt. 16 gr.
33. $\frac{4}{5}$ bu. = 3 pk. 1 qt. $1\frac{1}{5}$ pt.
34. 213 rd. 1 yd. $2\frac{1}{2}$ ft. = $\frac{2}{3}$ mi.
35. A pile of wood 4 ft. long, 3 ft. high, 4 ft. wide = $\frac{3}{8}$ cd.
36. ℥ ix ʒ j ℈ ij gr. viij = .76875 lb.
37. 3 lb. 10 oz. troy = 3 lb. $2\frac{1}{2}$ oz. avoirdupois nearly.
38. 528 chains = 6 mi. 192 rd.
39. $2\frac{3}{4}$ mi. = 220 ch.
40. 3 ch. 4 lk. = 200.64 ft.
41. $2\frac{1}{2}$ A. = 25 sq. ch.
42. A lot 12 ch. 24 lk. by 15 ch. 40 lk. = 18.8496 A.

$$12.24 \times 15.4 \div 10 = 18.8496.$$

43. 3 A. = 300000 sq. lk.
44. A lot 81 ch. 52 lk. by 34 ch. 2 lk. = 277.3 A.+.
45. 756 cu. ft. = 1306368 cu. in., or $5.9\frac{1}{16}$ cd.
46. .24 bu. wheat = 14 lb. 6.4 oz.
47. 75 sq. rd. 12 sq. yd. 4 sq. ft. = $20530\frac{3}{4}$ sq. ft.
48. 165888 cu. in. = $\frac{3}{4}$. cd.
49. 8410 lk. = 1 mi. 4.1 ch., or 1.05125 mi.
50. $24\frac{1}{2}$ lb. = $\frac{1}{8}$ bbl. flour.
51. 5 f℥ 36 ♏ = .7 f℥.
52. .008 mi. = 2 rd. 9.24 ft.
53. $68 = 680 d. = 6800 ct. = 68000 m.
54. 3256 m. = 325.6 ct. = 32 d. 5 c. 6 m. = $3.256.
55. 1250^{fr} = $241.25. $10 = 51^{fr} 81^{c}+.
56. 5^{Nap} = $19.30. $100 = 25^{Nap} 18^{fr} 13^{c}+.
57. 5 fathoms = —— ft.? 256 ft. = —— fathoms?
58. 2 mi. 34 rd. 3 yd. = —— yd.?
59. 200 yd. = —— mi.?
60. 1 mi. = yd.? 40 rd. = —— yd.?
61. 100 yd. = —— rd.? $\frac{2}{3}$ yd. = what part of a rd.?
62. A lot 5 rd. by 8 = what part of an acre?
63. $3\frac{3}{4}$ A. = —— sq. yd.? 5 sq. rd. = —— A.?
64. A lot 7.21 ch. by 3.40 ch. = 2.4514 A.
65. 5 A. 110 sq. rd. 1 sq. yd. = 5.6877 A. very nearly.
66. $\frac{3}{5}$ cd. = —— cu. ft.? 12 cu. ft. = what part of a cd.?
67. 5 cd. 120 cu. ft. = —— cu. yd.?
68. 1376 cu. yd. = $290\frac{1}{4}$ cd. 1 cu. yd. = —— cd.?
69. 3 bbl. 20 gal. = —— qt.?
70. 4000 qt. = 31 bbl. $23\frac{1}{2}$ gal. = $31\frac{47}{63}$ bbl.
71. 10 gal. 3 qt. 1 pt. = 43.5 qt.
72. $126\frac{3}{4}$ qt. = 31 gal. 2 qt. $1\frac{1}{2}$ pt. = 1 bbl. $1\frac{1}{2}$ pt.
73. 43 qt. = 5 pk. 3 qt., or $5\frac{3}{8}$ pk.
74. $\frac{1}{12}$ bu. = $5\frac{1}{3}$ pt.
75. 3.416 bu. = 3 bu. 1 pk. $5\frac{1}{3}$ qt. nearly.
76. 2 bu. 3 pk. 5 qt. $1\frac{1}{2}$ pt. = 187.5 pt.

77. 1 bu. 1 pk. 1 qt. 1 pt. = how many pecks?
78. 2 bbl. 15 gal. 110$\frac{1}{4}$ pt. = 91.78125 gal.
79. 5287 qt. = —— bbl. —— gal. —— qt. —— pt.?
80. 786 gal. = —— pt.?
81. 4 lb. 8 oz. 12 pwt. 16 gr. = 27184 gr.
82. 3 lb. 10 oz. 20 gr. = 46$\frac{1}{24}$ oz.
83. 342$\frac{2}{5}$ oz. = 28 lb. 6 oz. 8 pwt.
84. 1 T. 13 cwt. 58 lb. = 3358 lb.
85. 7129 lb. = 3 T. 11 cwt. 29 lb.
86. 7129 lb. = 3 T. 3 cwt. 73 lb. U. S. customs weight.
87. 52 rd. 4 yd. 2 ft. = $\frac{108}{660}$ mi.
88. 1$\frac{1}{12}$ bbl. = 34 gal. 1 pt.
89. 2 hr. 52. min. 48 sec. = $\frac{3}{25}$ da.
90. $\frac{5}{896}$ bu. = $\frac{5}{14}$ pt.
91. 11 da. = $\frac{11}{30}$ mo. = $\frac{11}{360}$ yr. in computing interest.
92. 2 mo. 13 da. = .2028− yr. in computing interest.
93. 2 yr. 5 mo. 7 da. = 2.436 yr.−, as above.
94. 11 yr. 10 mo. 21 da. = 11.89 yr.+, as above.
95. 4 yr. 10 mo. 24 da. = 58.8 mo., as above.
96. 3.725 yr. = 3 yr. 8 mo. 21 da., as above.
97. 7 mo. 6 da. = .6 yr., as above.
98. 1 yr. 8 mo. 12 da. = 1.7 yr., as above.
99. 1 yr. 4 mo. 24 da. = 1.4 yr., as above.

SECTION III.

ADDITION.

1. There are three casks which contain 2 gal. 3 qt. 1 pt., 5 gal. 2 qt. 1$\frac{1}{2}$ pt., and 4 gal. 1 qt. 1$\frac{1}{2}$ pt. How much do they all contain?

How many pints are 1$\frac{1}{2}$, 1$\frac{1}{2}$, and 1? How many quarts does this make? 2 qt., 1 qt., 2 qt., and 3 qt., make how many quarts? How *many gallons* does this make, and how many quarts over? 2 gal., *4 gal., 5 gal.,* and 2 gal., make how many gallons?

2. There are 5 pieces of rope whose respective lengths are 2 yd. 2 ft. 3 in., 4 yd. 1 ft. 7 in., 3 yd. 2 ft. 5 in., 5 yd. 2 ft. 10 in., and 3 yd. 2 ft. What is the entire length?

OPERATION.

2 yd.	2 ft.	3 in.
4 yd.	1 ft.	7 in.
3 yd.	2 ft.	5 in.
5 yd.	2 ft.	10 in.
3 yd.	2 ft.	
20 yd.	2 ft.	1 in.

EXPLANATION.

We write numbers of the same denomination in the same column, because such are more conveniently added together. We then begin the addition with the *lowest* denomination, because we can thus tell whether there will arise any of the higher denominations from adding the lower, and, if there does, can add it with the higher denominations as we go along. [Were we to *commence* with the highest denomination, we should have to revise our results after having added all the columns. Let the pupil try it.]

In this example the sum of the inches column is 25 in. = 2 ft. 1 in. The sum of the feet column, together with the 2 ft. from the inches column, is 11 ft. = 3 yd. 2 ft. The sum of the yards column, together with the 3 yd. from the feet column, is 20 yd.

To Add Compound Numbers.

263. Rule. — *Write the numbers so that like denominations shall stand in the same column.*

Beginning with the lowest denomination, add the numbers in it, and divide the sum by the number it takes of this denomination to make 1 of the next higher. Write the remainder under the column added, and add the quotient to the next denomination.

Proceed in this manner with each denomination in succession, writing the entire sum of the highest.

The pupil should give the reasons (Demonstration) in order. He is to tell (1) Why we write the numbers as the rule directs; (2) Why we begin with the lowest denomination; (3) Why we reduce the several sums to the higher denominations. (See "Explanation" above.)

3. What is the amount of £105 1s. 2d. 3 far., £218 11s. 5d. 2 far., £199 17s. 9d. 2 far., and £77 18s. 3d. 3 far.?

4. What is the sum of 8 lb. ℥xj ʒvj ℈ij, 9 lb. ℥x ʒvij ℈j, 4 lb. ℥vij ʒiij ℈j, 17 lb. ℥viij ʒiij ℈j, and 45 lb. ℥xj ʒiij ℈j?

5. What is the sum of 10 rd. 3 yd. 1 ft. 7 in., 7 rd. 2 yd. 2 ft. 5 in., 3 rd. 4 yd. 1 ft. 9 in., 5 rd. 2 yd. 1 ft. 10 in., and 13 rd. 4 yd. 11 in.?

6. What is the sum of 145 bu. 3 pk. 1 qt., 163 bu. 1 pk. 3 qt., 275 bu. 2 pk. 7 qt., 45 bu. 3 pk. 6 qt., and 73 bu. 1 pk. 5 qt.?

7. What is the amount of £13 17s. 11d. 1 far., £22 14s. 9d. 1 far., £37 18s. 6d. 3 far., and £46 13s. 7d. 2 far.?

8. A silversmith bought of A 3 lb. 9 oz. 14 pwt. 16 gr. of silver, of B 9 lb. 11 oz. 17 pwt. 18 gr., of C 1 lb. 8 oz. 19 pwt. 21 gr., and of D 3 lb. 7 oz. 12 pwt. 16 gr. How much silver did he buy?

9. What is the amount of 45 cd. 23 cu. ft. 25 cu. in., 273 cd. 75 cu. ft. 684 cu. in., 97 cd. 18 cu. ft. 384 cu. in., 250 cd. 64 cu. ft. 197 cu. in., and 264 cd. 84 cu. ft. 848 cu. in.?

10. What is the sum of 86 sq. yd. 7 sq. ft. 46 sq. in., 245 sq. yd. 8 sq. ft. 89 sq. in., and 265 sq. yd. 7 sq. ft. 128 sq. in.?

11. What is the amount of 5 bbl. 20 gal. 3 qt. $1\frac{1}{4}$ pt., 7 bbl. 25 gal. 2 qt., 28 gal. 1 qt., 3 bbl. 30 gal. $1\frac{3}{4}$ pt., 2 qt. $1\frac{1}{2}$ pt., 13 gal., 2 qt. $1\frac{3}{4}$ pt.?

12. What is the sum of $\frac{3}{4}$ of a bushel, 2.64 pecks, .5 bu., $12\frac{3}{5}$ qt., $10\frac{1}{2}$ pk., and 320 pt. in bushels?

13. What is the sum of 8 yd. 2 ft., $75\frac{1}{2}$ ft., 2005 in., 4.35 yd., $28\frac{2}{5}$ ft., $3\frac{2}{3}$ yd., .25 ft., and 226 ft. in rods?

14. Mr. E. Jones owned $1\frac{1}{2}$ sections of land in one township, 80 acres in another, a quarter section in another, a 40-acre lot in another, and a piece of land 40 rods by 40 chains in another. How much land had he in all?

SECTION IV.

SUBTRACTION.

1. From 3 lb. 8 oz. 16 pwt. subtract 1 lb. 3 oz. 12 pwt. 17 gr.

OPERATION.

3 lb.	8 oz.	16 pwt.	
1 lb.	3 oz.	12 pwt.	17 gr.
2 lb.	5 oz.	3 pwt.	7 gr.

EXPLANATION.

We write the denominations of the subtrahend under like denominations in the minuend, because it is more convenient to subtract a number of any denomination from another of like denomination. We begin to subtract at the lowest denomination; so that, if there should chance not to be as many of any particular denomination in the minuend as in the subtrahend, we can take one from the next higher denomination in the minuend, and put it with the number in this deficient denomination. Thus in this example there are *no* grains represented in the minuend; but we can take one of the 16 pwt. which makes 24 gr., and, subtracting 17 gr. from it, have 7 gr. left. Then 12 pwt. from 15 pwt. leaves 3 pwt., etc.

To Subtract Compound Numbers.

264. Rule. — *Write the subtrahend under the minuend so that its denominations shall fall under the corresponding denominations of the minuend.*

Begin with the lowest denomination, and take the number represented in each denomination of the subtrahend from the number in the corresponding denomination in the minuend, and write the remainder underneath.

If the number in any denomination in the minuend is less than the corresponding number in the subtrahend, take 1 of the next higher denomination of the minuend in which there are any, and reducing it to this lower denomination, and uniting it with what there may be in this denomination, perform the subtraction.

Observe, when passing to the higher denominations, how much remains in them.

2. From 14 bu. 3 pk. 4 qt. 1 pt. subtract 8 bu. 3 pk. 7 qt.
3. From £10 12*s*. 7*d*. take £6 8*s*. 5*d*.
4. From £17 0*s*. 3*d*. take £9 10*s*. 5*d*.

OPERATION.
£17 0*s*. 3*d*.
£ 9 10*s*. 5*d*.
£ 7 9*s*. 10*d*.

EXPLANATION.

As 5*d*. cannot be taken from 3*d*., we take £1 = 20*s*., and taking 1 of the 20*s*., which makes 12*d*., subtract 5*d*. from 12 + 3*d*., or 15*d*. Then we have 10*s*., which we subtract from the 19*s*. remaining of the £1. Finally we take £9 from £16.

5. From 10 bu. 3 pk. 4 qt. take 4 bu. 1 pk. 2 qt.
6. From 1 bu. 1 pk. 1 qt. take 2 pk. 1 qt. 1 pt.
7. From ½ bu. take 3 qt. and 2 pt.
8. From 5 bbl. 24 gal. 2 qt. take 1 bbl. 27 gal. 3 qt.
9. From 4 lb. 8 oz. 12 pwt. take 2 lb. 5 oz. 17 gr.
10. From 1 lb. take 15 pwt. 20 gr.
11. From 5 mi. 100 rd. 12 ft. take 2 mi. 30 rd. 15 ft.
12. From 27 A. take 110¼ sq. rd.
13. From 2½ cords take 120.32 cu. ft.
14. From 5 yd. take 5 ft.
15. From 2 T. 12 cwt. take 3420 lb.
16. From ½ section take 51 A. 45 sq. rd.
17. From ℥ vij ʒ iij ℈ j take ℥ iij ʒ v ℈ ij.
18. Sold from a barrel of molasses 10 gal. 3 qt. 1 pt. How much remained?
19. From the sum of 3 lb. 13 oz., 2 lb. 5 oz., and 6 lb. 11 oz., take 12 lb. 10 oz.
20. From ¼ yd. take 7 in.

To Find the Time between Two Dates.

265. There are three methods in use for estimating the time between two dates; viz.: —

(1) To find the exact time in days.

(2) To find the time as commonly reckoned in computing interest; that is, reckoning 30 days as a month, and 12 months as a year.

(3) To find the time in calendar years, months, and days.

266. EXACT TIME IN DAYS.

1. What is the *exact time in days* between Jan. 12, 1856, and May 15, 1874?

SOLUTION. — Between Jan. 12, 1856, and Jan. 12, 1874, are 1874 − 1856 = 18 yr., in which there are 5 Februaries containing 29 days each.[1] Hence in these 18 yr. there are 365 × 18 + 5 = 6575 days. Then from Jan. 12 to May 15, 1874, there are 19 + 28 + 31 + 30 + 15 = 123 days. Therefore there are in all 6575 + 123 = 6698 days between these dates.

2. What is the *exact time in days* between April 5, 1860, and Nov. 24, 1875?

3. Find the *exact time in days* between the 4th of July and Christmas.

4. Between May 6, 1878, and Aug. 10, 1878.
5. Between July 17, 1875, and April 11, 1876.
6. Between Oct. 7, 1878, and Jan. 23, 1879.
7. Between March 21, 1879, and May 18, 1879.
8. Between June 7, 1876, and Aug. 15, 1876.
9. Between Jan. 5, 1872, and April 1, 1872.
10. Between Jan. 5, 1873, and April 1, 1873.

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267. TO FIND THE TIME BETWEEN TWO DATES, RECKONING 30 DA. = 1 MO., AND 12 MO. = 1 YR.

1. Reckoning 30 da. a month, and 12 mo. a year, what is the time between Aug. 17, 1871, and Feb. 3, 1875?

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[1] That is, those in the leap years 1856, 1860, 1864, 1868, and 1872.

OPERATION.	
1875 yr. 2 mo. 3 da.	
1871 yr. 8 mo. 17 da.	
3 yr. 5 mo. 16 da.	

EXPLANATION.

The later date (larger number) is the 1875th year, 2d month, 3d day. The earlier is the 1871st year, 8th month, and 17th day. These dates are subtracted as in ordinary subtraction, calling 30 days a month, and 12 months a year.[1]

Find the time between the following dates, reckoning 12 mo. a year, and 30 da. a month: —

2. March 16, 1850, and Dec. 5, 1871.
3. Aug. 23, 1846, and April 2, 1827.
4. Dec. 22, 1620, and Jan. 19, 1875.
5. May 5, 1872, and July 15, 1873.
6. April 17, 1878, and July 7, 1880.
7. Oct. 9, 1872, and May 30, 1876.
8. Dec. 27, 1874, and June 30, 1878.

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268. TO FIND THE TIME, RECKONING CALENDAR MONTHS AND YEARS.

1. Mrs. J. was born Dec. 5, 1825. What was her exact age, in calendar years, months, and days, on the 20th day of January, 1875?

SUGGESTION. — Such a question should be answered by giving the entire years and *calendar* months between the dates, and adding the remaining days. Thus, in this case, from Dec. 5, 1825, to Dec. 5, 1874, is 49 years; from Dec. 5, 1874, to Jan. 5, 1875, is 1 mo.; and from Jan. 5 to Jan. 20 is 15 days. Hence Mrs. J.'s age is 49 yr. 1 mo. 15 da.

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[1] A full explanation of this process requires that we understand that the earlier date is really 1874 yr. 1 mo. 3 da. after the Christian era, and the later 1870 yr. 7 mo. 17 da. after the same time. Hence we should have

1874 yr. 1 mo. 3 da.	
1870 yr. 7 mo. 17 da.	
3 yr. 5 mo. 16 da.	

But it is a little more convenient to take the dates as we are accustomed to name them, and subtract. That this process gives the correct difference is evident, since it is *only* increasing both minuend and subtrahend by 1 yr. 1 mo., and hence leaves the *difference the same*.

2. What is Mr. O.'s age March 12, 1875, he having been born July 24, 1827?

All such problems can be solved by subtraction if we observe the following —

269. Rule. — In reckoning calendar months and exact days, when subtracting the days, "borrow," and, when adding, "carry," *according to the number of days which make the month next preceding that in which the period terminates;* and when obtaining *a date* by subtraction, should a 0 fall in months order, transfer 1 yr. = 12 mo. to that order.

3. What is Mr. O.'s age March 12, 1875, he having been born July 24, 1827?

1875	3	12
1827	7	24
47	7	16

The month "borrowed" is February, 28 da.

4. What is Mary's age Aug. 15, 1875, she having been born July 20, 1868?

The month "borrowed" is July, 31 da.

5. A note dated July 25, 1857, matures in 5 yr. 3 mo. 24 da. When is it due?

The month "carried" (filled out) is the 10th, 31 da.

6. June 8, 1877, I have a note which has run 2 yr. 5 mo. 10 da. What is its date?

Subtracting, we have 1875 0 29, which is Dec. 29, 1874.

7. Feb. 12, 1876, Henry is 10 yr. 1 mo. 15 da. old. When was he born?

8. What is the exact time in days from Sept. 7, 1873, to Dec. 10, 1875?

Subtracting, we have 2 yr. 3 mo. 3 da. The 2 yr. = 730 da., and the 3 mo. = 91 da.

SECTION V.

270. MULTIPLICATION.

1. Multiply 5 yd. 2 ft. 8 in. by 7.

OPERATION.
5 yd. 2 ft. 8 in.
7
41 yd. 0 ft. 8 in.

EXPLANATION.

The multiplier is written under the lowest denomination of the multiplicand as matter of custom. We commence the multiplication with the lowest denomination, since by so doing we can find how many of the next higher denomination any particular product makes, and thus add it in with the next product as we pass along. Thus 7 times 8 in. are 56 in. = 4 ft. 8 in. Writing the 8 in. under inches, we reserve the 4 ft. to be added to the next product. 7 times 2 ft. are 14 ft., which, with the 4 ft. from the preceding product, make 18 ft. = 6 yd. 0 ft. Finally, 7 times 5 yd. = 35 yd., which, with the 6 yd. from the preceding product, makes 41 yd. Hence 7 times 5 yd. 2 ft. 8 in. are 41 yd. 8 in.

2. Multiply 5 lb. 13 oz. by 8.

(1) Where do you write the multiplier? Why?

(2) Where do you begin to multiply? Why?

(3) What do you do with the product arising from multiplying any particular denomination by the multiplier? Why?

Pupil *write* the rule, and the reasons for it; that is, the *Rule* and the *Demonstration.*

3. Multiply £7 9*s.* 5*d.* by 6.

4. Multiply 1 cd. 112 cu. ft. by 10.

5. Multiply 12 gal. 3 qt. 1 pt. by 7. By 100.

6. Multiply 4 oz. 12 pwt. 21 gr. by 16. By 132.

	4 oz.	12 pwt.	21 gr.	
			132	
	9	5	18	Product by 2.
11	7	6	6	" " 30.
38	8	7	12	" " 100.
51 lb.	0 oz.	19 pwt.	12 gr.	Product by 132.

7. Multiply 17 A. 35 sq. rd. 4 sq. yd. 5 sq. ft. by 23.

8. Multiply 4 mi. 110 rd. 17 ft. by 127. By 250.

9. Multiply 5 da. 15 hr. 13 min. 20 sec. by 341.

10. Multiply 10° 12′ 14″ by 45. By 6. By 13. By 10.
11. Multiply 3 lb. ℥ viij ʒ ij ℈ j by 4. By 12. By 24.

SECTION VI.

DIVISION.

1. Divide 23 da. 15 hr. 51 min. by 7.

OPERATION.

7) 23 da. 15 hr. 51 min.

3 da. 9 hr. 7 min. $17\frac{1}{7}$ sec.

EXPLANATION.

We begin the division with the *highest* denomination, since, if there is any remainder, it can be reduced to the next lower denomination, combined with what is expressed in this denomination, and the whole sum divided at once. Thus 23 da. $\div 7 = 3$ da. and 2 da. ($= 48$ hr.) remainder. Then 48 hr. $+$ 15 hr. $= 63$ hr., which, divided by 7, gives 9 hr. 51 min. $\div 7 = 7$ min. and 2 min. (or 120 sec.) remainder. 120 sec. $\div 7 = 17\frac{1}{7}$ sec.

2. Divide 12 lb. 15 oz. by 8.
3. Divide 125 bbl. 17 gal. 3 qt. by 36.

OPERATION.

36) 125 bbl. 17 gal. 3 qt. (3 bbl. 15 gal. $1\frac{17}{36}$ qt.
108
17 bbl. *remaining.*
$31\frac{1}{2}$
$8\frac{1}{2}$
17
51
17
$552\frac{1}{2}$ gal.
36
192
180
$12\frac{1}{2}$ gal. *rem.*
4
50 qt.
3 qt.
53 qt.
36
17 qt. *rem.*

EXPLANATION.

Dividing 125 bbl. by 36, we find the quotient 3 bbl., and a remainder 17 bbl. Reducing this to gallons, and adding the 17 gal. in the dividend, we have $552\frac{1}{2}$ gal. This, divided by 36, gives a quotient 15 gal., and a remainder $12\frac{1}{2}$ gal. This, reduced to quarts, makes, with the 3 qt. of the dividend, 53 qt. Dividing this by 36, we have $1\frac{17}{36}$ qt.

To Divide Compound Numbers.

271. Rule. — I. *Write the divisor on the left hand of the dividend, and the quotient underneath the dividend, or at its right, according as you divide by short or long division.*

II. *Beginning with the highest denomination, divide it, and write the quotient of this denomination in its place. Reduce the remainder (if any) to the next lower denomination, and add to it the number of this denomination in the dividend. Divide as before, reducing the remainder to the next lower denomination.*

Proceed in this manner till the division is complete.

Demonstration. — The general principle involved in this operation is the same as that involved in simple division; viz., the quotient is found by dividing the parts of the dividend separately, and adding the quotients. (See rule for Simple Division and its demonstration.)

The relative position of dividend, divisor, and quotient, is mainly matter of custom or convenience.

The division is commenced at the left hand, in order that the several remainders which may arise may be reduced and combined with the lower denominations as the work proceeds.

4. Divide £25 10*s*. 8*d*. by 9.
5. Divide 28 lb. 14 oz. by 5.
6. Divide 3 lb. ℥ vij ʒ iv ℈ ij by 6.
7. Divide 20 A. 100 sq. rd. by 10. By 27. By 13.
8. Divide 6 mi. by 115.
9. Divide 1 bbl. by 22.
10. Divide $\frac{1}{2}$ a bushel by 7.
11. Divide $7\frac{3}{4}$ cords by 3.
12. Divide 5 yd. 1 ft. 8 in. by 4.

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13. How many times is 2 bu. 3 pk. 7 qt. contained in 17 bu. 2 pk.?

Suggestion. — Reduce both dividend and divisor to the lowest denomination in either, and then divide.

14. Divide 3 bbl. by 6 qt. By 20 gal. 2 qt. By 5 gal. 3 qt. 1 pt.

SECTION VII.

PRACTICAL EXPEDIENTS.

[The purpose of this section is to exhibit a few of the practical expedients by which common arithmetical operations may be abridged. While the pupil should perform the operation intelligently, and hence should *occasionally* be asked to give a "solution," the chief purpose is to secure facility in getting results.]

CONTRACTIONS IN MULTIPLICATION.

272. *To Multiply by 25.*—Multiply by 100, and then divide by 4.

273. *To Multiply by 125.*—Multiply by 100, and add to this product $\frac{1}{4}$ itself.

Ex. Multiply 2346 by 25. 780 by 25. 800 by 25. 5432 by 25, 5437, 1000.

OPERATIONS.

$$\begin{array}{r} 234600 \\ \hline 58650 \end{array}$$

Ex. Multiply 5082 by 125; also 3702, 540, 100, 64827, 34000.

$$\begin{array}{r} 508200 \\ 127050 \\ \hline 635250 \end{array}$$

274. *To Multiply by a Number represented by two Digits, one of which is 1.*—Multiply by the digit which is not 1, and write the product under the multiplicand; removing this product one place to the left if this digit is 10's, and one to the right if it is units, adding these numbers.

Ex. Multiply 78579 by 81. By 61, 91, 71, 21, 31, 11, 41.

$$\begin{array}{r} 78579 \\ 628632 \\ \hline 6364899 \end{array}$$

Ex. Multiply 78579 by 18. By 13, 15, 17, 16, 14, 15.

$$\begin{array}{l} 78579 \\ 628632 \\ \hline 1414422 \end{array}$$

275. *To Multiply by a Number represented by 9's.*— Multiply by 1 with as many 0's at the right as there are 9's, and then subtract the multiplicand. If the right-hand figure of the multiplier is 8, subtract twice the multiplicand; if 7, three times; if 5, *five* times, etc.

Ex. Multiply 857639 by 999.. Also 786042 by 9999; 3760048 by 99; 2856437 by 9999; 987654 by 999; 9999 by 9999.

$$\begin{array}{r} 857639000 \\ 857639 \\ \hline 856781361 \end{array}$$

Ex. Multiply 857639 by 998. Also 64738 by 998; 64820 by 9998; 43725 by 98.

$$\begin{array}{r} 857639000 \\ 1715278 \\ \hline 855923722 \end{array}$$

Ex. Multiply 8764987 by 997. By 99. By 95. By 98. By 995.

276. *To Square*[1] *any Number ending in ½.*—Multiply the integral part by 1 more than itself, and to the product add (annex) ¼.

Ex. Square 34½. 46½. 87½. 93½. 16½.

Ex. Square 17½. 128½. 478½. 647½. 273½.

Ex. Square (mentally) 5½. 4½. 1½. 2½. 3½. 7½. 8½. 9½. 10½.

$$\begin{array}{r} 34 \\ 35 \\ \hline 170 \\ 102 \\ \hline 1190\tfrac{1}{4} \end{array}$$

The reason for this rule will appear if we consider, that, when we multiply (in the ordinary way) by the ½ in the multiplier, we get ½ of the integer in the multiplicand + ¼. Then, when we multiply the ½ in the multiplicand by the integer in the multiplier, we get ½ the integer. Hence the entire effect of the ½ in each factor is to give a product larger than the integer by *once* the integer and ¼ more.

$$\begin{array}{r} 8\tfrac{1}{2} \\ 8\tfrac{1}{2} \\ \hline \left\{\begin{array}{r} 4\tfrac{1}{4} \\ 4 \end{array}\right. \\ 64 \\ \hline 72\tfrac{1}{4} \end{array}$$

CONTRACTIONS IN DIVISION.

[The reasons for the following are so simple, that it is not deemed necessary to state them. We give the rules, and a few examples for practice.]

[1] To square a number is to multiply it by itself. Thus the square of 32 is $32 \times 32 = 1024$.

277. *To divide by* 25. — Multiply by 4, and divide by 100; i.e., remove the decimal point 2 places to the left.

278. *To divide by* 15, 35, 45, 55, *or* 65. — Double the dividend, and divide by 30, 70, 90, 110, or 120.

279. *To divide by* 125. — Multiply by 4, and divide by 1000.

280. *To divide by* $3\frac{1}{3}$. — Multiply by 3, and divide by 10.

281. *To divide by* $12\frac{1}{2}$ *or* $16\frac{2}{3}$. — Multiply by 8 or by 6, and divide by 100.

Ex. 1. — Divide 7856 by 25. By 15, 35, 45, 55, 65, 125, $3\frac{1}{3}$, $12\frac{1}{2}$, $16\frac{2}{3}$.

2. Divide 8463, 963, 78427, 80056, and 7000, by each of the above numbers.

Aliquot Parts.

282. An **Aliquot Part** of a number is any number (integral or mixed) which will exactly divide it.

[The above methods are really methods by "*Aliquot Parts;*" but it is not customary to speak of them as such.]

283. The Aliquot Parts of $1 are, —

50¢ = $\frac{1}{2}$.	$33\frac{1}{3}$¢ = $\frac{1}{3}$.
25¢ = $\frac{1}{4}$.	$16\frac{2}{3}$¢ = $\frac{1}{6}$.
20¢ = $\frac{1}{5}$.	$12\frac{1}{2}$¢ = $\frac{1}{8}$.
10¢ = $\frac{1}{10}$.	$6\frac{1}{4}$¢ = $\frac{1}{16}$.

Mental Exercises.

1. What cost 27 yd. of cloth at 25¢ per yard?

PRACTICAL OPERATION.[1] $\frac{1}{4}$ of 27 is $6\frac{3}{4}$. ∴ $6.75.

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[1] SOLUTION. — At $1 per yard, 27 yd. would cost $27: hence, at $$\frac{1}{4}$ per yard, 27 yards cost $\frac{1}{4}$ of $27, or $$6\frac{3}{4}$ = $6.75. (See note at the beginning of the section.)

2. What cost 42 yd. of calico at $12\frac{1}{2}$¢ per yard? At $16\frac{2}{3}$? At $6\frac{1}{4}$? At $33\frac{1}{3}$? 25? 20? 10? 50?

OPERATION. — For the first $42 \div 8 = 5\frac{1}{4}$. ∴ The cost is $5.25.

3. What cost 35 lb. of butter at $33\frac{1}{3}$¢ per pound? At 25¢? At 50¢ ? At 20¢? $16\frac{2}{3}$? 10? $12\frac{1}{2}$?

4. What cost 15 lb. of tea at 1.33\frac{1}{3}$ per pound? At $1.10? At $1.50? At $1.25? At 1.12\frac{1}{2}$? At $1.20?

OPERATION. — For the first, $15 + 5 = 20$. ∴ The tea cost $20.

5. At $12\frac{1}{2}$¢ per pound, how much sugar can bought for $1? For $3? For $1.25?

OPERATION. $8 + 2 = 10$. ∴ 10 lb. can be bought for $1.25.

6. At $33\frac{1}{3}$¢ per pound, how much coffee can be bought for $1? For $2? For $5?

OPERATION. $2 \times 3 = 6$. ∴ 6 lb. can be bought for $2.

7. What cost 15 yd. of cloth at $3.20 per yard? At $2.10? At 4.33\frac{1}{3}$?

8. What cost 25 yd. of cloth at $4.25 per yard? At $3.40? At $11.25?

OPERATION. — For the first, $425 \div 4 = 106.25$.

EXPLANATION. 100 yd. would cost $425, and 25 yd. would cost $\frac{1}{4}$ as much.

9. What cost $3\frac{1}{3}$ lb. of butter at 32¢ per pound? At 36¢? At 45¢? At 21¢?

At 21¢ 10 lb. cost $2.10, and $3\frac{1}{3}$ cost $2.10 ÷ 3 = 70¢.

10. What cost $12\frac{1}{2}$ yd. of cloth at 40¢ per yard? At $1.12? At 16¢? At $2.20?

11. What cost 48 gal. of molasses at $66\frac{2}{3}$¢ per gallon?

OPERATION. $48 - 16 = 32$. ∴ $32. Why?

12. What cost 120 bu. of potatoes at $37\frac{1}{2}$¢ per bushel?

13. What cost 256 bu. of onions at $87\frac{1}{2}$¢ per bushel?

$256 — $\frac{1}{8}$ of $256, or $256 — $32 = $224. Why?

14. What cost 75 cords of wood at $5.50 per cord.

---

284. The Aliquot Parts of a Year are,—

6 mo. = $\frac{1}{2}$; 4 mo. = $\frac{1}{3}$; 3 mo. = $\frac{1}{4}$; 2 mo. = $\frac{1}{6}$; 1 mo = $\frac{1}{12}$.

Also, when 30 da. are called a month, 1 mo. 6 da. = $\frac{1}{10}$; 1 mo. 10 da. = $\frac{1}{9}$; 1 mo. 15 da. = $\frac{1}{8}$; 2 mo. 12 da. = $\frac{1}{5}$.

285. The Aliquot Parts of a Month (30 da.) are,—

15 da. = $\frac{1}{2}$; 10 da. = $\frac{1}{3}$; 6 da. = $\frac{1}{5}$; 5 da. = $\frac{1}{6}$; 3 da. = $\frac{1}{10}$.

Mental Exercises.

Ex. 1. A man's wages are $450 per year. What are they for 6 mo.? 4 mo.? 3 mo.? 2 mo.? 1 mo.? 1 mo. 6 da.? 1 mo. 10 da.? 1 mo. 15 da.? 2 mo. 12 da.?

2. The interest on a note is $84 per year. What is it for each of the times in Ex. 1?

3. The interest on a note is $45 per year. What is it per month? What for 15 da.? 10 da.? 6 da.? 5 da.?

Written Exercises.

4. At $360 per year, what is the rent of a house for 2 yr. 7 mo. 25 da?

The work in the margin is all that should be written. The pupil *thinks* thus: "For 2 yr. the rent is $720; for 6 mo., $180; for 1 mo., $30 (i.e., $\frac{1}{6}$ of $180); for 15 da., $15; for 10 da., $10."

Operation.

$720
180
30
15
10
$955

5. At $450 per year, what is the rent of a house for 2 yr. 8 mo. 25 da.? For 1 yr. 5 mo. 11 da.? For 3 yr. 2 mo. 13 da.?

What part of the interest for 2 mo. is the interest for 10 da.? What for 3 da.?

Operation for Last.

$1350
75
12.50
3.75
$1441.25

6. At $480 per year, what is the rent of a house for 8 mo. 13 da.?

7. If John's salary is $560 per year, how much does he receive for 5 mo. 15 da.? For 1 yr. 10 mo. 18 da.? For 2 yr. 6 mo. 10 da.? For 9 mo. 14 da.?

Solutions of the Last.

By Aliquot Parts.

	$280
$46.666	$140
	15.555
	3.111
	3.111
	$441.777

14 da. $= \frac{1}{3}$ mo. $+ \frac{2}{5}$ of $\frac{1}{3}$ mo.

By Cancellation.

$$\frac{\$\overset{140}{\cancel{560}} \times \overset{3}{\cancel{9}}}{\cancel{12}} = \$420.00$$

$$\frac{\$\overset{14}{\cancel{560}} \times 14}{\underset{3}{\cancel{12}} \times \cancel{30}} = \frac{196}{9} = \$21.78$$

$\$441.78$

By Decimals.

30|14

$9.4\frac{2}{3}$

$46.666+
$9.4\frac{2}{3}$
15555
15555
186666
419999
$441.78

Observe in the method by decimals, that, in multiplying a repetend, we write the first figure of each partial product as it would be if the repetend were indefinitely extended.

286. What number of ounces are aliquot parts of an avoirdupois pound?

What number of pecks are aliquot parts of a bushel?

8. What cost 6 lb. 12 oz. of butter at 28¢ per pound? What 5 lb. 14 oz. at 30¢? 4 lb. 10 oz. at 24¢? 7 lb. 13 oz. at 40¢? Same at 32¢ per pound?

$$\$2.80 + .20 + .10 + .02\tfrac{1}{2} = \$3.12\tfrac{1}{2}.$$

9. At 14¢ per pound, what is the cost of a dressed turkey weighing 12 lb. 14 oz.? What one weighing 8 lb. 11 oz.? Same at 16¢ per pound?

10. What cost 5 bu. 3 pk. 2 qt. of grain at $1.20 per bushel? What 2 bu. 2 pk. 3 qt.?

$$\$6.00 + .60 + .30 + .07\tfrac{1}{2} = \$6.97\tfrac{1}{2}. \quad \text{Why?}$$

11. What cost 15 yd. of cloth at $3.20 per yard? At $2.10? At $4.33\frac{1}{3}$? $3.12\frac{1}{2}$? $1.66\frac{2}{3}$?

Solve the following by cancellation (see 2d solution, Ex. 7): —

12. \$56 per year is how much for 8 mo. 24 da.? 10 mo. 18 da.? 9 mo. 20 da.? 25 da.?

13. \$320 per year is how much for 2 yr. 8 mo. 18 da.? 1 yr. 10 mo. 21 da.? 5 yr. 9 mo. 15 da.?

14. \$20 per year is how much for 5 mo. 13 da.?

15. \$150 per year is how much for 2 yr. 11 mo. 13 da.?

SOLUTION OF 14TH.

$$\frac{\overset{4}{\$\cancel{20}} \times 5}{\cancel{12}} = 6\tfrac{2}{3} \qquad = \$6.67$$

$$\frac{\overset{3}{\cancel{20}} \times 13}{\underset{6}{\cancel{12}} \times \underset{3}{\cancel{30}}} = \frac{4.333+}{6} = .72$$

$$\$7.39$$

SOLUTION OF 15TH.

$$\frac{\overset{5}{\$\cancel{150}} \times 13}{12 \times \cancel{30}} = \frac{65}{12} = \$\ 5\tfrac{5}{12}$$

$$\frac{\overset{25}{\cancel{150}} \times 11}{\underset{2}{\cancel{12}}} = \frac{275}{2} = \$137\tfrac{6}{12}$$

$$300$$

$$\$442.92$$

16. At \$160 per year, what is the interest for 2 yr. 7 mo. 17 da.?

$$\frac{\overset{4}{\$\cancel{160}} \times 17}{\underset{3}{\cancel{12}} \times \cancel{30}} = \frac{68}{9} = \$7\tfrac{5}{9}, \text{ for 17 da.}$$

$$\frac{\overset{40}{\cancel{160}} \times 7}{\underset{3}{\cancel{12}}} = \frac{280}{3} = \$93\tfrac{1}{3}, \tfrac{3}{9}, \text{ for 7 mo.}$$

$$\$160 \times 2 = \$320, \quad \text{for 2 yr.}$$

$$\$421.11, \text{ for 2 yr. 7 mo. 17 da.}$$

17. Solve as above the following: —

\$80 per year for 3 yr. 11 mo. 18 da.

\$70 per year for 2 yr. 10 mo. 16 da.

\$130 per year for 1 yr. 8 mo. 21 da.

\$84 per year for 5 yr. 9 mo. 20 da.

\$20 per year for 11 mo. 15 da. For 7 mo. 21 da.

\$200 yer year for 1 yr. 4 mo. 23 da.

SECTION VIII.

THE METRIC[1] OR DECIMAL SYSTEM OF WEIGHTS AND MEASURES.

DIAMETER, 2 CENTIMS.

WEIGHT, 5 GRAMS.

287. The metric system can be readily learned, if the student will first fix in mind a definite conception of

The Units.

The **Meter** (*mee′-ter*) is the unit of *Length*, and is the basis from which all the rest are deduced.

A **Meter** = 39.37 *inches.*

The **Liter** (*lee′-ter*) is the unit of *Measures* of *Capacity.*

A **Liter** = { 1.0567 *liquid quarts*, or .908 *dry quarts.*

The **Gram** is the unit of weight.

A **Gram** = 15.432 *grains.*

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[1] This system takes its name *metric* from the *meter*, the unit of linear measure established by the French Government, and made the basis of all the others. — See *Appendix.*

Subdivisions and Multiples of the Units.

288. These units are divided and subdivided into 10ths, 100ths, and 1000ths, and multiplied by 10, 100, 1000, and 10,000, to make the other denominations. Hence the system is a *Decimal System.*

The names of the denominations *lower* than the unit are formed by prefixing the Latin syllables *Deci* ($\frac{1}{10}$), *Centi* ($\frac{1}{100}$), and *Milli* ($\frac{1}{1000}$), to the name of the unit.

The names of the denominations *higher* than the unit are formed in like manner by prefixing the Greek syllables *Deka* (dek′a) (10), *Hekto* (100), *Kilo* (1000), *Myria* (10,000), to the name of the unit.

289. COMPLETE TABLE OF THE METRIC SYSTEM.

Relative Values.	Length.	Weight.	Capacity.	Surface.	Solidity.
10,000	Myriam (Mm)	..	..	..	..
1,000	Kilōm (Km)	Kilōg (Kg)	Kilōl (Kl)	..	..
100	Hektōm (Hm)	Hektōg (Hg)	Hektōl (Hl)	Hektar (Ha)	..
10	Dekam (Dm)	Dekag (Dg)	Dekal (Dl)	..	Decaster (Ds)
Unit.	***Meter*** (m)	***Gram*** (g)	***Liter*** (l)	***Ar*** (a) [1]	***Ster*** (s) [1]
.1	Decim (dm)	Decig (dg)	Decil (dl)	..	Decister (ds)
.01	Centim (cm)	Centig (cg)	Centil (cl)	Centar (ca)	..
.001	Millim (mm)	Millig (mg)	Millil (ml)	..	..

This table contains all the denominations in use, with the spelling and abbreviations approved by the Metric Bureau, Boston, and the American Metrological Society, New York. The abbreviated names will be seen to consist of the prefixes with the first letter of the principal word, or name of the unit. Thus we have *decim* for decimeter, *kilōg* for kilogram, *centil* for centiliter, etc. The *accent* is always on the first syllable: *c* is soft (*s*), *e* in the prefixes short, and *o* long (ō). Thus *decim* is dĕcĭm; *centig* is sĕntĭg; *kilol,* kĭlōl, etc. Ar is like *are.*

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[1] 1 ar = 1 square dekam; 1 ster = 1 cu. meter.

290. The abbreviations of the names of the units and the multiples are written as superiors. The abbreviations of the multiples are written with capitals. The submultiples are written without capitals, and in line with the figures. Thus 5 meters, written 5^{m}; 3 liters, 3^{l}; 7 hektols, 7^{Hk}; 8 dekags, 8^{Dk}. So 4 decims is written 4 dm; 6 centigs, 6 cm; 9 millims, 9 ml, etc. The tendency now is to write all the abbreviations without the period.

LEGALIZED EQUIVALENTS.

291. For purposes of reduction to and from our common measures, the pupil should fix in mind the following equivalents as legalized by the United-States Government: —

1^{m} = 39.37 in.	1^{Kg} = 2.2 lb. Av.
1^{l} = 1.06 liquid qt.	1^{a} = 3.95 sq. rd.
or .908 dry qt.	1^{s} = 35.52 cu. ft.
1^{g} = 15.432 gr., or .035 oz. Av.	

MEASURES OF LENGTH.

292. THE INSTRUMENTS USED in place of our common 2 ft. ruler or carpenter's square, and the yard-stick or measure, are the *Meter*, a ruler 39.37 in. long, and divided into 10ths (decims) and 100ths (centims); and a *Double-decim* ruler, 2 dm in length, graduated into centims, and these again divided into tenths, making millims. The meter is folded into 4 parts, or 10 parts, for a pocket-measure; and the 2 dm ruler into 2 parts.

Such lengths as we usually indicate by yards, feet, and inches, are indicated by meters and centims.[1] Such as we indicate by miles are indicated by kilōms (kilometers). Very small dimensions, as those used in microscopy, are indicated in millims.

1. To which of our common measures is a meter nearest equal? How many meters in a rod? What is the length of

[1] There seems to be a well-defined tendency to use the decim as a common unit *for smaller* measures. The use of the *double-decim* ruler, and the near commensurability of the decim with one foot, will facilitate this.

a 12 ft. board expressed in meters? Express the dimensions of a room 20 ft. by 24 ft. in meters.

2. What is the stature of a 6 ft. man expressed in the metric system? What of one 1.8^{m} expressed in feet and inches?

Is a man's stature any more likely to be conveniently expressed in feet and inches than in meters and decimals? Is a man any more likely to be just 6 ft. in height than 1.8^{m}?

3. How does the ten-folded pocket-meter compare in length (when folded) with the common four-folded 2 ft. pocket-ruler? How does the four-folded meter compare with the two-folded 2 ft. pocket-ruler?

4. When the metric system comes into common use, what lengths of boards will probably take the place of our 12 ft.? 14 ft.? 16 ft.? What thicknesses will probably take the place of our 1 in., 2 in., and 3 in. stuffs, respectively? What dimensions of scantling our 2 in. by 3 in.? Our 3×4? Our 2×8 joist? Our 2×12?

As an inch is very nearly $\frac{1}{4}$ of a decim, we shall probably speak of "quarter-decim" stuff, or simply "quarter-stuff," instead of "inch-stuff," "half-decim" stuff, or "half-stuff," for "2 in. stuff," etc. Of course, positive answers cannot be given to such questions as the above. Nevertheless, the student will get a better appreciation of the relation of the metric to the common system by exercising his judgment on such questions than by any mere reductions.

5. With what in our common measure will a 2 dm by 3 dm timber most nearly correspond? What a 3 dm by 4 dm? What a 3 dm square?

6. What will 12 in. by 16 in. glass be in the metric system?—that is, what size will be likely to replace this? What 18 in. by 24 in.?

7. What is the distance from Albany to New York, expressed metrically, it being 145 miles? What the distance from Detroit to Chicago *viâ* the M. C. R.R. (288 mi.)?

8. What simple fraction of a mile is a kilōm (approximately)?

9. A railroad-train, running 40^{Km} per hour, runs how many miles?

10. How will the rate 1 mile in 2 min. 4 sec. be expressed metrically? One mile in 5 min.?

11. Which is the faster rate, 1^{Km} in 2 min. 20 sec., or 1 mi. in 2 min. 40 sec.?

12. What part of an inch is a millim? What is the approximate value in hundredths of an inch?

13. Glass is ruled for microscopic measurements in parallel lines from $\frac{1}{100}$ to $\frac{1}{1000}$ mm apart. What are these distances in inches?

14. Animal cells vary in diameter from $\frac{1}{40}$ mm to $\frac{1}{80}$ mm, and vegetable from $\frac{5}{8}$ mm to $\frac{1}{250}$ mm. Express these facts in inches, calling a millim .04 of an inch.

MEASURES OF WEIGHT.[1]

293. For the ordinary purposes of the grocery and market, the *kilōg* (called kilō in Europe) is used. For jewellers' and apothecaries' purposes, and for the chemical laboratory, the *gram* and *millig* are the units used.

The standard government weights at Washington are of brass and platinum. The brass weights are a five-kilōg, double-kilōg, kilōg, demi-kilōg, double-hektōg, hektōg, demi-hektōg, double-dekag, dekag, demi-dekag, double-gram, and gram. The platinum are a demi-gram, double-decig, decig, demi-decig, double-centig, centig, demi-centig, double-millig, and millig.

1. When steak is 14¢ per lb., what is it per Kg? Sugar at 30¢ per Kg is what per lb.? At 25¢ per Kg?

2. What is the weight of a bushel of wheat in Kgs? Of oats? Of corn? Of a barrel of flour?

[1] The measures of *length* and *weight* are the two of the metric system of the most practical importance in our country at present: hence this arrangement, and the fuller attention given them.

3. One lb. av. equals how many kilōgs? One kilōg is how many pounds?

4. How many grams in an ounce of gold? In a pennyweight? How many milligs make a grain troy or apothecaries'?

5. The United-States post-office allows 15^g as the weight of a single letter, or $\frac{1}{2}$ oz. troy. Which is the greater?

6. What would be the *practical* equivalent for the apothecaries' grain, scruple, and dram, in metric weights?

7. How many grams in an ounce avoirdupois? What would be the practical equivalent in grams for $\frac{1}{4}$ lb. avoirdupois?

8. One-eighth of a grain is the common dose of morphine. What would be the prescription in the metric system?

9. Quinine is frequently given in 4 or 5 grain doses. What would be the prescription in the metric system?

10. In weighing a quantity of sugar, I find it is balanced by a double-kilōg, a demi-hektōg, and a dekag weights. What is the weight in kilōgs?

N.B. — It is one purpose of a number of the preceding exercises to suggest, that, when the metric system comes into use, most of our common specifications of quantities for practical purposes will undergo slight changes to conform to the units of the new system, so as not to involve troublesome fractions. Thus, instead of a *rod*, we shall speak of 5 meters; and, instead of laying out village-lots 4 rods by 8, we shall lay them out 20^m by 40. Instead of 12 by 16 in. glass, we shall have 3 by 4 dm glass. Instead of prescribing ʒ ij, the physician will write 8^g, etc.

11. Instead of 1 ʒ, what amount will probably be substituted in the metric system?

294. The *Tonneau*, or *ton*, of the metric system, is 2,204.6 lbs., and is, consequently, so nearly equivalent to our *long ton* (2,240 lbs.) as to take its place without difficulty. The name *Ton* will doubtless be used instead of the French *tonneau*.

12. Coal at $9 per common ton (2,000 lbs.) would be how much per metric ton? Hay at $12 per common ton would be how much per metric ton?

MEASURES OF CAPACITY.

295. For such quantities of liquids or dry substances as we usually designate by the pint, quart, or gallon, the *liter* is used, as 7.5^l, 15^l, $\frac{1}{2}^l$, etc.; but for larger quantities the *hektol* is used, as 8.2 hl, 10.5 hl, etc.

The United-States Government standards at Washington are a double-liter, liter, demi-liter, double-decil, decil, demi-decil, double-centil, and centil.

The double-liter, liter, and demi-liter are respectively so nearly equivalent to our ½ gal., quart, and pint measures for liquids, as to take their places without embarrassment.

A double-dekal and a dekal would take the place of our ½ bu. and peck measures for grain very readily.

1. To what common measure is a liter nearly equal? How many liters in a gallon? In a barrel of 31½ gallons? How many does a common pail (2½ gal.) hold?

2. How many liters in a peck? In a half-bushel?

3. If a bushel of wheat is to weigh 60 lb., what should be the weight of a hektōl?

4. Wheat at $3.50 per Hl is what per bu.?

5. Molasses at $1.25 per gal. is how much per Dl?

296. One of the principal advantages which the metric system offers for scientific purposes is the facility which it affords for passing from measures of capacity to those of weight, and *vicè versa.* Thus a *liter* is a *cubic decim*, and a *gram* is a *cubic centim* of pure water at the temperature of melting ice. Hence, knowing the specific gravity of any substance (i.e., its weight as compared with water), we can readily pass from weight to volume, and *vicè versa.*

6. What is the weight of a liter of distilled water at the *temperature* of melting ice?

7. The specific gravity of linseed-oil is .94. How much would a cask of 2^{Hl} weigh?

8. I find that a liter of alcohol weighs 8^{Hg}. What is its specific gravity?

The weight of a liter of any liquid expressed in kilōgs is its specific gravity, or the weight of a millil expressed in grams, etc.

9. The specific gravity of milk is 1.032. What does 1^{Dl} weigh?

10. A pail containing 1^{Dl} of cider is filled, and the cider found to weigh 10.18^{Kg}. What is the specific gravity of cider?

11. One millil of sulphuric acid is found to weigh 1.842^{g}. What is its specific gravity?

MEASURES OF AREA.

297. For measuring *Boards* and small areas the square meter, decim, etc., are used; but for *Land* the *Ar*, which is a square dekam, and even the Hektar, which is 100 ars, is used. $1^{Ha} = 2.471$ acres, and the *Ar* is about $\frac{1}{40}$ of an acre. Since 1 mile is but a little over 16^{Hm}, a section of land is about 256^{Ha}. It is common, in surveying, to speak of 16^{Hm} as a mile.

1. How many hektars in a section of land?
2. Land at \$250 a hektar is how much per acre?
3. How many square feet in a square meter?
4. Land at 8^{Nap} a hektar is how many dollars an acre?
5. How many hektars in a rectangular piece of ground 1000^{m} by 400?
6. How many square meters in 6 boards 4^{m} in length and 5 dm in width?

MEASURES OF VOLUME.

298. The *Ster*, which is a cubic meter, is the proposed unit of volume; but it has fallen into general disuse. — *President* BARNARD, *in Johnson's Cyclopædia.*

The chief interest, therefore, which attaches to the metric measurement of volume at present, is as a means of defining the measures of weight and capacity. See (**296**) and also Appendix.

1. How many hektols of water does a cylindrical cistern contain which is 2 meters in diameter and 2.5 meters deep?
2. In 200 sters how many cords?
3. 500 cu. yd. are how many sters?
4. How many cubic meters (sters) of earth are removed in digging a ditch 2^m wide, 1.5 deep, and 4^{Km} long?

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299. ***Examples in the Use of the Metric Measures.***

[By permission, the following (46) examples are taken from an excellent little manual by Supt. Henry E. Sawyer of the schools of Middletown, Conn., now Associate Principal of the Connecticut State Normal School at New Britain. This manual will be found specially helpful in showing how to teach the Metric System so as to make it interesting and practically useful.]

1. Add 23 cm, 47 cm, 9 cm, 38 cm, 74 cm.
2. Add 237^m, 53 cm, 17^m3, 24^m07.
3. Add 239^m47, 98 cm, 19^m, 70^m07.
4. From 97 cm take 39 cm.
5. From 7^m35 take 4^m86. From 9^m08 take 57 cm.
6. The lengths of the principal buildings at the Centennial Exposition were as follows: Main Building, 573^m024; Machinery Hall, 426^m72; Horticultural Hall, 116^m738; Art Gallery, 111^m25; Women's Pavilion, 58^m52; Agricultural Building, 249^m936. How far would they have extended if ·ed in a row, end to end?

7. How much longer was the Main Building than Machinery Hall? The Agricultural Building than the Art Gallery?

8. Multiply 24 cm by 3. 87 mm by 9.

9. $29^{m}43 \times 7 = ?$ $216^{m}329 \times 8 = ?$

10. Divide 78 cm by 13. 168^{m} by 8.

11. $16^{m}8 \div 8 = ?$ $436^{m}324 \div 4 = ?$

12. $9864^{Km} \div 8 = ?$ $96^{Hm} \div 7 = ?$

13. If Mary lives 753^{m} from the schoolhouse, how far must she travel, in coming to school and going home, in one week?

14. How far must a man walk in a day to travel $237^{Km}6$ in 6 days?

15. From Quebec to the mouth of the Saguenay River is about $216^{Km}18$. At what rate must a steamer run to make the trip in 9 hours?

16. In what time will a train run from Boston to Albany at the rate of $34^{Km}3$ per hour, the distance being about $325^{Km}85$?

17. How many revolutions will an engine-wheel $4^{m}5$ in circumference make in running $416^{Km}7765$?

18. Read the following: $6^{l}3$; $19^{Hl}15$; $3^{cl}7$; $93^{Dl}47$.

19. In 27^{l} how many cl? Dl?

20. In 3954 cl how many Dl? Hl? l? dl?

21. In 697^{Hl} how many l? cl? dl?

22. At 3 cents a cl, what is the price per l?

23. At 25 cents per l, what cost 2 cl?

24. If a Dl of oil is worth \$1.63, what is it per l?

25. When corn is worth \$2.24 per Hl, what is it per l?

26. At \$1.35 per Dl, what is a liter of molasses worth?

27. If a chicken eats 2 dl of corn in a day, what will it cost to keep 173 chickens a week when corn is worth \$2.15 per Hl?

28. Read $31^{g}7$; 23^{Kg}; $17^{T}396$; 37^{g}; $3^{Kg}7$.

29. Reduce $27^{T}3$ to Kg; to g.

30. Into a basket weighing $4^{Kg}3$ were put the following articles: 2^{Kg} of coffee, $\frac{1}{2}^{Kg}$ of tea, $3^{Kg}5$ of sugar, 100^{g} of pepper, 50^{g} of nutmegs, 500^{g} of turnip-seed, and $1^{Kg}5$ of rice. What was the weight of the basket and its contents?

31. A car weighing $7^{T}832$ contains 136 barrels of flour, each weighing $96^{Kg}16$. Find the weight of the car and its contents. What is the difference in weight?

32. At $1.15 per Kg, what cost $7^{T}3$?

33. What cost a Kg of sugar at $265 per T?

34. What cost $29^{Kg}5$ at 16¢ per g?

35. What is the weight of 27^{l} of water?

36. What is the weight of 36 cl of water?

37. $3^{l}5$ of water weighs how many g?

38. $1^{l}93$ of water weighs how many g?

39. What is the weight of 173^{l} of water?

40. A man carting water from a river has two casks, one holding 136^{l}, and the other 125^{l}. When both casks are full, what weight of water has he on his cart?

41. If milk is 1.03 times as heavy as water, how much should 8 liters of milk weigh?

42. If you buy 42 liters of milk, and find that it weighs only $42^{Kg}75$, is it pure?

43. Wishing to find the capacity of a bottle, and having no measures at hand, I weighed it. Empty, it weighed 520^{g}; and filled with water, $1^{Kg}29$. How much did it hold?

44. What is the capacity of a conical glass which weighs when empty 540^{g}, and when filled with water $2^{Kg}225$?

45. A test-tube weighing 19^{g} was filled with water, when the tube and water together were found to weigh $64^{g}5$. How much water was in the tube?

46. The same tube, filled with sulphuric acid, weighed $103^{g}5$. The acid was how many times as heavy as water?

Business Rules.

CHAPTER V.

SECTION I.

PERCENTAGE.

Definitions and First Principles.

1. DURING a severe winter a farmer lost 5 sheep out of every 100 of his flock. What part of his flock did he lose?

2. John's father agreed to give him $8 for every $100 he would earn for himself. To what part of his earnings was his father's gift equal?

300. **Per Cent** means *By the Hundred.*

To say that a man lost 5 per cent of his sheep is to say that he lost 5 out of every hundred of them, or .05 of them. Again: to say that a father gives his son a sum equal to 8 per cent of the son's earnings is to say that he gives the son $8 for every $100 he earns, or a sum equal to .08 of his earnings.

3. A nurseryman lost by drought 6 per cent of his trees. What part of his trees did he lose? If he had 2150 trees, how many did he lose? i.e., .06 of 2150 = how many?

4. A man's house was damaged by fire to an extent estimated at 15 per cent of its value. If the house was worth $6530, what was the amount of damage?

5. A speculator bought a car-load of wheat for $450, and sold it at a profit of 4 per cent. How much did he make by the speculation?

This means that he made .04 of $450, which is $18.

6. What is 9 per cent of 250? 10 per cent of 48? 17 per cent of 53? 11 per cent of 1437?

301. Rate[1] is the number by which we multiply to obtain any required per cent of a given number.

Thus, to obtain 7 per cent of 250, we multiply 250 by .07. Hence .07 is the *Rate* (not *Rate per cent*). See footnote.

302. The result obtained by taking a certain per cent of a number is called *the Percentage.* The term *Percentage* is also used as a general designation for all processes involving this method of reckoning by the hundred.

303. The **Base** is the number upon which the percentage is estimated. In the last illustration, 250 is the *base.*

7. Mr. Smith, having a flock of 340 sheep, found that in 1 year they increased at the rate of 50 in a hundred. What was the per cent (or rate per cent) of increase? What was the *rate* of increase? How *much* was the increase? How many sheep had he after the increase?

304. The **Amount** is the sum of the base and percentage.

In Ex. 7 what is the Base? What the *Rate per cent?* What the *Rate?* What the *Percentage?* What the *Amount?*

8. Having $350, I used it in buying grain, which I sold so as to gain $5\frac{1}{2}\%$. How much had I then?

$5\frac{1}{2}\%$ of $350 = $19.25. Hence I had $350 + $19.25, or $369.25.

What is the *Base?* The *Rate?* The *Percentage?* The *Amount?*

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[1] The use of the term *Rate per cent* in this sense is inadmissible; but we may so use *Rate:* in fact, this is the common meaning of the word *rate* in mathematics. An allowance of 7 on a hundred is not at a rate of .07 *per cent*, although it is at a *rate of .07:* the *Rate per cent* is 7. Per cent is used by ellipsis for *Rate per cent.*

9. If the *base* is 784, and the *rate per cent* 6, what is the *percentage?* *What the amount?*

305. The character % is used as a substitute for the words *per cent.*

Thus 4% means "4 per cent."

10. Mr. A. used $500 speculating in wheat, and lost 8% of it. How much did he lose? How much had he left?

306. The **Difference** is what remains of the *Base* after the percentage is taken out.

11. Mr. Smith had an orchard of 320 trees; and, during a severe winter, 5% of them died. How many died? How many remained? What is the *Base? Rate? Percentage? Difference?*

12. What part of a number is 60% of it?

60% means 60 on a hundred. Now, 1 on a hundred is $\frac{1}{100}$ part. Hence 60 on 100 is $\frac{60}{100}$, or $\frac{3}{5}$ part. Hence 60% of any thing is $\frac{3}{5}$ of it.

13–29. What part of a number is 10% of it? 20%? 50%? $12\frac{1}{2}$%? $16\frac{2}{3}$%? 40%? 11%? 15%? 7%? 6%? 4%? $\frac{1}{2}$%? $\frac{3}{4}$%? $\frac{5}{8}$%? 100%? 200%? 150%?

$\frac{1}{2}$%, or $\frac{1}{2}$ per cent, is $\frac{1}{2}$ on a hundred, just as 2% is 2 on a hundred. At $\frac{1}{2}$% the rate is .005.

30–40. What *rate* is $\frac{3}{5}$%? $\frac{3}{2}$%? $\frac{1}{3}$%? $1\frac{1}{2}$%? $\frac{3}{4}$%? $\frac{5}{8}$%? $\frac{4}{5}$%? $\frac{5}{6}$%? $1\frac{1}{4}$%? $12\frac{1}{2}$%? $8\frac{3}{4}$%?

$\frac{3}{5}$% is at the rate $.00\frac{3}{5}$, or .006. $\frac{3}{2}$% is at the rate $.01\frac{1}{2}$, or .015. The per cent being $\frac{1}{3}$, the rate is $.00\frac{1}{3}$. $\frac{3}{4}$% is at the rate .0075.

41–48. What is the *per cent* corresponding to each of the following rates: .05? .10? 1.12? .03? .007? .005? $.00\frac{1}{4}$? $.\frac{1}{2}$?

$.\frac{1}{2}$ is an absurd expression, and has no meaning. It is not $\frac{1}{2}$ of $\frac{1}{10}$, for that is written $.0\frac{1}{2}$; nor is it $\frac{1}{2}$ of 1, for that is simply $\frac{1}{2}$.

307. ***Rules of Percentage.***

$$\left.\begin{array}{rl} \text{I.} & \textit{Base} \times \textit{Rate} = \textit{Percentage}. \\ \text{II.} & \textit{Base} + \textit{Percentage} = \textit{Amount}. \\ \text{III.} & \textit{Base} - \textit{Percentage} = \textit{Difference}. \end{array}\right\} \text{PRIMARY RULES.}$$

$$\left.\begin{array}{rl} \text{IV.} & \dfrac{\textit{Percentage}}{\textit{Rate}} = \textit{Base}. \\ \text{V.} & \dfrac{\textit{Percentage}}{\textit{Base.}} = \textit{Rate}. \\ \text{VI.} & \dfrac{\textit{Amount}}{\textit{Base}} = 1 + \textit{Rate}. \\ \text{VII.} & \dfrac{\textit{Amount}}{1 + \textit{Rate}} = \textit{Base}. \\ \text{VIII.} & \dfrac{\textit{Difference}}{\textit{Base}} = 1 - \textit{Rate}. \\ \text{IX.} & \dfrac{\textit{Difference}}{1 - \textit{Rate}} = \textit{Base}. \end{array}\right\} \text{SECONDARY RULES.}$$

The *Three Primary Rules* are but direct applications of definitions (**301**), (**304**), and (**306**).

IV. and V. are deduced from I. on the principle that the product of two factors divided by either factor gives the other. Thus, —

$$\textit{Base} \times \textit{Rate} = \textit{Percentage}. \therefore \left\{\begin{array}{l} \dfrac{\textit{Percentage}}{\textit{Rate}} = \textit{Base}. \\ \dfrac{\textit{Percentage}}{\textit{Base}} = \textit{Rate}. \end{array}\right.$$

VI. and VII. are obtained from II. on the same principle that IV. and V. are from I., by observing, that, to obtain the *Amount*, we multiply the *Base* by the *Rate*, and then add the *Base*, whence we obtain (1 + *the Rate*) times the *Base*. Thus, —

$$\textit{Base} \times (1 + \textit{Rate}) = \textit{Amount}. \therefore \left\{\begin{array}{l} \dfrac{\textit{Amount}}{\textit{Base}} = 1 + \textit{Rate}. \\ \dfrac{\textit{Amount}}{1+\textit{Rate}} = \textit{Base}. \end{array}\right.$$

VIII. and IX. are obtained from III. as VI. and VII. are from II., by observing, that, to obtain the *Difference*, we take the *Rate* times the

Base out of the *Base*, and hence have (1 — *the Rate*) times the *Base*. Thus, —

$$Base \times (1 - Rate) = Difference. \quad \therefore \begin{cases} \dfrac{Difference}{Base.} = 1 - Rate. \\ \dfrac{Difference}{1 - Rate} = Base. \end{cases}$$

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308. *Methods of Explanation.*

[There are two principal methods of applying the principles of percentage in practice: one being to observe which of the elements — base, rate, percentage, amount, or difference — are given, and which required, and then select and apply the proper rule; while the other method teaches to examine into the nature of each problem, and, discovering the relations existing between the quantities, analyze accordingly. We give illustrations of both methods.]

1. How much is 7% of $350?

SOLUTION BY RULE. — In this case $350 is the *base*, and .07 is the rate, and the percentage is required. Hence, by (307), I., we multiply the base by the rate, and find the percentage to be $24.50.

OPERATION.

$$\begin{array}{r} \$350 \\ .07 \\ \hline \$24.50 \end{array}$$

SOLUTION BY ANALYSIS. — Since 7% is 7 on every hundred, it is .07 of any sum. .07 of $350 is $24.50.

2. A speculator bought a car-load of wheat for $450, and sold it at a profit of 4%. How much did he receive?

BY RULE. — In this case $450 is the base, .04 is the rate, and the amount is required. Hence we multiply the base by the rate, and have the percentage by (307), I. Adding the percentage to the base, we have the amount by (307), II.

OPERATION.

$$\begin{array}{rr} & \$450 \\ & .04 \\ \hline \text{Percentage,} & \$18.00 \\ \text{Base,} & 450 \\ \hline \text{Amount,} & \$468.00 \end{array}$$

BY ANALYSIS. — Since a profit of 4% is an increase of 4 on a hundred (i.e., of .04), the speculator gained .04 of $450 by the operation. .04 of $450 is $18. Hence he gained $18 on $450, and consequently received $450 + $18 = $468.

3. By investing a certain sum, I made a profit of 12%, and thereby gained $100.80. What was the sum invested?

By Rule. — Here we have the *percentage* ($100.80) and the *rate* (.12) given to find the *base*. Hence, according to (**307**), IV., we divide the percentage by the rate, and obtain the base, $840.

Operation.

$$\frac{\$100.80}{.12} = \$840.$$

Analysis. — A gain of 12% is a gain of 12 cents on $1. Hence to gain $100.80 will require as many dollars as $0.12 is contained times in $100.80, which is $840.

4. Bought cloth at $3.50 per yard, and sold it at 70¢ per yard more than I gave. What per cent profit did I make?

By Rule. — Here we have the base $3.50, and the percentage $0.70 to find the rate. Hence we divide the percentage (.70) by the base (3.50), and have the rate, .20, by (**307**), V. The rate being .20, the rate per cent is 20.

Operation.

$$\frac{.70}{3.50} = .20.$$

Analysis. — If, in spending $3.50, I gain $0.70, in spending $1 I should gain .70 ÷ 3.50, or .20 dollars, and in spending $100 I should gain $20. Hence I gain 20 on 100, or 20%.

5. If I buy land at $50 per acre, and sell it at $60, what per cent do I make?

By Rule. — In this case the $60 includes the base and percentage: hence it is the *amount*. Having given base and amount, we divide the amount by the base, and get 1.20, which, by (**307**), VI., is 1 + rate. ∴ The rate is .20, and the rate per cent 20.

Operation.

$$\frac{6\cancel{0}}{5\cancel{0}} = 1.20.$$

Analysis. — If I receive $60 for $50 laid out, for $1 I receive 60 ÷ 50, or $1.20. Hence I make 20¢ on $1, or 100¢; i.e., 20%.

6. By selling a certain piece of land for $560, I make 12% on the cost. What was the cost?

By Rule. — Given the amount (560) and rate (.12) to find the base, by (**307**), VII., we divide the amount by 1 + the rate (i.e., 1.12), and have the base, *500.*

Operation.

$$\frac{560}{1.12} = 500.$$

ANALYSIS. — As I make 12%, I make 12¢ on \$1. Hence, for every dollar the land cost me, I get \$1.12. But in all I receive \$560. Hence the land cost me as many dollars as \$1.12 is contained times in \$560; i.e., \$500.

7. I sell a fine horse for \$540, and thereby lose 25%. What did the horse cost me?

BY RULE. — Here are given the difference (540) and the rate (.25). Hence, by (307), IX.,

OPERATION.

$$\frac{540}{.75} = 720.$$

ANALYSIS. — As I lose 25%, I lose 25¢ on each dollar the horse cost me, and hence receive 75¢ for each \$1 cost. But in all I receive \$540. Therefore the horse cost me as many dollars as 75¢ is contained times in \$540, or \$720.

8. Mr. A. bought a house for \$7500, and sold it for \$6000. What per cent was his loss?

BY RULE. — There are given base (\$7500), difference (\$6000), to find rate. Hence, by (307), VIII., etc. Finally as .80 is 1 — rate, that is the remainder after subtracting the rate from 1, if we take this remainder from the minuend, 1, we shall have the subtrahend, .20, which is the rate. Hence the rate per cent was 20.

OPERATION.

$$\frac{60\cancel{00}}{75\cancel{00}} = \tfrac{4}{5} = .80$$

$$1 - .80 = .20$$

ANALYSIS. — He received \$6000 for an outlay of \$7500. Hence for an outlay of \$1 he received 6000 ÷ 7500, or \$0.80. Hence he lost 20¢ on \$1, or 20%.

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309. *Examples for Practice.*

Solve the following with or without writing according to the simplicity of the problem.

N.B. — The rate should always be expressed in the most convenient form, not necessarily in the form of a *decimal fraction.* Thus, to find $33\frac{1}{3}$% of \$360, we would not multiply by $.33\frac{1}{3}$, but by $\frac{1}{3}$; i.e., *divide* by 3. But, to obtain 7% of any number, it is most convenient to mul-

tiply by .07. Again: to find 25%, we would divide by 4, which is the same as multiplying by .25, or $\frac{1}{4}$. Always use as few figures as practicable in the "operation." Use cancellation whenever it will aid.

1. 5% of 780 = 39.
2. 12% of 475 yd. = 57 yd.
3. 10% of 860 trees = ?
4. 35% of 1840 = ?
5.[1] $33\frac{1}{3}$% of \$234.54 = ?
6. 45% of $18\frac{3}{4}$ = ?
7.[2] $\frac{3}{4}$% of \$348 = ?
8. $\frac{4}{5}$% of $\frac{15}{16}$ lb. = ?
9. 7% of \$47 = ?
10. 100% of \$58 = ?
11. 1% of \$58 = ?
12.[3] 120% of \$150 = ?
13. 110% of \$60 = ?
14. $112\frac{1}{2}$% of \$16 = ?
15. 150% of \$365.20 = ?
16. 125% of \$7.50 = ?
17. 115% of \$11.37 = ?
18. 200% of \$247 = ?
19. 300% of \$75.50 = ?
20. $\frac{1}{2}$% of \$360 = ?
21. 50% of \$360 = ?
22. 1% of \$37 = ?
23. 100% of \$37 = ?
24. 7% of \$43.20 = ?
25. 9% of \$162.43 = ?
26. Find 6% of \$1.75.
27. Find 6% of \$350.
28. Find 7% of \$140.
29. Find 8% of \$1.
30. Find 9% of \$100.
31. Find $\frac{2}{3}$% of \$0.75.
32. Find $1\frac{1}{2}$% of \$$\frac{5}{9}$.
33. Find $\frac{5}{8}$% of \$1540.
34. Find $\frac{1}{2}$% of \$2500.
35. Find 7% of \$34.28.
36. Find 11% of \$15.17.
37. Find 6% of \$42.18.
38. Find $33\frac{1}{3}$% of 465 gal.
39.[4] Find $37\frac{1}{2}$% of 816 mi.
40. Find 35% of \$21.75.
41. Find 48% of 13.42.
42.[5] Find 7% of $\frac{3}{4}$. $\frac{5}{6}$.
43. 8% of 7. $4\frac{1}{2}$. 5.
44. $12\frac{1}{2}$% of 16. $3\frac{1}{3}$. 7.
45.[6] $66\frac{2}{3}$% of $1\frac{1}{2}$. $4\frac{1}{5}$. 6.
46. 7% of $\frac{1}{2}$. $\frac{2}{3}$. $\frac{4}{5}$.
47. 9% of $4\frac{3}{4}$. $5\frac{1}{6}$. 7.
48. 3% of $2\frac{2}{3}$. $1\frac{1}{2}$. $5\frac{1}{3}$.
49. 11% of $\frac{5}{6}$. $3\frac{1}{4}$. $2\frac{2}{7}$.
50. 111% of $\frac{5}{17}$. $\frac{1}{13}$. $3\frac{2}{7}$.

---

[1] $33\frac{1}{3}$% is $.33\frac{1}{3}$, or $\frac{1}{3}$ of any thing.

[2] 1% of \$348 is \$3.48, and $\frac{3}{4}$% is $\frac{3}{4}$ of \$3.48.

[3] 120% is 1.20, or $1\frac{1}{5}$ times any thing. $\therefore$ \$150 + \$30 = \$180.

[4] $37\frac{1}{2}$% is $\frac{3}{8}$ of any thing. $\therefore$ $\frac{3}{8}$ of $816 = 3 \times 102 = 306$.

[5] 7% of $\frac{5}{6}$ is $\frac{7}{100}$ of $\frac{5}{6} = \frac{7}{120}$.

[6] $66\frac{2}{3}$% of $1\frac{1}{2}$ is $\frac{2}{3}$ of $\frac{3}{2} = 1$.

310. What per cent of —

51. \$60 is \$20?
52. 55 is 11?
53. 148 is $24\frac{2}{3}$?
54. $47\frac{1}{2}$ is $7\frac{1}{2}$?
55. 40 is 15?
56. $\frac{1}{2}$ is $\frac{1}{3}$?
57. $\frac{1}{3}$ is $\frac{1}{2}$?
58. $\frac{3}{4}$ is $\frac{2}{5}$?
59. 37 yd. is 37 yd.?
60. 37 yd. is .37 yd.?

What per cent is —

61. \$180 of \$360?
62. \$1.80 of \$360?
63. $\frac{1}{2}$ of $\frac{1}{3}$?
64. $\frac{3}{4}$ of $\frac{2}{5}$?
65. \$547.80 of \$365.20?
66. \$66 of \$60?
67. \$226.50 of \$75.50?
68. 8¢ of 5¢?
69. 4¢ of $12\frac{1}{2}$ ¢?
70. 1¢ of \$1? \$1 of 1¢?

Such questions are equivalent to asking what part one number is of another (**158**), with the added condition that the answer shall be given in hundredths. Thus 15 is $\frac{15}{40}$, or $\frac{3}{8}$ of 40, and $\frac{3}{8} = 37\frac{1}{2}$ hundredths.

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311. Of what number is —

71.[1] 385 $12\frac{1}{2}\%$?
72. 245 10%?
73. 125 15%?
74. 7.15 $33\frac{1}{3}\%$?
75. \$53.25 10%?
76. 27.5 bu. 8%?
77. 168 men 8%?
78. 231 oxen 7%?
79.[2] $\frac{2}{5}$ $53\frac{1}{3}\%$?
80. 15 $37\frac{1}{2}\%$?

Of what number is —

81. 146 lb. 8%?
82. 240 men $12\frac{1}{2}\%$?
83.[3] $5\frac{1}{3}$ $1\frac{1}{2}\%$?
84.[4] $7\frac{3}{4}$ $\frac{3}{4}\%$?
85. $\frac{1}{2}$ 150%?
86. \$37 1%?
87. \$37 100%?
88. \$37 300%?
89. \$78.18 $33\frac{1}{3}$?
90. 5 8%?

---

[1] Avoid mechanical processes. $12\frac{1}{2}\%$ is $\frac{1}{8}$. If 385 is $\frac{1}{8}$, 8-eighths is 8 times 385.

[2] $53\frac{1}{3}\%$ is $\frac{53\frac{1}{3}}{100}$, or $\frac{160}{300} = \frac{8}{15}$. $\frac{2}{5} \div \frac{8}{15} = \frac{2}{5} \times \frac{15}{8} = \frac{3}{4}$.

[3] Operation. $\dfrac{5\frac{1}{3}}{\frac{1\frac{1}{2}}{100}} = \dfrac{1600}{4\frac{1}{2}} = \dfrac{3200}{9} = 355\frac{5}{9}$.

[4] $\dfrac{7\frac{3}{4}}{\frac{\frac{3}{4}}{100}} = \dfrac{3100}{3} = 1033\frac{1}{3}$.

312. What sum amounts to —

91.[1] \$360 at 20%?	101. \$540 at 8%?
92.[2] \$210 at 75%?	102. \$234 at 6%?
93. \$61.53 at 5%?	103. \$1000 at 9%?
94. \$36.54 at $1\frac{1}{2}$%?	104. $4\frac{1}{2}$ at $2\frac{3}{4}$%?
95. \$40.23$\frac{1}{4}$ at $4\frac{1}{2}$%?	105. \$100 at 100%?
96. \$129.368 at 3%?	106. \$100 at 1%?
97.[3] \$3519.15 at 1.12$\frac{1}{2}$%?	107. \$56.85 at 10%?
98. \$82 at 2.5%?	108. \$145 at 4%?
99. \$408.20$\frac{1}{2}$ at 7%?	109. $\frac{3}{4}$ at $12\frac{1}{2}$%?
100.[4] $\frac{3}{5}$ at $\frac{2}{3}$%?	110. \$4 at $112\frac{1}{2}$%?

313. What sum gives a difference of —

111. \$483.60 at 7%?	121. 273 at 25%?
112.[5] \$24.50 at $12\frac{1}{2}$%?	122. 520 at 20%?
113. \$300 at $6\frac{1}{4}$%?	123.[7] 320 at $33\frac{1}{3}$%?
114. 730 yd. at 4%?	124. \$138 at 7%?
115. 584 lb. at 8%?	125. \$24.50 at 8%?
116. 347 ft. at 10%?	126. \$160 at 12%?
117. 46 at $3\frac{1}{2}$%?	127. \$300 at 3%?
118. 112 at $2\frac{3}{4}$%?	128. $\frac{3}{5}$ at $\frac{7}{8}$%?
119.[6] $\frac{3}{4}$ at $37\frac{1}{2}$%?	129. $4\frac{2}{3}$ at $3\frac{1}{2}$%?
120. $5\frac{1}{3}$ at $63\frac{2}{3}$%?	130. 0 at 100%?

314. General Suggestion. — In solving problems in percentage, a good general method is to *indicate all operations before performing any, and then put the work in the best form for cancellation.*

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[1] Keep the work down to a minimum. $360 \times \frac{5}{6} = 300$.

[2] $210 \times \frac{4}{7} = 120$.

[3] $\frac{3519.15}{1.01125} = 3480$.

[4] $\frac{\frac{3}{5}}{1.00\frac{2}{3}} = \frac{60}{100\frac{2}{3}} = \frac{30}{50\frac{1}{3}} = \frac{90}{151}$.

[5] $1 - .12\frac{1}{2} = 1 - \frac{1}{8} = \frac{7}{8}$. $\therefore \frac{24.50}{\frac{7}{8}} = \frac{196}{7} = 28$.

[6] $\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \times \frac{8}{5} = 1\frac{1}{5}$.

[7] $\frac{320}{\frac{2}{3}} = 160 \times 3 = 480$.

SECTION II.

APPLICATIONS OF SIMPLE[1] PERCENTAGE.

315. Business calculations based on percentage are of two classes: (1) those which do not involve the element of *time* in the computations, and (2) those which do.

Of the 1st class are the simpler problems of *Profit* and *Loss*, *Commission*, *Brokerage*, *Bankruptcy*, *Stocks*, *Insurance*, *Taxes*, and *Duties*. Of the 2d class are *Interest*, *Discount*, *Annuities*, including many problems in *Insurance*, *Exchange*, and *Equation of Payments and Accounts*.

Problems of the 1st class are solved directly by the principles, or by the formulas (**307**).

Profit and Loss.

316. **Profit**, or **Gain**, is the excess of what is received for an article over its total cost. **Loss** is the excess of the total cost of an article over what is received for it.

1. Bought a flock of sheep numbering 350. In one season it increased 24%. How many had I then? Had it *decreased* 24%, how many should I have had?

2. Bought 12 crocks of butter, weighing 35 lb. each net,[2] for 19¢ per lb., and sold it at a profit of 2%. What did I receive for the whole? How much did I *gain*?

3. My house, which is valued at $5500, was damaged by fire 15%. What was the total damage?

4. Invested $3560 in town-lots in a new village in Kansas. In the course of a year they increased in value $62\frac{1}{2}$%. What were they worth then?

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[1] By "Simple" Percentage is meant Percentage which does not involve the element of *time*.

[2] This means exclusive of the crocks which contain the butter.

5. Bought 20 horses at an average price of $225 each. I lost 25% of them, and sold the remainder at an advance of 30% on the cost price. Did I gain, or lose, by the transaction? How much?

6. What must be the selling price of cloth which cost $4 per yard, in order to realize a profit of 10%? Of $12\frac{1}{2}$%? Of 25%? Of 30%? Of 15%? Of 20%?

7. At what must calico, which cost 6¢ per yard, be sold, in order to realize a profit of 8%? Of $\frac{5}{8}$%? Of 1%? Of $2\frac{1}{2}$%? Of $\frac{1}{2}$%? Of $1\frac{3}{4}$%? Of 5%? Of $4\frac{1}{2}$%?

8. Bought a piece of cloth containing 30 yd. at $3 per yard. 10 yd. of it were damaged, so that I had to sell it at a loss of 50%. The remainder I sold at 20% profit. How much did I lose on the whole?

9. The standard for gold coin in the United States is 9 parts pure gold, and 1 part alloy. What % is alloy? What % is gold?

10. If I buy cloth at $5 per yard, and sell it at $5.50, what % profit do I make?

11. When I sell goods at $1\frac{1}{4}$ their cost, what % profit do I make?

I make a profit of $\frac{1}{4}$ the cost; i.e., on every $1 spent in buying I make $\frac{1}{4}$ of a dollar. The question then is, "$\frac{1}{4}$ is what % of 1?"

12. If I buy land at $27 per acre, and sell it at $36, what % do I make? What, if I sell it at $30? At $90?

13. A fruit-grower shipped 300 baskets (pecks) of peaches to Chicago; but, on the way, 75 baskets spoiled. What % did he lose? What % was left?

14. When I sell goods at $\frac{2}{3}$ the cost, what % do I lose?

The loss is $\frac{1}{3}$ of the cost. Then the question is, "$\frac{1}{3}$ is what % of 1?"

15. When I sell goods at twice the cost, what % do I *make*? When at $1\frac{1}{5}$ the cost? At $1\frac{1}{10}$ the cost? At $2\frac{1}{2}$ the *cost*?

16. What % is made by buying tea at 80 cents per pound, and selling it at $1? At 90¢? At 85¢? At $1.10?

17. Bought a span of horses for $575, and sold them at $650. What % did I make?

18. Bought a house and lot for $11500, and sold them at $13640, after having expended $350 in repairs. What % did I make?

19. A merchant marked prints which cost him 7¢ to be sold at 9¢. What % advance on cost was this?

20. Bought 2560 lb. of coffee at 31¢ per pound, and paid $1.50 per hundred for freight and $1 for cartage. What % did I make by selling it at 45¢ per pound?

21. By selling cloth at $5.50 per yd., I make 10 % on the cost? What was the cost?

22. By selling a horse for $230, I lost 8% on the cost. What was the cost?

23. What was the cost of cloth marked $3.50 per yd., this being 15% advance on the cost?

24. A merchant having marked down his goods $33\frac{1}{3}$% from his usual retail price, which was 20% advance on cost, what was the cost of an article now marked 20¢?

25. A merchant who had marked a certain lot of goods to sell at 15% advance on cost, in consequence of a rise in the market marked them up 5% on the former retail price. At what % advance on cost were they now marked?

26. From what price can I fall $33\frac{1}{3}$% on goods which cost $3.20 per yard, and still make 20%?

27. At what must I purchase nails by the keg (100 lb.) to sell them at 5¢ per lb., and make 15%?

28. I have marked goods which cost me $2.50 per yd. to sell at 25% advance. What % can I fall on this selling price, and make 20% on the cost?

29. A grocer, by retailing sugar at $12\frac{1}{2}$¢ per lb., made 10% on the cost. What was the cost per bbl. of 200 lb.?

30. If, by selling nails at 6¢ per lb., I lose 4%, will I gain, or lose, by selling at 7¢? What % on cost?

31. At what must I buy boots by the case (1 doz. pairs) to make 15%, and sell at $4.60 per pair?

32. Coffee, which cost me 14¢ per lb., I sell at 6 lb. for $1. What % do I make?

33. If tea at 75¢ per lb. gives a profit of 20%, what would it yield at 56$\frac{1}{4}$¢? What at 50¢? What was the cost?

34. What % does a man make who sells a horse for $100 which was given to him? What, if he sells it for $250?

35. Owning $\frac{3}{5}$ of a factory, I sold 16$\frac{2}{3}$% of my interest for $800, which was considered to be 10% less than its real value. What was the estimated value of the factory?

36. What % is made by buying berries by dry measure, and selling at the same price per quart liquid measure? What % is lost if I buy by liquid measure, and sell by dry, at the same rate per quart?

37. A druggist buys a certain drug at $7.00 per lb. av., and sells it at $1.00 per oz. apothecaries' weight. What % profit does he make?

38. How must an article be sold by the dram (apothecaries'), which cost $5.00 per lb. av., to make 50% profit?

39. I made $1750 in a certain business in 1876, which was 15% more than I made in 1875. How much more did I make in '76 than in '75?

40. If I sell $\frac{3}{4}$ of an article for the cost of the whole of it, what % gain do I make on the part sold?

If I sell 3 parts for what 4 parts cost, I sell at $\frac{1}{3}$ more than cost; i.e., at 33$\frac{1}{3}$% advance.

41. If I sell $\frac{3}{4}$ of an article for what $\frac{1}{2}$ of it cost me, what % do I lose on the part sold?

If I sell 3 parts ($\frac{3}{4}$) for what 2 parts ($\frac{1}{2}=\frac{2}{4}$) cost me, I sell at $\frac{2}{3}$ cost, and hence lose $\frac{1}{3}$ cost, or 33$\frac{1}{3}$%.

Or suppose the article cost me $100: I sell for $50 what cost me

$75. Hence I lose $25, which is $\frac{1}{3}$ the cost of the part sold. ∴ I lose $33\frac{1}{3}$%.

42. If $\frac{4}{5}$ of the buying price equals the selling price, what is the loss per cent?

43. If $\frac{4}{5}$ of the selling price equals the buying price, what is the gain per cent?

44. B lost 5 per cent by selling a hectoliter of turpentine which cost $15. For what did he sell it a liter?

45. Sold cloth which cost me 8 francs per yard at 8 marks per yard. What % did I make?

COMMISSION, BROKERAGE, AND BANKRUPTCY.

317. An **Agent, Broker,** or **Commission-Merchant,** is a person who does business for another.

Commission or **Brokerage** is the percentage paid an agent, broker, or commission-merchant, and is estimated at a certain rate % on the amount of business done.

The distinction between Commission and Brokerage is not very clearly defined: but in a general way it may be said that the term *Broker* is more exclusively applied to persons dealing in money, stocks, exchanges, or other more characteristically monetary matters; while a Commission-Merchant deals in some other kind of property. The term *Agent* seems to be getting into use as a general term, covering all classes of business-men away from the central office who render service in business-affairs for others.

318. The **Amount** of money *received* or *expended* in behalf of another is usually the *base* on which commission is reckoned, except in case of dealings in stocks and exchange; in which cases it is customary to estimate brokerage on the *par value* of the paper.

319. A **Bankrupt** is a person who, having failed in business, is unable to pay his debts.

In such cases it is customary for the bankrupt to transfer his property to another person, called an *Assignee*, whose duty it is to settle with the creditors. The *Liabilities* are the sum of the debts. The *Assets* are the value of the property, including money, notes, and accounts due the bankrupt, etc.

1. Mr. Smith left with merchant Jones 5 doz. pr. gloves to be sold at $1.50 per pair, agreeing to allow Mr. Jones 10% for selling. What was Mr. Jones's commission (percentage) on 4 doz. which he sold? How much would he pay over to Mr. Smith?

2. Mr. Smith left with merchant Jones a certain number of doz. pr. gloves to be sold at $1.50 per pair, agreeing to allow him 10% for selling. On settlement, Mr. Jones's commission was $12.60. How many dozens were sold?

3. $13.86 commission for selling $198 worth of goods is what % commission?

4. A dealer in real estate sold a farm for Mr. A., charging him 5%. His commission was $375. For what did he sell the farm?

5. A real-estate dealer charges me 5% for selling my farm of 320 acres at $58 per acre. How much do I receive for the farm?

6. A grain-dealer sells 2000 bu. of wheat for me, and pays me $2450, his commission for selling being 2%. At what price per bushel did he sell it?

7. An attorney collects a claim of $650.50. He pays $23.75 costs, and charges 5% for collection. What does he pay the owner of the claim?

8. A bankrupt's assets were found to be $33000, and his liabilities $86000. What % can he pay? What will a creditor receive whom he owes $650?

9. I receive $850 on a claim of $1250 against a bankrupt estate. What % does the estate pay? If the liabilities are $95000, what are the assets?

10. Paid an attorney \$57.82$\frac{1}{2}$ for collecting a claim at 4$\frac{1}{2}$%. What was the claim?

11. A commission-merchant charged \$25.50 commission at 2$\frac{1}{2}$% for selling 120 bbl. of flour. What did he sell it at per barrel? and how much did he pay over to the owner?

12. What amount of goods can be purchased for \$318 if the agent retain 6% on the amount expended?

13. A real-estate agent received \$3000 to invest in land, after deducting his commission of 6$\frac{1}{4}$%. What amount did he invest?

14. A commission-merchant receives and sells 12600 bu. of wheat at \$1.37 per bushel on a commission of 3$\frac{1}{2}$%. What was his commission?

15. A commission-merchant charged \$17.28 for selling 640 bu. of potatoes at 60 cents a bushel. What was the rate per cent?

16. A commission-merchant sold 127 bbl. of flour for me at \$7.50 per bbl. He paid \$12.85 freight; and this, with his commission, amounted to \$70. What was the rate per cent of his commission?

17. How many barrels of flour, at \$7 a barrel, can an agent buy for \$441, after taking out his commission of 5%?

18. A country merchant forwarded 160 bbl. of flour to be sold at \$6.25 a barrel, the agent receiving a commission of 3% for selling. After paying \$5.45 for cartage, and deducting his commission of 1$\frac{1}{2}$%, he invested the proceeds in plaster at \$19 a ton. How many tons did he buy?

19. I sent a note of \$2500 to a lawyer in Hudson, with instruction to secure what he could upon it, as I understood that the firm against which the note was had gone into bankruptcy. He secured 62$\frac{1}{2}$% on the face of the note, and charged me 5% commission. How much did he remit to me?

The base is 62$\frac{1}{2}$% of 2500. Why? (See **318.**)

20. What amount of goods can be bought for $8758.25, allowing $2\frac{1}{2}$ per cent commission?

21. An agent received $5650 to invest in wheat, at a commission of $3\frac{1}{3}$ per cent. How much was expended in wheat? and what was the agent's commission?

22. A Michigan merchant sent to a commission-merchant in Chicago 12 tons of maple-sugar during the season. The Chicago merchant paid railroad charges at 50¢ per cwt., and $3.00 in all for cartage. His commission was $2\frac{1}{2}$ per cent. What would he remit the Michigan merchant, he having sold the sugar at 25¢ per pound?

23. A bankrupt's assets are found to be land valued at $3750; notes and accounts due him, $750; cash, $1250. He owes in New York, Mr. A., $4500; in Toledo, Mr. R. $1575, and Mr. C. $3000. I have a claim of $2540. Allowing the assignee 5% on the assets, and court expenses $350, what per cent is my claim worth?

STOCKS.

320. **A Company** is an association of persons for transacting business.

Business Companies are of two general classes, —*incorporated* and *unincorporated.* The former are spoken of as *Corporations,* and the latter as *Firms, Houses,* or *Partnerships.*

321. **Capital Stock,** or *Joint Stock,* is the amount of money paid, together with that subscribed, but not yet paid in, for the purpose of carrying on the business of the company or corporation.

322. **Stocks** are the certificates of a corporation, signed by the proper officers, and showing that the holder owns *so many* shares in the capital stock of the company.

Stocks are usually reckoned by *Shares* of $100 each. Thus, if the capital of a bank is $200,000, and I own 50 shares of the stock, I own $\frac{5,000}{200,000}$, or $\frac{1}{40}$ of the capital, and hence am entitled to $\frac{1}{40}$ of the net profits of the business. An owner of *stocks* is called a *Stockholder*.

323. The **Gross Earnings** of a company is the total amount of money, or its equivalent, received in the transaction of its business.

324. The **Net Earnings** of a company is the amount that is left after deducting from the gross earnings the expenses of conducting its business, losses, and accrued interest upon its bonds or other obligations.

325. An **Assessment** is a sum required of stockholders to meet the losses or expenses of the company.

326. A **Dividend** is an amount paid out of the *Net Earnings* to the *Stockholders*. It is usually reckoned at a certain per cent on the nominal or face value of the stocks.

327. The **Par Value** of stock is the face of the certificate or bond.

328. The **Market Value** of stock is the price per share at which it can be bought.

329. When stocks sell for more than their par value, they are said to be at a **Premium**; when for less, they are at a **Discount**.

330. **Premium, Discount,** *and* **Brokerage** *are always reckoned on the* **Par Value.**

331. **Stock-jobbing** is the business of buying and selling Stocks and Bonds, with a view to speculation.

332. **Quotations** are public statements made in the newspapers and otherwise of the *market value* of stocks, bonds, etc.

I read in the daily paper to-day the following *Quotations:* —

Kansas Pacific	$49\frac{1}{2}$.
Michigan Central	$82\frac{1}{2}$.
New-York Central	$116\frac{1}{2}$.
Illinois Central	$84\frac{1}{2}$.
Western Union Telegraph	$105\frac{1}{4}$.
Wells, Fargo, & Co.'s Express Stock	100.

This means that I can buy stocks in these companies at the annexed rates: i.e., Kansas Pacific at $\$49\frac{1}{2}$ per *share;* Michigan Central, $\$82\frac{1}{2}$ per share; New-York Central, $\$116\frac{1}{2}$ per share, etc. The stock of the 1st, 2d, and 4th are below par, the 3d and 5th above, and the 6th at par.

1. What cost 50 shares Michigan Central Railroad at $49\frac{1}{2}$?

2. At what per cent discount is Illinois Central Railroad stocks, quoted $84\frac{1}{2}$?

3. How much % below par is Kansas Pacific when quotations are $49\frac{1}{2}$?

4. I send to a broker in New York $1000 with which to buy New-York Central stocks, quoted at $116\frac{1}{2}$. If he can buy only whole shares, and charges me brokerage $\frac{1}{4}\%$, how many shares can he buy for me? and how much money will he return? What is his brokerage?

As every share will cost $\$116\frac{1}{2} + \$\frac{1}{4}$ for brokerage, it will cost $\$116\frac{3}{4}$. Hence he can buy $1000 \div 116\frac{3}{4} = 8$ shares and $66 remaining. His brokerage is $\frac{1}{4}\%$ of $800, or $2.

5. A gas company whose capital stock is $650000 declares a semi-annual dividend of $4\frac{1}{2}\%$. How much will be Mr. A.'s dividend, who holds 20 shares? What was the entire dividend?

6. A railroad corporation whose capital stock is $5000000 wishes to raise by assessment $45000. What will be the rate per cent assessed? and what Mr. Z.'s assessment, who owns 15 shares?

7. How much money must I remit to Chicago to buy 42

shares in Illinois Central, quoted at $84\frac{1}{2}$, allowing the broker there $\frac{1}{3}\%$ for buying?

8. If the Illinois Central declares a dividend of 5% per annum, what % do I make on my investment in Ex. 7?

As I pay $\$84\frac{5}{8}$ for each share, and receive \$5 per annum on a share, the question is, What % of $\$84\frac{5}{8}$ is \$5?

9. If New-York Central pays 4% semi-annually, and the quotation is $116\frac{1}{2}$, what per cent per annum shall I make on my money by investing in it, brokerage for buying being $\frac{1}{2}\%$ (calling 4% semi-annually the same as 8% per annum)?

10. At what rate must I buy stock in a company which pays $4\frac{1}{4}\%$ semi-annual dividends to make the investment yield me 12% per annum, brokerage being $\frac{1}{2}\%$, and calling $4\frac{1}{4}\%$ semi-annually the same as $8\frac{1}{2}$ annually? What would such stock be quoted?

The simplest way to think of this question is this: If I knew what a share cost me, I would *divide* the annual receipt from it, \$8.50, by this cost, and get .12, if I was making 12%. Hence \$8.50 divided by the cost of 1 share is .12; then $\$8.50 \div .12 = \$70\frac{5}{6}$ is the cost of 1 share. But $\$\frac{1}{2}$ of this is required for brokerage. Hence the quotation would be $70\frac{1}{3}$.

11. I have \$2500 to invest for a couple of orphan-girls. Will it be better for them that I invest it in stock quoted at $81\frac{1}{2}$, which pays $2\frac{3}{4}\%$ semi-annual dividends ($5\frac{1}{2}\%$ per annum), brokerage at $\frac{1}{4}$, or in real estate paying 7% per annum?

INSURANCE.

333. Insurance is a branch of business in which companies called *Insurance Companies* make contracts to pay specified sums of money to other parties in the event of certain losses to which the latter may be liable, the company receiving a percentage on the sum guaranteed.

334. The contract is called a **Policy**. The sum which the party insured pays to the company is called the **Premium**.

335. There are two principal departments of the insurance business; viz., *Fire Insurance* and *Life Insurance*.

336. As to the *Constitution of the Company*, Life-Insurance companies may be *proprietary*, *mutual*, or *mixed*, — *Proprietary* when the stock is subscribed, and the company constituted in the ordinary way of organizing business corporations; *Mutual* when each person insured becomes a member of the company, and hence is both insured and insurer; *Mixed* when both features are combined.

337. *Policies* are *Life Policies* when the amount guaranteed is due on the death of the insured; *Term Policies* when this sum is payable upon the death of the insured, provided it occurs within a specified time; *Endowment Policies* when the guaranty is payable when the insured reaches a certain age, or at his death if it occurs before he reaches that age.

338. In addition to these *principal* forms of insurance, there are various others; as *Marine*, *Accident*, *Health*, etc. The character of the ordinary operations and computations will be understood from the following problems. Any attempt to explain the principles upon which life insurance is calculated, or the theory of *probabilities* on which all insurance depends, is quite beyond the scope of this work.

For a clear and simple exposition of the principles on which life-insurance calculations are made, see the author's "SCIENCE OF ARITHMETIC."

1. An insurance company gives me a contract agreeing to pay me $\frac{2}{3}$ the value of my house in the event of its being burned during the year, for which security I pay the company $\frac{5}{8}\%$ on the amount insured. What is the written contract called? My house being worth $3000, how much do I pay for the insurance? What is this called? If my house is burned during the time, how much do I receive?

2. What is the annual premium on a policy which insures *a house* worth $12000 for $\frac{5}{6}$ its value at $\frac{1}{2}\%$?

3. I insure my house for $3500, furniture for $1500, and library for $1000, at 80¢ per annum on $100, for 3 years, paying $1 for policy, and 75¢ for survey (i.e., examination of premises). What is the total premium?

4. A life-insurance company gives me a contract in which they agree to pay my wife $4000 at my death, in consideration of my paying them an annual premium of 3%. What is my annual payment?

5. What is the annual premium on a life policy for $3500 at $2\frac{3}{4}$%?

6. A ship was insured for $38000 for 2 months at 4%, and its cargo for $22400 for 3 months at $6\frac{1}{2}$% ; and, being at sea the 64th day afterwards, the ship and cargo were destroyed by fire. What sum more than the premium received would the insurance company have to pay?

7. What is the rate % for insurance of $5000 in an accident-insurance company for one day, if the premium paid is 10 cents?

8. The premium for insuring goods was $5.50; the rate was $1\frac{1}{2}$%. For what amount were they insured?

9. If at 8% the premium on a boat was $146, for what amount was the boat insured?

10. If $59.22 was the premium, and 7% the rate, what was the amount of insurance?

11. A premium of $129.60 was paid for the insurance of $17280 on a store. What was the rate % of insurance?

12. What was the rate % of insurance on a box of goods, if it was insured for $340, and the premium was $4.25?

13. Mr. A. insures his house at 5% for $3160, which covers $\frac{3}{4}$ the value of the house, and $2 which he pays for the execution of the policy. What is the value of the house?

14. A ship from New York, valued at $52650, was insured for a whaling-cruise at $8\frac{1}{4}$%. What was the premium?

15. A gentleman has a house worth $16500, and furniture

valued at $3675, both of which are insured for $\frac{2}{3}$ of their value at $1\frac{1}{2}\%$. What annual premium does he pay?

16. A, at the age of 40, effects an insurance on his life for 4 years, for the sum of $8000, at the rate of $1.80 on $100 per annum. What is the annual premium?

17. My house is valued at $5000, my furniture at $2000, and my library at $2000. If I get the whole insured for $\frac{2}{3}$ its value at $\frac{3}{8}\%$, what is my annual premium? In the event of damage by fire, if my house is injured to the extent of $1500, my furniture $600, and my library $300, what shall I receive from the company?

ANS. Premium, $22.50. Ordinarily policies make the company liable to pay all damages up to the amount insured. Hence in this case I should receive $2400.

Companies often reserve the right to restore the damaged building in lieu of paying its estimated damage in money to the owner.

18. My house is worth $1200. I have it insured at $\frac{2}{5}\%$, so as to cover $\frac{2}{3}$ of its value and the premium if it chances to burn within the year. What is the sum named in the policy? What the premium?

Of every $1 premium paid, how much applies on the property, and how much to refund premium?

19. A manufacturing establishment, worth $200000, is insured in Co. A for $\frac{1}{5}$ its value at $\frac{3}{4}\%$, in Co. B for $\frac{1}{4}$ its value at $\frac{3}{5}\%$, and in Co. C for $\frac{1}{10}$ its value at $\frac{2}{3}\%$. What is the total annual premium? In case the establishment is damaged by fire to the extent of $15000, what is due from each company?

20. Jan. 1, 1879, I took out a policy on my house, valued at $6000, covering $\frac{3}{4}$ its worth at $\frac{2}{5}\%$, and paid $1 for policy, and $1 for surveying. I carried the policy 6 years and 3 mo., when the house burned. What was my loss, not reckoning interest on the annual premiums?

21. B's house, worth $15880, is insured for $\frac{4}{5}$ of the value at 4%, so as to include the premium if burned. Required the sum stated in the policy.

22. For what sum must a stock of dry-goods, worth $7500, be insured in order to include the value and the premium, when the rate of insurance is $2\frac{1}{2}$%?

TAXES AND DUTIES.

Taxes.

339. A **Tax** is money required by the government to be paid by the people of the country for the support of government, or for public enterprises.

The general theory of taxes is that they are laid upon *property*, and not upon persons: so it is designed that every dollar's worth of property shall bear an equal part of the tax to be raised. Apportioning the tax to be raised, and determining the value of each person's property, is called *Assessing*.

There are, however, in some of the States, what are called *Poll Taxes*. A poll tax is usually a small sum (75¢ or $1) required of every male over 21 years of age.

1. In a certain school-district, the entire property of which is valued[1] at $125,000, a schoolhouse costing $5000 is to be built by public tax. How much is assessed on a dollar? What is Mr. Smith's schoolhouse tax, his property being assessed at $3000? Mr. Jones's, whose property is assessed at $4500? Mr. White's, whose property is assessed at $10000?

If $125000 worth of property is taxed $5000, $1 bears $\frac{1}{125000}$ of the tax. Hence the tax on $1 is $\frac{5000}{125000}$ of a dollar, or 4 cents. Mr. Smith's tax would therefore be 3000 times 4¢, or $120; Mr. Jones's, $180; and Mr. White's, $400.

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[1] Such valuation for the purpose of assessing taxes is often made at half or less than half the actual value of property, though the present tendency is to require that it be made at the cash value.

2. The total State tax of Michigan for 1875 was $903-435.50.[1] The total valuation of property in the State was $630000000. What was the State tax on $1 valuation? What was a man's State tax whose property was valued at $2000?

3. In a certain village the taxes were, for State purposes, $1\frac{1}{2}$ mills on $1; for county purposes, $\frac{1}{2}$ mill on $1; for school, 3 mills; for township purposes, 2 mills; for corporation (village expenses), 2¢. What were a man's taxes whose property was listed (put on the tax-list by the assessor) at $3000? One whose property was listed at $600? What was the school tax of each? What the village tax?

4. A certain State legislature levies $\frac{1}{20}$ of a mill on a dollar as a tax for a particular interest. The valuation being $630000000, what will this yield?

5. In a State in which the entire valuation is $2500-000000, what amount will a tax of .01 of a mill on a dollar raise?

6. In a certain school-district the people desire to raise $5000 for the purpose of building a schoolhouse. It is estimated that 6% of the amount levied will not be collected, and 5% is to be allowed for collecting. What amount must be levied?

What will $1 tax levied net for the building of the house? The collector has 5% on all he collects, including his own percentage.

7. Verify the above. That is, in a certain school-district there was levied $5599.10 tax for building a schoolhouse. Of this 6% was uncollectible, and the collector was allowed 5% for collecting. What did the tax net for the building?

8. A bridge costing $250000 is to be built between two

[1] This was for such objects as the support of the State government; providing for interest and payments on the State debt; for the various State institutions, as the University, Agricultural College, Normal School, State Public School, the various *asylums and prisons*; and for carrying forward the building of the State Capitol.

cities. The larger city is to bear $\frac{3}{5}$ of the expense. The taxable property of this city being \$80000000, what will be a man's bridge tax who is taxed on 15000, assuming that the levy is made to include $3\frac{1}{2}\%$ for collecting, and 10% as uncollectible?

9. Verify the last. That is, having all the facts given, including the answer, except the cost of the bridge, to find the cost of the bridge.

10. The valuation of the taxable property of a town being \$2100000, what will a levy of $1\frac{1}{2}$ mills on a dollar net, 8% being uncollectible, and 3% being paid for collecting?

11. In a certain city a man pays \$35.22 tax on a valuation of \$1050. The entire tax levied being 425000, what is the total valuation of the city property? What is the actual value of the city property, if this man's property is worth \$4500?

12. The total valuation in a certain city is \$1500000, and a man taxed on \$1050 valuation pays \$35.22. What is the total tax levied?

13. The net proceeds of a certain assessment was \$150-000. Allowing $4\frac{1}{2}\%$ of the levy as having proved uncollectible, and $3\frac{1}{2}\%$ commission for collection, what was the total tax levied?

14. The assessed value of property in the State of Illinois for 1873 was \$1355401317. The total tax assessed was \$21963821.29. Of the latter 5023609.50 was for State purposes, \$5533091.20 for county purposes, \$1583942.32 for city purposes, and \$9823178.27 for town, district, and other local purposes. The school tax for this year was \$999-587.91.

15. What was the tax on \$1 valuation in the State of Illinois for 1873?

16. From the data in 5 and 6 fill out the following

TABLE.

PROPERTY.	TAX.	PROPERTY.	TAX.	PROPERTY.	TAX.	PROPERTY.	TAX.
$1		$10		$100		$1000	
2		20		200		2000	
3		30		300		3000	
4		40		400		4000	
5		50		500		5000	
6		60		600		6000	
7		70		700		7000	
8		80		800		8000	
9		90		900		9000	

17. From the above table, when filled out, determine by mere addition what a man's tax would be who was taxed on $5680 real estate and $1050 personal property. A man who was taxed on $548 real estate and $85 personal property. A man taxed on $8572 real estate and $12765 personal property.

18. How much property was a man assessed upon who paid $125 tax in Illinois in 1873? How much of this $125 was for State purposes? How much for school tax? If his property was assessed at $\frac{3}{5}$ its real value, what was this man worth?

19. What school tax did a man in Illinois who was worth $20000 pay in 1873, supposing his property to have been assessed at $\frac{1}{2}$ its real value?

United-States Revenue.

340. The expenses of the General Government[1] are provided for by a *Tax on Imported Goods*, and by the *Internal Revenue*.

341. Duties, or **Customs,** are taxes levied on imported articles, and are either *Specific* or *Ad Valorem*.

342. *Specific Duties* are duties levied on particular articles irrespective of their value. *Ad Valorem Duties* are duties levied on articles bought in foreign markets, and are estimated at a certain *per cent* on the net cost.

In transporting goods from one country to another there is more or less liability to loss, and consequently there are certain deductions made for such losses in cases in which specific duties are charged. The principal of these are *Draft*, an allowance by weight for waste; *Tare*, a deduction from gross weight, made for the weight of the box, bag, or other thing containing the goods; *Leakage*, a deduction made for actual loss through leakage from casks; *Breakage*, a loss by breakage from things imported in bottles.

In order to collect *Duties*, or *Customs*, Congress determines on what articles and at what rates they shall be charged; and the schedule embracing these facts is called a *Tariff*.[2] Congress also designates *Ports of Entry*; that is, ports where imported goods can be landed, and where *Custom-Houses* are built, and officers of government kept to collect customs.

343. Internal Revenue is revenue derived from sale of public lands; from sale of postage and other stamps; from taxes on certain manufactures, as distilled and malt liquors, etc.

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[1] That is, the government of the United States as a whole. These expenses are provided for by Congress, and embrace the salaries of United-States officers, the expenses of the mail, army, and navy service, the improvement of navigation, expenses for national buildings, etc.

[2] From Tarifa, a fortress established by the Moors at the Straits of Gibraltar, where they exacted duties from all vessels entering or leaving the Mediterranean Sea.

Laws regulating the internal revenue are called ***Excise Laws,*** in distinction from ***Tariff Laws,*** which regulate ***Duties.***

Ex. 1.—M. S. Smith, Detroit, Mich., importer of watches, etc., received an invoice of 3 cases of Swiss watches, costing 22800 francs; duty, 25%; cost of transportation, 35 francs; commission to agent in Geneva, 2½%. What was the total cost?

SOLUTION.

Net cost in Geneva,	22800	francs.
Duties paid in New York,	5700	"
Commission to agent in Geneva,	570	"
Transportation,	35	"
Cost in francs,	29105	
	.193	
Total cost,	$5617.265	

2. A dry-goods importer received at Boston from Liverpool the following invoice:—

650 yd. broadcloth	@	13*s.*
1246 yd. lace	@	2*s.*
1200 yd. coach lace	@	11*d.*
1950 yd. ingrain carpet	@	3*s.*
2560 yd. drugget	@	2*s.* 4*d.*

The duty on the broadcloth, carpeting, and drugget was 30%, and on the laces 25%. What were the customs?

3. What is the duty, at 40%, on 110 chests of tea, each containing 67 lb., and invoiced at 90¢ a lb., the tare being 9 lb. a chest?

4. What is the duty on 50 hhd. of molasses, 63 gal. each, at 20¢ per gal., leakage 3%?

5. What is the duty on 15 casks of Malaga wine, each holding 54 gallons, invoiced at 45 cents per gallon, allowing 2 per cent for leakage, the custom-house rate being 20 cents per gallon, and 25 per cent *ad valorem*.

On some articles both specific and *ad valorem* duties are charged, as is supposed in the last example.

6. A wholesale merchant in Boston imported 80 dozen bottles of Cologne water, invoiced at $5.25 per dozen. Allowing 5 per cent for breakage, and regarding a dozen bottles as equivalent to $2\frac{3}{4}$ gallons, what is the duty, the rate being $3 per gallon, and 50 per cent *ad valorem?*

7. What is the duty on 20 hhd. sugar, invoiced at 1160 lb. each, tare being 10%, and the duty 3¢ per lb.?

8. What is the duty on 3 lots linen hdkfs., which the appraiser classifies as follows? —

1 lot,	value	£34 4*s.* 6*d.*,	duty	35%.
1 lot,	"	£14 12*s.* 6*d.*,	"	40%.
1 lot,	"	£36 11*s.* 3*d.*,	"	40%.

9. How much will it take to pay the duties on 3 cases of French *mousseline de laine*, containing 7563.8 meters, invoiced at .88 francs, the impost being 8¢ per yd., and 40% *ad valorem*.

10. What is the duty on 5 T. 16 cwt. 3 qrs. 20 lb. of steel, invoiced at 25¢ per lb., the duty being 20%?

In the United-States custom-houses 112 lb. is called a hundredweight, whence 28 lb. is a quarter; i.e., the *Long Ton* is in use.

SECTION III.

INTEREST.

Ex. 1. — Mr. Smith lends me $250 for a year, and I agree to pay him back the $250 at the close of the year, and 6% additional for the use of the money. How much do I pay for the use of the money? How much do I pay Mr. Smith in all at the end of the year?

344. Interest is money paid for the use of money.[1]

345. The **Principal** is the sum for the use of which *interest* is paid.

It will be seen that *Principal* corresponds to *Base*, as heretofore used, and *Interest* to Percentage; so, also, the *Amount* is the sum of principal and interest.

346. Simple Interest is interest which is considered as falling due only when the principal is paid, or when a partial payment is made. It is usually reckoned at a certain per cent per annum (year).

According to this principle, — viz., that the interest does not fall due till a payment is made on the principal, — no interest is allowed on accrued interest.

[For information about USURY LAWS, see (**364**)].

2. What is the simple interest on $125 for 3 yr. at 7% per annum? What the amount?

OPERATION.	EXPLANATION.
$125 .07 8.75 3 $ 26.25 125.00 $151.25	Since 7% is .07 of the principal, the interest for 1 yr. is $125 × .07, or $8.75; and the interest for 3 yr. is 3 times the interest for 1 yr., or $8.75 × 3 = $26.25. The amount, being the sum of principal and interest, is $151.25.

3. What is the interest on $250.60 for 2½ yr. at 10% per annum? What the amount?

4. If I borrow of Mr. White $325 for 1 yr. 8 mo. 15 da. at 7%, what shall I have to pay him at the expiration of the time?

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[1] As the basis on which interest is computed is always money, it is not deemed best to cumber the definition with any allusion to any thing else.

EXPLANATION.

Since 7% is .07 of the principal, the interest for 1 yr. is \$325 × .07 = \$22.75. For 6 mo. the interest is ½ of the interest for a year, and the interest for 2 mo. is ⅓ the interest for 6 mo. 15 da. is ½ of 1 mo., or ¼ of 2 mo. Hence the interest for 15 da. is ¼ the interest for 2 mo. Adding these results, we have the interest for 1 yr. 8 mo. 15 da.

OPERATION BY ALIQUOT PARTS.

\$325	
.07	
\$22.75	Int. for 1 yr.
11.375	Int. for 6 mo. = ½ int. for 1 yr.
3.792	Int. for 2 mo. = ⅓ int. for 6 mo.
.948	Int. for 15 da. = ¼ int. for 2 mo.
\$38.86	Int. for 1 yr. 8 mo. 15 da.
325.00	Principal.
\$363.86	Amount.

OPERATION BY DECIMALS.

30	15.	da.
12	8.5	mo.
	$1.7\frac{1}{12}$	yr.

```
  $325
   .07
 22.75
 1.7 1/12
   189
15 925
22 75
38.864
325
$363.86
```

The interest for 1 year is \$22.75, and 1 yr. 8 mo. 15 da. = $1.7\frac{1}{12}$ yr. Hence the interest for 1 yr. 8 mo. and 15 da. is \$22.75 × $1.7\frac{1}{12}$.

To Compute Simple Interest.

347. Rule. — *Multiply the principal by the rate, and this product by the time in years.*

To find the amount, add the interest to the principal.

While this rule is perfectly general, some modifications in the form of the work arise from different methods of treating the *months* and *days*. We give three: 1st, Considering the months as aliquot parts of a year, and the days as aliquot parts of a month (30 da.); 2d, Reducing the months and days to decimals of a year; and, 3d, Calling the months 12ths of a year, and the days 30ths of 12ths, *indicating the operations, and cancelling.* (See **348, 349.**)

The method of computing interest by aliquot parts is in more general use, notwithstanding that the method by decimals usually requires less work. The most expeditious method is by cancellation. (See **348, 349.**)

Business-men, who have frequently occasion to compute interest, — as bankers, —*generally make use of Tables.*

5. What is the amount of \$350 for 3 yr. 10 mo. 19 da. at 8%?

By Aliquot Parts.

\$350	
.08	
28.00	
3	
84.00	
14.00	
7.00	
2.3333+	
1.1666+	
.2333+	
.0777+	
\$108.81	Interest.
350.00	
\$458.81	Amount.

By Decimals.

30	19.		\$350
12	10.633+		.08
	3.886+		\$28.00
	28		
	31 088		
	77 72		
	\$108.808	Interest.	
	350		
	\$458.81	Amount.	

Solve the following by each of the above methods, and observe which is the more expeditious. Find both interest and amount.

6. \$52.80 at 6% for 2 yr. 8 mo. 12 da.
7. \$235.50 at 10% for 3 yr. 6 mo. 10 da.
8. \$245.60 at 8% for 2 yr. 7 mo. 21 da.
9. \$500 at 6% for 2 yr. 5 mo. 12 da.
10. \$750.50 at 7% for 1 yr. 8 mo. 20 da.
11. \$436.75 at 5% for 1 yr. 2 mo. 15 da.
12. \$230 at 6% for 11 mo. 15 da.
13. \$1385.50 at 15% for 23 da.
14. \$14.30 at 8% for 2 yr. 9 mo.
15. \$325.25 at 6½% for 2 yr. 9 mo. 12 da.
16. \$2360.25 at 8% for 7 mo.
17. \$18.28 at 5% for 5 yr. 9 da.
18. \$87.50 at 7% for 3 yr. 3 mo.
19. \$480 at 15% for 6 yr. 3 mo.
20. \$18.20 at 5¾% for 9 yr. 9 mo. 9 da.

348. The last example solved by cancellation:—

Int. for 9 mo., $\dfrac{1.05 \times \overset{3}{\cancel{9}}}{\underset{4}{\cancel{12}}} = \dfrac{3.15}{4} = .79-$

Int. for 9 da., $\dfrac{1.05 \times \overset{3}{\cancel{9}}}{\cancel{12} \times \underset{10}{\cancel{30}}} = 0.26+$

For the parts of 1 yr. the interest will be found with sufficient accuracy if we take the interest for 1 yr. to the nearest cent.

	\$18.20
	.05¾
	910
	455
	9100
Int. 1 yr.,	1.0465
	9
Int. 9 yr.,	9.4185
" 9 mo.,	.79
" 9 da.,	.026
	\$10.23

349. Rule. — *To find the interest for months by cancellation, write as the numerator of a fraction the interest for 1 year, taken to the nearest cent, and the number of months as factors. For the denominator write 12, and then cancel.*

For days write as the numerator the interest for 1 year, and the number of days in the same way, and for the denominator 12 × 30, and then cancel.

That the three methods of considering the time are all embraced under the general rule (**347**) may be shown by indicating the three methods of solving the last problem thus:—

PRINCIPAL.	RATE.	TIME.	INTEREST.
\$18.20	× .05¾	× $(9 + \frac{1}{2} + \frac{1}{2} \text{ of } \frac{1}{4} + \frac{1}{10} \text{ of } \frac{1}{4})$	= \$10.23.
\$18.20	× .05¾	× (9.7¾)	= \$10.23.
\$18.20	× .05¾	× $(9 + \frac{9}{12} + \frac{9}{12 \times 30})$	= \$10.23.

These multiplications by the time in the first two methods would appear in the practical work thus (for the 3d, see above):—

Int. for 1 yr.,	1.0465	1.0465
	9	9.7¾
	9.4185	5232
Int. for ½ yr.,	.5232	2616
" ½ of ½ yr.,	.2616	73255
" $\frac{1}{10}$ of ¼ yr.,	.0261	94185
	\$10.23	\$10.23

21. \$64.50 at 7% for 2 yr. 16 da.
22. \$725 at $3\frac{1}{2}$% for 5 yr. 2 mo. 18 da.
23. \$5000 at 7% from May 6, 1875, to July 7, 1877.

Find the time by subtracting dates (**265**).

24. \$81.25 at 6% from Aug. 6, 1873, to Nov. 4, 1876.
25. \$105.23 at 10% from June 10, 1871, to Oct. 1, 1875.
26. \$76.42 at 5% from May 9, 1874, to Aug. 9, 1874.
27. \$18.00 at 8% from Aug. 8, 1875, to Aug. 30, 1875.
28. \$5600 at $4\frac{1}{2}$% from April 1, 1876, to April 1, 1878.
29. \$43.60 at $3\frac{3}{4}$% from July 12, 1874, to June 1, 1877.
30. \$150.30 at 10% from May 8, 1875, to Nov. 6, 1876.
31. \$400 at 10% from Sept. 6, 1876, to Sept. 6, 1878.
32. \$350 at 12% from Nov. 9, 1877, to Dec. 9, 1877.
33. \$820 at 7% from Dec. 5, 1876, to July 24, 1877.
34. \$1000 at 7% from Feb. 7, 1870, to Aug. 9, 1876.
35. \$125.41 at 7% from July 10, 1873, to June 10, 1874.
36. \$93.25 at 10% from Jan. 1, 1877, to July 1, 1877.
37. \$48.50 at 5% from Oct. 3, 1877, to Jan. 3, 1878.
38. \$150.40 at 7% from May 23, 1876, to Oct. 1, 1878.
39. \$741.50 at $5\frac{1}{2}$% from Nov. 29, 1875, to Aug. 30, 1877.
40. \$13.50 at 10% from May 7, 1876, to Sept. 10, 1876.
41. \$250 at 6% from July 1, 1873, to April 1, 1874.
42. \$450 at 5% from Aug. 7, 1875, to Aug. 7, 1877.
43. \$158.23 at 8% from Dec. 25, 1877, to Sept. 23, 1878.
44. \$354.40 at 8% from June 30, 1870, to Nov. 1, 1876.
45. \$700 at 9% from May 20, 1873, to March 6, 1875.
46. \$60 at $12\frac{1}{2}$% from March 17, 1875, to April 6, 1877.
47. \$4000 at 5% from Jan. 25, 1872, to Feb. 18, 1874.
48. \$250 at 10% from March 6, 1872, to April 30, 1873.
49. \$175.50 at 7% from Feb. 7, 1876, to Aug. 11, 1878.
50. \$300 at 8% from July 1, 1876, to Jan. 16, 1878.

[For further exercises and various forms of *notes*, see p. 263 *et seq.*]

To Find the Simple Interest on any Principal by means of Interest Tables.

There are several different volumes of such tables in use by bankers and accountants; but the general principle is the same. We have space to give only one page of such tables, and select that which gives the simple interest on $1, for any time less than 6 years, at 5%, 6%, 7%, 8%, 10%, and 12%. Such volumes generally contain tables which enable us to take the interest on any sum directly from the table, requiring no arithmetical process but addition.

350. Rule. — *To find the interest on any sum from the following table, take from the table the interest on* $1 *for the given number of years, months, and days, and add these results. Multiply this sum by the given principal.*

1. Find from the table the simple interest on $143.25 for 3 yr. 7 mo. 22 da. at 7% per annum.

INTEREST ON $1.

.21	For 3 yr.
.0408	For 7 mo.
.0043	For 22 da.
.255	For 3 yr. 7 mo. 22 da.

The interest on $143.25 is 143¼ times the interest on $1: hence

```
     .255
     143¼
       64
      765
    1020
    255
  $36.53  Int. required.
```

Solve the following by the table on the next page, finding both interest and amount: —

2. $340 at 5% for 2 yr. 5 mo. 11 da.
3. $28 at 10% for 93 da. (3 mo. 3 da.)
4. $12.50 at 8% for 63 da.
5. $135.37 at 7% for 5 mo. 13 da.
6. $81.40 at 8% for 1 yr. 17 da.
7. $471 at 10% for 2 yr. 6 mo. 5 da.
8. $251.13 at 7% for 30 da. For 1 yr. 3 mo.
9. $125.10 at 12% for 340 da. For 3 yr.
10. $2000 at 10% for 2 yr. For 3 yr. 6 mo. 10 da.
11. $57.35 at 7% for 2 yr. 8 mo. 10 da.

12%.	6%.	7%.		10%.	5%.	8%.
			YEARS.			
.12	.06	.07	1	.10	.05	.08
.24	.12	.14	2	.20	.10	.16
.36	.18	.21	3	.30	.15	.24
.48	.24	.28	4	.40	.20	.32
.60	.30	.35	5	.50	.25	.40
			MONTHS.			
.01	.005	.00583	1	.00833	.00416	.00666
.02	.01	.01166	2	.01666	.00833	.01333
.03	.015	.01750	3	.02500	.01250	.02000
.04	.02	.02333	4	.03333	.01666	.02666
.05	.025	.02916	5	.04166	.02083	.03333
.06	.03	.03500	6	.05000	.02500	.04000
.07	.035	.04083	7	.05833	.02916	.04666
.08	.04	.04666	8	.06666	.03333	.05333
.09	.045	.05250	9	.07500	.03750	.06000
.10	.05	.05833	10	.08333	.04166	.06666
.11	.055	.06416	11	.09166	.04583	.07333
			DAYS.			
.00033	.00016	.00019	1	.00027	.00013	.00022
.00066	.00033	.00038	2	.00055	.00027	.00044
.00100	.00050	.00058	3	.00083	.00041	.00066
.00133	.00066	.00077	4	.00111	.00055	.00088
.00166	.00083	.00097	5	.00138	.00069	.00111
.00200	.00100	.00116	6	.00166	.00083	.00133
.00233	.00116	.00136	7	.00194	.00097	.00155
.00266	.00133	.00155	8	.00222	.00111	.00177
.00300	.00150	.00175	9	.00250	.00125	.00200
.00333	.00166	.00194	10	.00277	.00138	.00222
.00366	.00183	.00213	11	.00305	.00152	.00244
.00400	.00200	.00233	12	.00333	.00166	.00266
.00433	.00216	.00252	13	.00361	.00180	.00288
.00466	.00233	.00272	14	.00388	.00194	.00311
.00500	.00250	.00291	15	.00416	.00208	.00333
.00533	.00266	.00311	16	.00444	.00222	.00355
.00566	.00283	.00330	17	.00472	.00236	.00377
.00600	.00300	.00350	18	.00500	.00250	.00400
.00633	.00316	.00369	19	.00527	.00263	.00422
.00666	.00333	.00388	20	.00555	.00277	.00444
.00700	.00350	.00408	21	.00583	.00291	.00466
.00733	.00366	.00427	22	.00611	.00305	.00488
.00766	.00383	.00447	23	.00638	.00319	.00511
.00800	.00400	.00466	24	.00666	.00333	.00533
.00833	.00416	.00486	25	.00694	.00347	.00555
.00866	.00433	.00505	26	00722	.00361	.00577
.00900	.00450	.00525	27	.00750	.00375	.00600
.00933	.00466	.00544	28	.00777	.00388	.00622
.00966	.00483	.00563	29	.00805	.00402	.00644

12. $145 at 8% for 3 yr. 11 mo. 5 da.

13. $280 at 6% for 7 mo. 16 da.

14. A note[1] of $65.80, dated Feb. 20, 1868, and bearing interest at 7%, was paid June 25, 1870. What was the amount paid?

Find the time by subtracting dates (265).

15. On the 21st day of January, 1874, for value received, I promise to pay to John Jones, or order,[2] $350, with interest at 7% per annum.

AUBURN, Dec. 5, 1869. HENRY FISH.

What was the amount of this note Jan. 21, 1874?

16. One day after date, for value received, I promise to pay John Smith, or bearer,[3] one hundred and twenty-five and $\frac{25}{100}$ dollars, with interest at 10%.

ROCHESTER, MICH., May 6, 1875. HENRY HOYT.

What was the amount of this note March 5, 1877?

17. Jan. 6, 1877, for value received, I promise to pay Enos Ames[4] five hundred and fifty dollars, with interest at 7%.

PERRYSBURG, O., May 7, 1875. M. C. PETERS.

What is the amount of this note Jan. 6, 1877?

18. Due Charles Minton, or order, thirty-six dollars, with interest at 6%, value received.

WESTON, O., April 6, 1874. JOHN PIPER.

What was the amount of this due bill Jan. 15, 1876?

---

[1] A *note* is a written contract by which one party agrees to pay another party a specified sum.

[2] The words "or order" in this connection prevent John Jones from selling the note, without putting his name on it; i.e., indorsing it. When indorsed, it is said to be "negotiable;" and John Jones can be made to pay it, if Henry Fish does not.

[3] This note is negotiable without being indorsed. Anybody can collect it who may chance to have it. But, if John Smith or anybody else does indorse it, the indorser becomes liable for it.

[4] As this note is payable to nobody but Enos Ames, no one else can collect it. It is not "negotiable," and Enos Ames cannot sell it even by indorsing it. This is the common law. There are statutes in some States, as in Illinois, making such paper negotiable by *indorsement*.

19. One day after date, for value received, we jointly and severally agree to pay Sarah Miner, or order, seven hundred dollars, with interest at 7%.

SOLOMON PIKE.
CHICAGO, ILL., June 6, 1875. JAMES NOAH.

What was the amount of this note March 29, 1877?

Such a note as the above is called a "*Joint and Several*" note, and either signer is equally liable for it. The holder may take his choice as to which he will collect it from, or he may proceed against both signers.

20. Two years from date, for value received, I promise to pay Stephen Ely, or order, three hundred and seventy-five dollars, with interest. (See **365.**)

PLYMOUTH, MICH., June 7, 1875. SMITH PHILLIPS.

What was the amount of this note June 7, 1877?

What would the amount have been if the note had been dated Plymouth, Mass.? If in Wisconsin? If in Ohio? In Illinois? In Minnesota? In Iowa?

21. Sold my house and lot Aug. 21, 1875, for $5500, receiving $2500 cash, and a 7% note for 3 yr. secured by mortgage for the balance. I immediately let the $2500 at 10% for 3 yr. When both became due, I bought a house and lot for $8560. How much money besides the avails of the house and lot sold did I have to raise?

22. Bought a bill of goods amounting to $750, $\frac{1}{3}$ payable in 30 da., $\frac{1}{3}$ in 60 da., and $\frac{1}{3}$ in 90 da., at 6%. What was the entire cost of the goods?

23. What is the amount of $83.25 at 8% from May 6, 1861, to Nov. 10, 1870?

At 10% from July 8, 1871, to April 17, 1873?

At 6½% from Sept. 13, 1870, to Feb. 13, 1875?

Other Methods of Reckoning the Time.

351. The foregoing method of obtaining the time by subtracting the earlier date from the later, calling 12 mo. a year and 30 da. a month, though the method in common use, does not usually get the exact time when months and days are involved. There are *three* other methods in use: —

1st, That which requires that a day be reckoned $\frac{1}{365}$ part of a year, and that the time be reckoned in years and days, the month unit being excluded. All interest transactions with the United-States Government are computed on this basis. It is called *Exact Interest.*

2d, A number of States have statutes requiring that the time be reckoned in *calendar* years and months, and that the days in excess be reckoned as 30ths of a month.

3d, *Bankers*, and sometimes other business-men, reckon interest on *short-time paper* by taking the exact number of days, and calling them 360ths of a year.

There is never more than one or two days' difference between the time as found by the *common method* and by the *second* above given.

From the interest obtained by the *Banker's Method* for a given number of days, $\frac{1}{73}$ must be subtracted to give the *Exact Interest*, and to the latter $\frac{1}{72}$ must be added to produce the former.

1. What is the exact interest on $450 at 10% from May 25, 1868, to Jan. 8, 1869? What by the Banker's Method?

The interest for 1 yr. is $45, and the exact time 228 da. Hence

$$\text{The } \textit{Exact Interest} \text{ is } \frac{\$\overset{9}{\cancel{45}} \times 228}{\underset{73}{\cancel{365}}} = \frac{2052}{73} = \$28.11-.$$

$$\text{By the } \textit{Banker's Method,}\ \frac{\$\cancel{45} \times \overset{114}{\cancel{228}}}{\underset{\underset{4}{\cancel{8}}}{\cancel{360}}} = \$28.50.$$

352. Days of Grace. — When money is borrowed at a

bank, the interest is required in advance (i.e., when the money is borrowed), and is reckoned for *three days* (in Pennsylvania four days) more than the nominal time. These three days are called DAYS OF GRACE. The interest in such a case is usually called *Discount*. (See **371.**)

This custom of allowing days of grace has become well-nigh universal with reference to *business paper* (obligations for the payment of money). The general rule is, that a suit at law cannot be instituted for the collection of any such paper until three days after its nominal maturity. Hence, in discounting such paper, it has become customary to compute the amount including these days. When the interest is paid at the time of settlement, it is, of course, reckoned to the date of such settlement.

---

353. The Maturity of a note is 3 days after it is nominally due.

When the date at which a note *falls due* is specified, the days of grace are always included.

2. What is the exact interest and what the bank discount on \$140.40 from Aug. 29, 1864, to Nov. 28, 1864, at 6%?

Time 60 da.

$$\frac{\overset{2.808}{\$\cancel{140.40}} \times 6 \times \cancel{60}}{\underset{73}{\cancel{365}} \times \cancel{100}} = \frac{16.848 \times 6}{73} = \frac{101.088}{73} = \$1.38+ \text{ Exact Interest.}$$

All the work appears here except the last division.

$$\frac{\$140.40 \times \cancel{6} \times \cancel{60}}{\cancel{360} \times 100} = \$1.40, \text{ Banker's Discount.}$$

3. What is the exact interest on \$1580 from June 10, 1874, to Feb. 17, 1875, at 10%?

4. What is the exact interest on a \$1000 United-States bond, at 5%, from Oct. 1 to May 6 following? From March 13 to Dec. 12 following?

Compute the exact interest on the following, and find the *amounts*: —

	Date.	Principal.	%.	When Due.
5.	May 10, 1876,	$45.25,	7,	Aug. 8, 1877.
6.	Sept. 20, 1876,	$82.10,	8,	June 5, 1877.
7.	Feb. 10, 1876,	$125.80,	5,	May 11, 1877.
8.	Jan. 1, 1871,	$530.00,	$4\frac{1}{2}$,	Nov. 10, 1873.
9.	April 7, 1874,	$1000.00,	4,	July 17, 1876.
10.	Aug. 13, 1876,	$250.00,	4,	Mar. 19, 1877.
11.	May 1, 1876,	$125.00,	7,	Sept. 6, 1876.
12.	Aug. 17, 1875,	$35.50,	10,	Sept. 21, 1875.

---

13. By the banker's method, what is the interest on a 7% $350 note dated May 11, 1876, and *nominally* payable Sept. 10, 1876? What by the common method? What by the method of exact interest? (All with grace.)

The interest for 1 yr. is $24.50.

By the *Banker's Method* the time is 125 da., or $\frac{125}{360}$ of a year, and the interest is $8.51—.

By the *Common Method* the time is 4 mo. 2 da., and the interest is $8.30+.

By the *Exact Method* the time is 125 da., or $\frac{125}{365}$ of a year, and the interest is 8.39+.

14. For value received, I promise to pay George Van Horn, or order, $500, Nov. 6, 1877, with interest at 10%.

PONTIAC, MICH., June 1, 1877. AMOS WHITE.

What is the interest on this note by the Banker's Method? When does the note *mature?*

Other Methods.

354. THE 6% METHOD. — When the time is to be reckoned in the common way (i.e., 12 mo. = 1 yr., and 30 da. = 1 mo.), *call $\frac{1}{2}$ the number of months* CENTS, *and $\frac{1}{6}$ the number of days* MILLS, and the sum will be the interest on $1 for the given time at 6%.

The reason for this is evident; since at 6% the interest on $1 for

1 yr. is 6c., or ½c. per mo. Again: as the interest on $1 for 1 mo. is 5 *mills*, it is 1 *mill* for every 6 days.

Ex. What is the interest on $245.50 at 6% for 2 yr. 7 mo. 21 da.?

½ the months is 15.5, and ⅙ the days 3.5. Hence the interest on $1 for the time at 6% is $0.1585. Multiplying this by 245½ gives $38.91, the interest required.

NOTE.—Having the interest at 6%, that at 5% can be obtained by *deducting* ⅙ of the interest at 6%, at 7% by *adding* ⅙, at 4% by *deducting* ⅓, at 8% by *adding* ⅓, etc.

355. THE 1% METHOD. — *Remove the decimal point in the principal 2 places to the left. Multiply this result by the rate per cent, and the time.*

Moving the decimal point 2 places to the left gives the interest on the principal for 1 yr. at 1%. Multiplying this by 7 gives it for 7%, etc.

Ex. Solve the above example in this way: —

Interest for	1 yr., at 1%,	$2.455
		6
" "	1 yr., at 6%,	$14.73
" "	2 yr.,	29.46
" "	6 mo.,	7.365
" "	1 mo.,	1.227
" "	10 da.,	.409
" "	10 da.,	.409
" "	1 da.,	.041
" "	2 yr. 7 mo. 21 da.,	$38.91

BY DECIMALS. — The time is 2.64⅙ year. Hence we multiply the interest for 1 yr. by the number of years.

$14.73
2.64⅙
245
5892
8838
2946
$38.9117

356. THE 12% METHOD. — *Remove the decimal point 2 places to the left, and then multiply by the time in months.*

This gives the interest for 12%, which is readily changed to any other rate. Thus, for 6%, take $\frac{1}{2}$ the interest at 12%; for 8%, deduct $\frac{1}{4}$; for 9%, deduct $\frac{1}{3}$; for 10%, deduct $\frac{1}{6}$; for 7%, add $\frac{1}{6}$ to $\frac{1}{2}$.

The reason for this rule is, that 12% is 1¢ on a dollar of principal for 1 mo.

Ex. — The last example computed in this way appears as in the margin.

```
   $2.455
     31.7
    17185
    2455
   7365
 2)77.8235
  $38.91
```

This is an excellent method. It will be observed that all the work which needs to be written for this solution appears in the margin.

[Practice in the use of any of these methods can be secured by the foregoing examples, if desired.]

COMPOUND INTEREST.

357. Compound Interest is interest considered as falling due at regular intervals of time, and to be reckoned as increasing the interest-bearing debt from such times.

This method of reckoning interest allows interest on interest accrued; and hence the term *compound*, meaning *interest on interest.*

1. What is the amount of $350 at annual compound interest for 3 years at 7%?

OPERATION.

$350	1st Prin.
.07	
24.50	Int. for 1 yr. on 1st Prin.
350.00	1st Prin.
$374.50	Amount for 1st yr., or 2d Prin.
.07	
26.2150	Int. on 2d Prin.
374.50	2d Prin.
$400.715	Amt. of 2d Prin. for 1 yr., or 3d Prin.
.07	
28.05005	Int. on 3d Prin.
400.715	3d Prin.
$428.76	Amt. at end of 3d yr.

EXPLANATION. — As the interest is considered as falling due at the end of each year, at the end of the first year the debt is \$374.50. This is, therefore, to be on interest for the next year. Again: as the interest on this for a year, \$26.215, falls due at the end of the year, it is added to the principal for this year, and makes the interest-bearing sum for the 3d year \$400.715. This sum on interest for a year amounts to \$428.77; which is, therefore, the amount of \$250 on compound interest for 3 yr. at 7%.

2. What is the amount of \$152 at semi-annual compound interest for 2 years at 6% per annum?

OPERATION.

```
      $152
       .03
      4.56
     152
    156.56
       .03
    4.6968
   156.56
   161.257
       .03
   4.83771
  161.257
  166.095
      .03
  4.98285
 166.095
$171.08
```

EXPLANATION.

As the interest is considered as falling due at the end of each ½ year, we compute the interest for ½ a year, and then add it to the principal, thus making a new principal for the next ½ year. Instead of multiplying by .06 in this instance, and dividing the product by 2 to get the interest for ½ a year, we simply multiply by .03, which gives the same result.

358. Rule. — *To compute Compound Interest, reckon the interest on the principal for the first interval of time, add it to the principal, and consider this as a new principal for the next interval, etc.*

Or, *Find from the interest tables the amount of* \$1 *for the given rate and time, and multiply this by the given principal.*

The result thus found is the AMOUNT. *The Compound Interest is the remainder after the first principal is subtracted from this amount.*

COMPOUND INTEREST TABLE.

YEAR.	3%.	4%.	4½%.	5%.	6%.	7%.
1	1.030000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.060900	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.092727	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.125509	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.159274	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.194052	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.229874	1.315932	1.360862	1.407100	1.503630	1.605781
8	1.266770	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.304773	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.343916	1.480244	1.552969	1.628898	1.790848	1.967151

3. Find the amount of \$243.12 at annual compound interest for 3 yr. at 4%, both with and without the use of the table. Also the interest.

By the Table. — The amount of \$1 at 4% for 3 yr. is \$1.12486. Now, \$243.12 amounts to 243.12 times as much as \$1, or \$273.48.

Hence the interest is \$273.48 — \$243.12 = \$30.36.

```
    $1.12486
      243.12
    --------
      224972
     112486
    337458
   449944
   224972
 -----------
$273.4759632
```

Find the compound interest of the following sums for the respective times and rates, both by the use of the table and without it: —

4. \$340 for 2 yr., compounded semi-annually, at 6%.[1]
5. \$100 for 7 yr. at 4½%.
6. \$230 for 6 yr. at 8%. At 5%. At 4%.
7. \$125 for 3 yr., compounded quarterly, at 12%.
8. \$270 for 4 yr., compounded semi-annually, at 8%.
9. \$250 for 3½ yr., compounded semi-annually, at 10%.
10. What is due on a note of \$200, bearing semi-annual compound interest at 9%, 2 yr. 10 mo. from date?

[1] This means, "at 6% per annum," but compounded (i.e., interest added to principal) every 6 mo.: hence it is the same as 3% for 4 years, or \$42.67.

SUGGESTION. — For $2\frac{1}{4}$ yr. the amount is \$249.2364. This is then on interest for 4 mo., which makes the whole amount \$256.71.

11. What is the difference between the simple interest of \$500 at 10% for 3 years and the compound interest on the same sum for the same rate and time?

12. What is the amount of \$325 at quarterly compound interest, at 2% per quarter, for 2 yr. 5 mo. 10 da.?

13. What is the interest of \$540.20, interest compounded semi-annually, at 5% per semi-annum, for 4 years? What is the difference between this and the interest compounded annually at 10%?

14. What is the difference between the interest of \$100, compounded quarterly at 6% per annum, for 2 yr., and the simple interest of the same sum for the same time at 7%?

15. What is the compound interest of \$480, at 5% per annum, from May 6, 1873, to July 13, 1875?

16. What is the amount of a note for \$500 Jan. 15, 1877, which draws 8% semi-annual compound interest, and is dated Aug. 18, 1874?

The laws of the States usually do not allow the collection of compound interest. In some States such notes as the above would be collectible with simple interest, while in others the taking of such a note would forfeit all interest; and in other States it would entail a still heavier loss, in some even the entire debt.

ANNUAL, SEMI-ANNUAL, AND QUARTERLY INTEREST.

359. Contracts are often made in which it is agreed that the interest shall be paid annually, semi-annually, or even quarterly. This is, in fact, compounding the interest thus often; but, if the payments of interest are not made as they fall due, the general rule is that only simple interest can be collected, although the statutes of some of the States allow simple interest on *the deferred payments of such interest.*

1. On a note for \$150, bearing interest at 7%, payable

annually, the debtor had neglected to pay the interest for 3 yr. Allowing simple interest on the deferred payments, what was then due on the note?

OPERATION.

$150	
.07	
$10.50	
.07	
.7350	
3	
$2.205	Int. on int.
31.50	Int. on prin.
150.00	
$183.71+	Amt.

As the first year's interest, $10.50, was not paid when due, it was subject to 2 yr. interest; and in like manner the 2d year's interest on principal was subject to 1 year's interest. Hence we have 3 years' interest on the interest of the principal for 1 year ($10.50) as the interest upon interest.

2. On the same principle as in the last, what is due on a note for $525, bearing 6% interest, payable annually, the interest payments having been deferred 4 years?

3. On the same principle, what is due on a $275 10% note, interest payable semi-annually, but deferred 3 yr. 8 mo. 17 da.?

4. As above, what is due on a $100 8% note, interest payable quarterly, but deferred 1 yr.?

5. As above, what is due on a note for $200, interest payable annually at 10%, but deferred 10 years?

In this there are 45 yr. interest on the interest for 1 yr.

Find the amount due on the following sums at the respective rates and times, the interest payable as indicated, but considered as deferred: —

6. $350, 7%, annually, for 3 yr. 5 mo. 10 da.
7. $820, 5%, semi-annually, for 3 yr. 9 mo. (See below.)
8. $85.30, 6%, semi-annually, for 1 yr. 10 mo.
9. $250, $4\frac{1}{2}$%, annually, for 5 yr. 8 mo. 12 da.
10. $500, 10%, semi-annually, for 2 yr. 7 mo.

The 7th gives $2\frac{1}{2}$% interest on $20.50 for $6\frac{1}{2}+5\frac{1}{2}+4\frac{1}{2}+3\frac{1}{2}+2\frac{1}{2}+1\frac{1}{2}+\frac{1}{2}=24\frac{1}{2}$ periods of 6 mo. each; i.e., $12.56 as the *interest on the interest.* To this add the amount of the $820 at simple interest for the entire time, and we have $973.75 + $12.56 = $986.31.

PARTIAL PAYMENTS.

United-States-Court Rule.

360. It frequently happens that a debtor does not pay his note all at one time. In such a case, whatever is paid at any time is indorsed (credited) on the back of the note, and is called a **Partial Payment** (or simply a payment).

There are several methods in more or less general use for computing interest on such notes. There are, however, but three that are in general use; viz., the one adopted by the U. S. Court and by most of the States, and that called the *Merchant's Rule*, which is much used by business-men for short-time paper, and the *Vermont Rule* for notes bearing interest payable annually.

The *U. S. Court Rule* is based on the two following principles: —

1. *The principal cannot be diminished until the accrued interest is paid.*
2. *Interest shall not draw interest.*

U. S. Court Rule for Computing Interest on Notes on which Partial Payments have been made.

361. Rule. — I. *Compute the Interest on the Principal from the date of the note to the time of the first payment. If this payment equals or exceeds this interest, find the amount, and subtract the payment. Treat this remainder as a New Principal, and proceed to the next payment. Continue the process till the time of settlement is reached.*

II. *If any payment is less than the accrued interest, add such payment to the next, and treat the sum as one payment made at the latter date.*

1. $350.

One day after date, for value received, I promise to pay

John Jay, or bearer, three hundred and fifty dollars ($350), with interest at 7% per annum.

ROCHESTER, N.Y., May 7, 1868. AMOS AMES.

On this note there were the following indorsements: —

Sept. 17, 1870, $100;
Feb. 10, 1872, $50.

How much was due on the note Oct. 25, 1872?

Interest at time of 1st payment	$57.85
As the payment ($100) exceeds this	350.00
we find the amount	$407.85
and subtract the payment	100.00
New Principal	$307.85
Interest on new principal at the time of the 2d payment .	30.11
As the payment ($50) exceeds this, we find the amount .	$337.96
and subtract the payment	50.00
2d *New Principal*	$287.96
Amount of this 2d new principal from date of last payment to Oct. 25, 1872	$302.24

2. $475. CHICAGO, ILL., Sept. 14, 1869.

On July 12, 1875, for value received, I promise to pay Peter Price, or order, four hundred and seventy-five dollars ($475), with interest at 6% per annum.

JAMES WHITE.

On this note were the following indorsements: —

April 12, 1871, $25; Aug. 20, 1873, $150;
Nov. 27, 1874, $100; May 1, 1875, $5.

What remained due on this note July 12, 1875?

SUGGESTIONS. — At the time of the 1st payment, the accrued interest was $44.97. If this be added to the principal, and the payment, $25, subtracted, *part* of this interest would be included in the *New Principal*, and hence interest would draw interest, and the 2d principle be violated. Hence we consider the 2d payment, made Aug. 20, 1873, as $175, and compute the interest on the face of the note, $475, up to this date.

This is $112.10; and, the payments ($175) being greater than the accrued interest, we find the amount, $587.10, and deduct these payments, leaving as a new principal $412.10. [N. B. — Each new principal must be less than the preceding, otherwise there will be interest on interest.]

The amount of this new principal at the time of the 3d payment was $443.49; and as this payment, $100, was more than the accrued interest, $31.39, our 2d new principal is $343.49.

Finally, it is evident, without careful computation, that the next payment, $5, over 5 mo. from the last, did not equal the interest then accrued. So we compute the interest on this 2d new principal, $343.49, to the time of settlement, and, finding the amount, deduct the $5. This leaves the amount due on the note $351.37.

3. $504. CLEVELAND, O., June 10, 1869.

On demand, for value received, I promise to pay Zenas White, or order, five hundred and four dollars ($504), with interest at 6% per annum. J. A. KING.

On this note were the following indorsements: —

Jan. 25, 1870, $84; May 15, 1870, $100;
Feb. 20, 1871, $200.

What was due July 5, 1871?

4. $450. LOUISVILLE, KY., Jan. 1, 1865.

Two years after date, for value received, I promise to pay to the order of James Jones four hundred and fifty dollars ($450), with interest at 8% per annum.

RANDALL WRIGHT.

On the back of this note were the following indorsements: —

March 16, 1865, $75; Jan. 1, 1866, $100;
April 4, 1866, $200.

What was due on the note Jan. 1, 1867?

5. Date of note, March 11, 1870; face, $58.50; rate per cent, 10. Payments: June 5, 1871, $12; Nov. 23, 1873, $6; Aug. 7, 1874, $5; Dec. 18, 1874, $20; May 10, 1876, $5. *How much* remained due July 1, 1876?

6. On a note of \$400, at 7%, there was paid \$100 annually for 3 years. How much remained due 3 yr. 4 mo. from the date of the note?

MERCHANT'S RULE.

362. It is a common practice with business-men to treat obligations maturing and settled in a year or less, and upon which payments have been made, according to the following

Rule. — *Find the amount of the principal from the date of the note to the time of settlement, find the amount of each payment from the time it was made to the time of settlement, and subtract their sum from the first result.*

1. \$250.60. ANN ARBOR, July 7, 1876.

For value received, I promise to pay Stephen Beckwith, or order, two hundred and fifty and $\frac{60}{100}$ dollars, April 15, 1877, with interest at 7%. EDWARD SNOW.

Indorsed Sept. 20, 1876, \$80.00.
" Jan. 1, 1877, \$50.00.
" Mar. 13, 1877, \$50.00.

What was the amount due April 15, 1877?

Reckoning the time by the Banker's Method (i.e., using exact time and the common interest tables, but without grace), we have, —

Amount of note April 15, 1877,						\$264.34
Amount of 1st	payment,	from	Sept. 20	to April 15,		\$83.22
"	2d	"	" Jan. 1	"		51.01
"	3d	"	" March 13	"		50.32
Total amount of payments,						\$184.55
Balance due April 15, 1877,						\$79.79

2. Date of note, Aug. 23, 1874; principal, \$420; rate, 10%; nominal maturity, May 1, 1875. Indorsements: \$100

Oct. 15, 1874; $200 Jan. 1, 1875. What was due May 1, 1875, reckoning the time by the Banker's Method?

3. $500. RICHMOND, Jan. 1, 1875.

Ninety days after date, for value received, I promise to pay to the order of Frank H. Ransom five hundred dollars, with interest at 6%. JOHN M. SABIN.

Indorsements: Jan. 20, $100; Feb. 10, $50; Feb. 25, $100; March 1, $150.

What was due at maturity (353), Banker's Method?

4. $400. BUFFALO, Jan. 1, 1874.

One year after date, for value received, I promise to pay N. Stacy, or order, four hundred dollars, with interest at 7%. M. M. DEYOUNG.

Indorsements: March 16, 1874, $200; July 1, 1874, $100.

What was due at maturity?

5. $700. DANBURY, Feb. 17, 1874.

Six months after date, for value received, I promise to pay John Gordon, or bearer, seven hundred dollars, with interest at 6%.

Received on the above, May 10, 1874, $350.
" " " June 25, 1874, $200.

When did the note mature, and what was due?

VERMONT RULE.[1]

Partial Payments on Notes with "Annual Interest."

363. When partial payments are made on notes, with *Interest* "payable annually," at other times than those at which the annual interest falls due, the method usually adopted is as follows: —

---

[1] The *Old Vermont Rule*, which was in quite general use in the country half a century ago, was the same as the *Merchant's Rule*, without limitation to "notes running a year or less." This rule, now known as the *Vermont Rule*, because first adopted by the courts of Vermont, has since been adopted by several other States, and is the one commonly used in computing interest on notes when the interest is payable annually, and partial payments have been made.

Find the interest on the note for 1 year; and find also the amount of the payments made during the year, from the times they were severally made to the end of the year.

If the payments amount to more than the interest due, take their amount from the amount of the note, and make the remainder a new principal.

But, if the amount of the payments does not equal the interest due, the principal remains unchanged; and the amount of the payments is taken from the interest, the remainder being treated as deferred interest.

Proceed in this manner with each year till the time of settlement, the last period being that from the time the last annual interest fell due to the time of settlement.

(*a*) The times at which interest falls due, and to which interest on payments is reckoned, and at which the amounts of the payments are applied, are called *Rests*. Courts have allowed these rests to be made at Jan. 1 on such notes, instead of at the time at which annual interest fell due. In some cases, banks have been allowed to make these *Rests* quarterly.

(*b*) In New Hampshire, if a payment made on a note bearing interest payable annually is less than the interest then due, it is carried forward, and added to the next payment *without interest*, and so on till the sum does exceed the interest, or to the time of settlement, when it is deducted; but when payments are made expressly on account of interest accruing, but not then due, they are applied when the interest falls due, *without interest* on so much of such payments as is necessary to cover the interest accruing.

Ex. 1. — On a 10% note for $600, with interest payable annually, and dated June 12, 1873, there were the following indorsements: —

June 12, 1874, $60;	Dec. 5, 1874, $100;
April 10, 1875, $50;	Nov. 4, 1876, $30.

What remained due Jan. 5, 1877, by the Vermont Rule?

Due June 12, 1874	$660.00
Paid June 12, 1874	60.00
Balance due June 12, 1874	$600.00
Interest for 1 year	60.00
Amount due June 12, 1875	$660.00
Amount of two payments made during this year; i.e., $100 for 189 da. and $50 for 63 da. ($150.178 + $50.863)	$156.041
Balance due June 12, 1875	$503.959
Interest on this for 1 year	50.396
Amount on interest from June 12, 1876, to settlement Jan. 5, 1877, 207 da.	$554.355
Interest on above for 207 da.	31.286
Amount due Jan 5, 1877	$585.792
Less amount of $30 payment for 62 da.	30.51
Balance due on settlement	$555.28

2. On a note for $1000, bearing 8% interest payable annually, and dated July 27, 1873, there were the following indorsements: Jan. 1, 1874, $50; Sept. 19, 1875, 150; July 27, 1876, $200; Feb. 3, 1877, $250. What was due Sept. 1, 1877? (Reckon calendar months.)

$500. CONCORD, N.H., June 7, 1873.

3. On demand, for value received, I promise to pay Enos Ames, or order, five hundred dollars, with interest at 6%, payable annually. AMOS WHITE.

Indorsements: Feb. 10, 1874, $15; Aug. 15, 1874, $25; May 17, 1875, $150; Jan. 13, 1876, $20. What was due, reckoning calendar months, Oct. 18, 1876? What, if the payments were made "on interest accruing"?

USURY LAWS.

364. Usury Laws are laws regulating the rate of interest, or manner of reckoning it.

365. Legal Interest.—*Legal Interest* is the rate per cent established by law as that which is to be implied in an interest-bearing obligation in which the rate is not specified.

In Louisiana the legal rate is	5%.
In the N. E. States (except Conn.), N. C., Penn., Del., Md., Va., W. Va., Tenn., Ky., O., Mc., Miss., Ark., Io., Ill., Ind., the Dist. of Columbia, and debts due the United States .	6%.
In N. Y., Conn., N. J., S. C., Ga., Mich., Minn., Kan., and Wis.	7%.
In Ala., Fla., and Tex..	8%.
In Col., Neb., Nev., Ore., Cal., and Washington Territory .	10%.
In England and France	5%.
In Canada, Nova Scotia, and Ireland	6%.

366. **Usury** is a higher rate of interest than is lawful. Most of the States allow interest above the legal rate, when it is agreed upon between the parties and specified in the contract. Thus O. and La. and Mich. allow any rate up to 8%; Ill., Io., Miss., Wis., Mo., and Tenn., up to 10%; Minn. and Tex., up to 12%; Neb., 15%; Kan., 20%; Mass., R. I., Fla., Ark., Cal., Nev., Col., any rate agreed upon.

Of course, in those States where parties are prohibited from making a contract for more than a certain rate, *Compound Interest* is illegal, and cannot be collected at law.

367. In computing interest or discount, "a year" is a calendar year, and "a month" a calendar month. To this there are no exceptions in the States. Also, when years, months, and days are mentioned in the contract, the days are reckoned as 30ths of a month. But, in transactions with the General Government, the month unit is dropped, and the time is reckoned in years and days, the days being called 365ths of a year. In New York, when time is specified in days, the days are to be reckoned as 365ths of a year.

368. Notes falling due on Sunday, or on a legal holiday, are in most of the States required to be paid on the preceding day. In Connecticut, if the day of maturity is a legal holiday falling on Sunday, the note is due on Monday. In Maine and Nebraska, if the day of maturity is a legal holiday falling on Monday, the note is payable on Tuesday; and in New York a note maturing on a legal holiday, or Monday observed as such holiday, is payable the following day.

369. In the following States, simple interest can be collected on unpaid annual interest; viz., Michigan (same rate as borne by the note), Ohio, Wisconsin, Vermont, New Hampshire, Iowa (6%). In Pennsylvania, Georgia, Illinois, and Indiana, by special contract (only). In Massachusetts such annual interest can be sued for when due; but no interest can be collected on it.

370. In Pennsylvania a note for 30 da. is discounted at bank for 34 da.; one for 60 da., for 64 da.; one for 90 da., for 94 da. This practice comes from counting both the day on which the note is drawn and the day on which it falls due. In the ordinary practice, only one of these is counted.

SECTION IV.

DISCOUNT.

371. Discount is a general term used by business-men to signify any deduction made from a *nominal* price or value. There are *three* principal uses of the term; as in what is called *Commercial or Trade Discount*, *Bank Discount*, and *True Discount*.

COMMERCIAL OR TRADE DISCOUNT.

Ex. 1.—I asked a bookseller the price of a certain book. He answered, "The list price is $15; but I can allow you a *discount* of 30%." What did he ask me for the book?

By a "*discount* of 30%," he meant 30% less that $15. 30% of $15 is $4.50. Hence he proposed to sell me the book for $15 — $4.50, or $10.50. The $4.50 may be called the *Commercial* or *Trade Discount.*

A slightly modified form of *Trade Discount* is illustrated by the following example:—

2. A Western shoe-merchant buys of a Boston dealer, on 60 days' time, the following bill: the understanding being, that, if payment is made in 30 da., he shall have "2% off;" and if in 10 da., 3%:—

2 cases boots,	$30	$60.00
3 cases "	$96	$288.00
½ case shoes,	$90	$45.00

What amount will pay the bill in 10 da.? What in 30 da.?

The purpose of this arrangement is to make it for the interest of the purchaser to pay as soon as possible. Thus, in this case, if he does not pay till the expiration of the 60 da., he must pay the full amount of the bill, $393.00; but, if he pays within 10 days, he gets a discount of 3%, or $11.79, having only $381.21 to pay. If he pays any time between 10 and 30 days, or on the 30th, he gets 2% off (that is, $7.96), and has to pay $385.04.

372. Commercial or Trade Discount is a deduction from the *nominal* price, or value, of an article, or from the amount of a bill of purchase for payment before it falls due.

[For the solution of the following, the principles of Simple Percentage are adequate.]

3. Having bought a bill of goods amounting to $250 on 90 days' credit, the tradesman says to me, "For cash I could discount you 10% on this bill." What amount of money would pay the bill *now?*

4. A certain article is marked to sell at 25% advance on cost; and the dealer gives me 10% off from retail price, and I pay $6.75 for it. What was the cost?

5. I buy of A. T. Stewart & Co., on 4 mo. time, a bill of goods amounting to $500; the rule of the house being to allow 6% off if payment is made in 10 da., and 5% if made in 30 da. What amount will pay the bill in 10 da.? What in 30 da.?

6. I paid $2.20 for a book on which the bookseller allowed me 20% discount from the retail price. What was the retail price?

7. What is saved by paying a 60 da. bill for $1200, ½ in 10 da. at 5% discount, and ½ in 20 da. at 4% discount? (Interest on the money not considered.)

8. I buy goods for $350 on 30 da., and for $500 on 60 da., and pay the former in 10 da. with 3½% discount, and the latter in 20 da. with 5% discount. How much better is this than 10% per annum for my money for the time I anticipate the payments? (Exact interest.)

9. A dry-goods merchant, finding a piece of cloth which cost him \$3.75 per yard somewhat damaged, offered it for sale at 10% discount. What did he ask per yard for it?

10. A merchant sold some damaged cloth at \$3.37½ per yard, which was at a discount of 10% from the cost. What was the cost per yard?

11. A merchant sold cloth, which cost him \$3.75 per yard, at \$3.37½. What per cent did he discount?

12. What is the cash value of a bill amounting to \$3750 at 10% discount, and 2½% off for cash?

By this is meant, that, for the usual time which the house allows credit, they will sell the purchaser a bill of goods of \$3750, reckoned at the regular rates, for 10% off; but for cash down they will deduct 2½% from this.

13. What is the cash value of a bill of goods amounting to \$2157.25 at 15% discount, and 3% off for cash?

BANK DISCOUNT.

Ex. 1. — John Smith desiring to borrow some money *at a Bank*, they tell him that they can "*accommodate*" him, and are "*discounting*" at 8%. They then furnish him a blank note, which, when filled out and signed, reads as follows: —

\$200. ANN ARBOR, May 5, 1879.

Sixty days after date I promise to pay to the order of James F. Royce two hundred dollars at the First National Bank, value received. JOHN SMITH.

Mr. Smith's friend, Mr. Royce, then writes his name on the back of the note, and becomes his *Indorser*. The note is then taken to the bank by Mr. Smith; and they take it, and pay him \$200, less the interest on \$200 for 63 days at 8%; i.e., \$200 — \$2.80, or \$197.20. The \$197.20 is called the *Proceeds*, or *Avails*; and the \$2.80 is the "*Discount.*"

373. Bank Discount is interest paid in advance, and for 3 days more than the nominal time. These 3 days are called *Days of Grace*. (See **352**.)

The above note "*Matures*" (i.e., falls due) July 7, 1879, 63 days after date. If Mr. Smith does not pay it before the close of business-hours on that day, the bank sends Mr. Royce a notice called a "*Protest*." This notice states that Mr. Smith has failed to pay his note, and that the bank now holds Mr. Royce responsible for it. This makes Mr. Royce liable, and he must pay the note if Mr Smith does not.

2. April 29, 1879, wishing to raise a little money, I find I have a good note against Mr. E. Wright for $240, dated Jan. 10, 1879, bearing 10% interest, and due 6 mo. after date. I take this to the bank, and find that they will discount it for me at 8% if I will indorse it. I put my name on the back, and hand it in. How much money do I receive?

Since Mr. Wright is not obliged to pay this note till 3 days after it is nominally due (i.e., until July 13), the bank will reckon interest on it to that date, and discount it accordingly. When it matures, the note will bring $252.20. The bank discount on this for the 75 da. from April 29 to July 13, at 8%, is $\frac{252.20 \times 8 \times 75}{360 \times 100} = \$4.20$. Hence I shall receive $248.

3. In many banks, as in those of New-York City, bank discount is reckoned as *Exact Interest* in advance. In such a bank, what is the discount on a note of $5000 for $^{60}/_{63}$ days at 10% per annum?

A note given at bank for 30 da. matures in 33, and the time is usually written $^{30}/_{33}$. So, also, $^{60}/_{63}$ da. means *nominally* due in 60 da., but *legally* in 63.

4. $175. ANN ARBOR, MICH., Feb. 23, 1879.

Sixty days after date I promise to pay to the order of Jas. F. Royce one hundred and seventy-five dollars at the Ann-Arbor Savings Bank, for value received, with ten per cent interest after due. EDWARD OLNEY.

This note, being indorsed by Mr. Royce, was discounted

at the savings bank on the day of its date. What were the proceeds?

5. I have a 7% note for $500, dated Jan. 25, 1874, and nominally due Dec. 10, 1875. I get it discounted at bank for 10% Sept. 6, 1875. What are the proceeds?

6. April 1, 1879, a merchant, being in need of ready money, finds that he has the three following notes: —

(1) Mr. Brown's note for $350, bearing 7% interest, dated Feb 6, 1879, and due 4 mo. after date;

(2) Mr. Jones's note for $300, bearing 6% interest, dated March 1, 1879, and due 2 mo. after date; and

(3) Mr. Smith's note for $150, bearing 8% interest, dated March 12, 1879, and due 3 mo. after date.

These being good notes, he indorses them (i.e., puts his name on the back of each), and, taking them to the bank, gets them discounted at 8%. How much ready money does he raise?

In finding the amount of a note in such a case, reckon *calendar* months, and any excess of days (with grace) as 30ths of a month. In computing the discount, reckon the exact number of days (with grace), and call them 360ths of a year. This would be the common practice of banks.

7. June 10, 1879, for value received, I promise to pay Enos White, or order, $320.

DETROIT, Jan. 5, 1879. JOHN EZRA.

By indorsing this note, Mr. White got it discounted at bank March 10, 1879, at 7%. What were the proceeds?

Such a note does not draw interest.

374. A **Negotiable** note is a note which the holder may sell to another person, who shall have legal power to collect it. The words "*or bearer*" make a note negotiable without indorsement. "*Or order*" requires indorsement. If no such phrase is contained, the note is not negotiable, either *with* or without indorsement.

8. $750. BOSTON, June 16, 1878.

Nine months after date, for value received, I promise to pay Mary Smith, or order, seven hundred fifty dollars, with interest at 6 per cent.

JOHN E. HOWE.

What is the discount, at 6%, Oct. 24, 1878?

How would this note be made negotiable?

9. $375. CHICAGO, ILL., Dec. 20, 1876.

Sixty days after date, for value received, I promise to pay E. D. Bronson, or bearer, three hundred seventy-five dollars, with interest at 10%, at the First National Bank, Chicago, Ill.

S. HOWARD BLACKWELL.

This note was discounted Jan. 23, 1877, at 10%. How was this note made negotiable? What were the proceeds?

Technically and legally, such a note is negotiable without indorsement; but it is the custom of *banks* to require indorsement, the same as when drawn payable to "order."

10. A note of $1400, dated July 19, 1877, due May 1, 1878, with interest at 6%, and discounted Jan. 17, 1878, at 10%. What were the proceeds?

11. A note of $2400, dated Oct. 16, 1877, due Jan. 1, 1879, with interest at 8%, and discounted July 26, 1878, at 10%. What were the proceeds?

12. CLEVELAND, Aug. 7, 1877.

Four months after date, for value received, I promise to pay Mr. Elisha Jones five hundred dollars, with interest at 8%.

JOHN GORTON.

Mr. Jones, having written his name on the back of this note, sells it to Peter Dull, who, wishing to get it discounted, presents it at a bank Oct. 10, 1878. What are the avails?

13. A note for $6000 was made May 10, 1868, payable in six months with interest at 9%, and discounted at a bank in Michigan, Oct. 3, 1868, at 7%. What were the proceeds?

14. What is the bank discount on a note for $250 (*Exact Int.*) at 7% for $^{60}/_{63}$ da.? For $^{30}/_{33}$ da.? At 10% for $^{90}/_{93}$ da.? For $^{30}/_{33}$ da.?

EXPEDITIOUS METHODS OF COMPUTING BANK DISCOUNT FOR 33, 63, AND 93 DAYS.

[As most Bank Paper is made for one of these three times, the following simple and elegant methods are worth knowing.]

375. ***General Method.* — *For 12 Per Cent.***

Calling 360 da. a year, to obtain the interest at 12% on any principal

For 33 da., take 11-1000ths of the principal.
For 63 da., take 21-1000ths of the principal.
For 93 da., take 31-1000ths of the principal.

DEMONSTRATION. — For 12% for 33 da., letting P represent the principal, we have

$$\frac{\overset{11}{\cancel{33}} \times \cancel{12} \times P}{\underset{\underset{10}{\cancel{120}}}{\cancel{360}} \times 100} = \frac{11}{1000} \text{ of P.}$$

The others are demonstrated in the same manner.

Observe that to take $\frac{11}{1000}$ is to multiply by 11, and remove the decimal point three places to the left.

To multiply by 11, write the principal under itself, removing it one place to the left, and add; to multiply by 21, write 2 times the principal in the same way; to multiply by 31, write 3 times the principal in the same way.

Ex. 1. — Find the interest on $5872 at 12% for 33 da., 63 da., 93 da.

$5 872	$5 872	$5 872
58 72	117 44	176 16
$64.59 Int. for 33 da.	$123.31 Int. for 63 da.	$182.03 Int. for 93 da.

376. ***For other Rates Per Cent than 12.***
First find 12% as above. Then,

For 6%, take $\frac{1}{2}$ of 12%.
For 7%, add $\frac{1}{6}$ to 6%.
For 8%, deduct $\frac{1}{3}$ from 12%.
For 9%, deduct $\frac{1}{4}$ from 12%.
For 10%, deduct $\frac{1}{6}$ from 12%.

NOTE.—When the principal is a round number of hundreds of dollars, 12% can be told at a glance. Thus 12% on $300 for 33 da. is $3.30; on $500, $5.50; on $700, $7.70, etc. Again: 12% on $300 for 63 da. is $6.30; on $800, $16.80; on $700, $14.70, etc. For 93 da. 12% on $200 is $6.20; on $100 is $3.10; on $400, $12.40. Thus it will be seen that for 33 da. the dollars in the discount are the hundreds of dollars in the principal; for 63 da., twice the hundreds; for 93 da., three times the hundreds,—the cents in each case being the principal with the right-hand 0 dropped.

2. Find the bank discount on $600 at 6% for 33 da.; 63 da.; 93 da.

$6.60	$12.60	$18.60
$3.30 for 33 da.	$6.30 for 63 da.	$9.30 for 93 da.

3. Find the bank discount on $200 at 8% for 33 da.; 63 da.; 93 da.

$2.20	$4.20	$6.20
.73	1.40	2.07
$1.47 for 33 da.	$2.80 for 63 da.	$4.13 for 93 da.

4. Tell mentally the bank discount on $200 for 33 da. at 12%; 6%; 7%; 8%; 9%; 10%. Also for 63 da. For 93 da.

5. Compute as above the bank discount on $756.80 for 33 da. at 12%. At 6%; 7%; 8%; 9%; 10%. Also for 63 da. and for 93 da.

To find the Face of a Note to be made at Bank, in order to obtain a Given Sum as Proceeds.

Ex. 1.—I wish to obtain \$500 at bank for $^{60}/_{63}$, and they are discounting at 10%. For what amount must I draw my note?

\$1 face of note gives \$1.00 — \$.0175 = \$0.9825 proceeds, since the interest on \$1 for $^{60}/_{63}$ at 10% is \$0.0175. Hence, to obtain \$500, I must make my note for $\frac{\$500}{.9825} = \$508.91$.

PROOF.—If I make a note for \$508.91 for $^{60}/_{63}$, the bank will deduct from the face of the note the *Bank Discount*, which is the interest in advance. Now, the interest for \$508.91 for $^{60}/_{63}$ da. at 10% is $\$508.78 \times \frac{1}{10} \times \frac{63}{360} = \$8.81$. Hence the proceeds of such a note are \$500.

377. Rule.—*Find the interest of \$1 for the given rate and time (including 3 da. grace), and, deducting this interest from \$1, divide the sum desired by the remainder. The quotient is the face of the note.*

[Reckon 360 da. a year.]

2. For what must I draw my note in order to obtain \$50 at bank for $^{30}/_{33}$ da., when they are discounting at \$8%? Give proof.

3. For what must I draw my note at bank for $^{90}/_{93}$ da. in order to obtain \$1000, when they are discounting at 7%? Give proof.

4. What is the bank discount on a note for $^{45}/_{48}$ da., which yields \$2500 proceeds, at 9%?

5. In order to obtain \$350 at bank for $^{30}/_{33}$ da., what must be the face of my note, discount being at 7%? What, to get \$150 for $^{90}/_{93}$ da. at 10%? To get \$750 for $^{60}/_{63}$ at 5%?

6. I owe a bill for flour and feed amounting to \$73.25, and give my note for 90 da. How must I draw it to cover the discount at 8%?

7. Sold a horse for $250, a carriage for $175, and a set of harness for $120, and took the purchaser's note for 90 da., so as to cover the discount at 6%. What was the face of the note?

8. Bought a bill of goods amounting to $2500 on 3 months' credit without interest. What should I be required to pay down, money being worth 10%?

9. A merchant buys a bill of goods, which he can have at $^{60}/_{63}$ da. credit, for $2850, or for $2800 cash. He can borrow at bank for 8%. Would it be better to do so? What would be the difference?

[Practically, banks do not use the above process. Were I in want of $300 for 60 da., and a bank was willing to "accommodate" me at 8%, the cashier would see from his tables that the discount on $300 would be $4.20, and would say, "Make your note for $305." The proceeds of this would be $300.73.]

TRUE DISCOUNT.

1. I have a note due 2 years hence, which will bring me at that time $228. I wish to obtain the money on it *now*. What is it worth, the use of money being worth 7% per annum?

The supposition is that the use of $1 for a year is worth to me $0.07, and for 2 years $0.14. Hence every $1.14 of the $228 due 2 years hence is worth to me $1 *now*. $228 \div 1.14 = 200$. Therefore the note is worth $200 *now*. The $200 is called the *Present Worth*. $228 — $200, or $28, is the *Discount*. To distinguish this from *Bank Discount*, it is usually called *True Discount*.

378. True Discount is a deduction made for the present payment of a sum of money due at some future time.

379. The Present Worth of a sum of money due at some future time is a sum which, put at interest at a rate agreed upon, will in the given time amount to the sum due.

2. I take a note for \$300, bearing interest at 7%, and due $3\frac{1}{2}$ years hence. What is its present worth, money being worth 10%?

SOLUTION. — The *amount* of the note at maturity will be \$373.50. Now, \$1 at 10% will amount to \$1.35 in the given time. Hence the *Present Worth* of said note is $373.50 \div 1.35 = 276.66\frac{2}{3}$, or \$$276.66\frac{2}{3}$.

PROOF. — That this is just appears from the fact, that, if I retain the note, I shall get \$373.50 at the expiration of $3\frac{1}{2}$ years; while if I sell it for \$$276.66\frac{2}{3}$, and the money is worth 10% to me, I shall realize the *amount* of \$$276.66\frac{2}{3}$ at 10% for $3\frac{1}{2}$ years, or \$373.50.

3. I take a note for \$300, bearing interest at 10%, due $3\frac{1}{2}$ years hence. What is its present worth, money being worth 7%?

The *amount* due on the note at the end of the time will be \$405. But, as money is now worth only 7%, \$1 in hand now will amount to \$1.245 in the $3\frac{1}{2}$ years. Hence the present worth is $405 \div 1.245$.

That this note is worth *more than its face* (\$300) is evident, since it is drawing a *higher rate of interest* than money is *now* worth.

380. The Face of a Note is commonly understood to be the principal, or that portion of the principal which is unpaid. Some, however, use the phrase as signifying what is due at the time; while others use it as signifying *the Amount of the Note at Maturity.*[1]

381. When the *True Present Worth* of a note exceeds the *face* of the note, this excess is called **Premium.**

382. The difference between the nominal present value (as the *face* of a note) and the *True Present Worth* is the **True Discount,** or **Premium,** as the case may be.

To find the Present Worth of a Sum of Money due at some Future Time.

383. Rule. — *Divide the sum due at the future date by the amount of \$1 at the rate agreed upon for the time from which it is proposed to discount the sum till the time said sum is due. The quotient is the Present Worth.*

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[1] I find no authority in the books for any thing but the former. — AUTHOR.

4. What is the true present worth of the following note, May 13, 1879, discounted at 8%?

$276. GRAND RAPIDS, MICH., Aug. 7, 1877.

For value received I promise to pay er White, or order, Dec. 10, 1879, two hundred seventy-six dollars, with interest at 6%. PETER DULL.

5. I have a 7% note for $186.50, dated Feb. 7, 1876, and due Sept. 20, 1879. What is its true present worth July 17, 1878, discounting at 10%?

6. Jan. 14, 1878, a speculator offered me $300 for a note of $350, dated May 7, 1877, payable Oct. 21, 1879, and bearing 6% interest, money being worth 10%. Did he offer me the full value of the note?

7. I have a 10% note for $280, dated Sept. 17, 1876, and due Feb. 6, 1879. May 23, 1878, Mr. C. proposes to buy it of me, discounting at 8%. What must he pay me?

8. Mr. C. gives me his note for $300, due 2 yr. hence, at 10%, and I sell it to Mr. B. the same day at 8% discount. What does B. pay me?

Why is this note worth more than its face?

9. Mr. C. gives me his note for $300, due 2 yr. hence, at 8%, and I sell it the same day to Mr. B. at 10% discount. What does B. pay me?

Why is this note worth less than its face?

10. Mr. C. gives me his note for $300, due 3 yr. hence, at 10% interest, which is all that money is worth. What is the present worth of the note on the day it is made? One year after its date, what is its present worth? Two and one-half years after date?

11. Mr. C. gives me his note for $300, due 3 yr. hence, without interest. What is it worth on the day it is given, money being worth 10%? What 1 yr. after date? What $2\frac{1}{2}$ yr. after date? What 3 yr. after date?

12. A merchant bought goods amounting to $4200 on a credit of 6 months, without interest. What sum in ready money would discharge the debt, money being worth 8% per annum?

13. A note, $52.25, dated Sept. 12, 1872, and bearing 10% interest, will be paid Jan. 1, 1879. What is it worth July 1, 1877, money being worth 6%?

14. Having bought a bill of goods amounting to $250 on 90 days' credit, the tradesman says to me, "For cash I could discount you 10% on this bill." What amount of money would pay the bill *now?*

15. Bought goods to the amount of $840.40 on 4 mo. credit, without interest. How much money would discharge the debt at the time of receiving the goods, discounting at 8% per annum?

16. July 1, 1878, I discount at 8% a 7% note for $275, dated Oct. 15, 1877, and due Jan. 12, 1880. What is the present worth?

17. April 10, 1879, I wish to raise some money, and find I have the three following notes: —

A 6% note for $225, dated Aug. 7, 1878, due Jan. 1, 1880;
A 7% note for $156, dated Nov. 1, 1877, due May 3, 1880;
A 7% note for $200, dated Feb. 3, 1878, due Oct. 14, 1878.

A friend is willing to buy the notes at 8% discount. How much money can I raise on them?

SECTION V.

GOVERNMENT BONDS.

384. The **Bond** of a corporation is its certificate of indebtedness, signed by the proper officers, and given under the corporate seal.

Such bonds are the notes of railroad, manufacturing, or other corporations, and are usually secured by mortgage upon their property. These bonds, like other notes, are made payable at a certain time, and bear a specified rate of interest. The bonds of a corporation are usually considered a surer investment than the stocks, since *dividends* on the latter are made only when the business receipts exceed the expenditures; whereas the *interest* on the former is due at the times named, whether there be profits in the business or not, and, when the bonds are secured by mortgage, the mortgage may be foreclosed like other mortgages. Nevertheless, in some very lucrative business, the stocks may be more valuable than the bonds.

385. Government Bonds are certificates of indebtedness issued by the government; as by the United-States or State government, by a county, city, school-district, or other government corporation.

Such bonds are usually made payable at a certain time, and bear a specified rate of interest.

The occasions for these bonds are such as the following: When a school-district wishes to build a fine house, but does not want to increase the taxes sufficiently to pay for it in a single year, but prefers to distribute the payment over several years; and, in like manner, when a county or state is called upon to expend more money than it is deemed expedient to raise by immediate taxation. But the fruitful cause of such government indebtedness is *war*. In consequence of our late war, the indebtedness of the United-States Government ran up from $88,995,810 in 1861 to $2,639,382,572 in 1867. In like manner, England and France have accumulated enormous debts.

386. English **Consols** are government stocks. **Rentes**[1] are **French** government stocks.

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[1] Pronounced "*rahnts*." In strict language, the term *Rentes* applies only to the *interest;* the principal — the debt itslf — being called *Nominal Capital*.

(*a*) English *Consols* are properly but perpetual 3% *annuities*, as the principal of the debt is not presumed to be payable. The consolidated debt of England is £731,413,523. The term *Consols* is applied also to certain United-States bonds.

(*b*) The principal part of the United-States interest-bearing debt in the market April 1, 1879, was,[1] —

5s of 81	$508,440,350
6s of 81	264,321,350
4½s (1891)	250,000,000
4s (1907)	556,467,950

(*c*) "5s of 81" means government bonds, which bear 5% interest, payable quarterly on Feb. 1, May 1, Aug. 1, and Nov. 1.

(*d*) "6s of 81" are 6% bonds, interest payable semi-annually on Jan. 1 and July 1.

Both of these classes of bonds are redeemable by the government in 1881, — the former May 1, and the latter June 30. No doubt the government will be ready to redeem them then, and interest on them will cease. There are also about 18½ million 6s of 80, which mature Dec. 31, 1880.

In this statement nothing is said of the old 10-40s and 5-20s, since the interest on the last of these expires July 1, 1879, and the government has already made ample provision for redeeming them. The total interest-bearing debt of the United States, April 1, 1879, including these outstanding 10-40s and 5-20s, was $1,968,962,800. This includes the 356 millions of 10-40s and 5-20s, and also the 4% which have been sold to redeem them. Hence our interest-bearing debt, after July 1, 1879, will be about $1,600,000.

(*e*) The 4½s bear 4½% interest, payable quarterly March 1, June 1, Sept. 1, and Dec. 1, and are redeemable Sept. 1, 1891.

(*f*) The 4s bear 4% interest, payable quarterly Jan. 1, April 1, July 1, Oct. 1, and are redeemable July 1, 1907.

387. A Coupon is a certificate of interest attached to a bond, which, on the payment of the interest, is cut off, and delivered to the payor.

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[1] The figures given above are from the official report of treasury April, 1879; but the immense sales of 4% during April and May make the 4s in the market now (*May 20*) nearer 800 million.

Stocks and bonds are bought and sold in the market just as wheat or cotton, and the prices fluctuate according to prosperity of business, the plenty or scarcity of money, and many other circumstances.

United-States bonds are of two classes, — *Registered* and *Coupon*. Registered bonds have to be indorsed by the owner when he sells them; and the government keeps an account of such transfers, and hence can at any time tell who owns a particular bond. The owner of a *Coupon* bond may lose it, or it may be stolen; and the person who has it in possession can collect the interest, or sell the bond. But registered bonds are not liable to such contingencies. Yet the impracticability of making transfers of registered bonds in foreign countries confines transactions abroad to coupon bonds.

388. Pacific Railroad Bonds are the bonds of these corporations guaranteed by the United States. Of these there are $64,623,512 in the market, bearing 6% interest, which is paid by the United-States Government. These are not reckoned a part of the United-States debt, since the government holds the first-mortgage bonds on the entire property of the companies to secure the payment.

389. Quotations are the statements made from day to day in the *newspapers*, giving the rates at which exchange, stocks, bonds, etc., are being bought and sold in the money market.

"Interest to buyer" means that the buyer has the advantage of whatever interest has accrued at the time of purchase. Thus, if I buy a $100 bond, "6s of 81," quoted at 106½ (which means $106.50 for a $100 bond), and there has accrued $2.10 interest on that bond at the time of my purchase, the bond really costs me $106.50 — $2.10, or $104.40. But if I buy a Michigan 7% bond for $100, quoted "112, interest to seller," 4 mo. after the interest was paid (i.e., when 4 mo. interest has accrued), it costs me $112 + ⅓ of $7, or $114.33⅓, since I have to pay to the seller the accrued interest.

Government bonds are always sold "interest to buyer." Other bonds are sometimes quoted "interest to seller;" which means that they are selling at the quotation plus the accrued interest at the time of sale.

390. *Examples.*

[We give examples showing how to find the cost of bonds, and what rate per cent an investment will yield. These are the two most important problems, and are solved by principles already made familiar.]

1. What cost a $500 United-States bond, 6s of 81, at 116½?

2. What cost a $1000 Tennessee 6% bond (old), quoted 34½?

3. United-States 6s of 81 are to-day (May 20, 1879) quoted at 107. What will a $500 bond cost me?

The quotation means that $100 of bond costs $107: but on this there is accrued (May 20) interest since Jan. 1 (see **386**, *d*); i.e., for 139 da. 6% on $100 for 139 da. is $2.29. This interest I shall receive July. Hence the bond actually costs me $107 — $2.29 = $104.71, saying nothing of the discount on the $2.29 from this time to July 1. The $500 bond, therefore, will actually stand me in $523.55, though really I pay at the purchase $535.

4. United-States 5s of 81 are quoted to-day (May 20, 1879) at 103⅝. What will a $1000 bond cost me exclusive of the interest accrued? (See **386**, *c*.)

5. What will a $500 bond, Michigan 7s (war bounty loan), quoted at "112, interest to seller," cost me 2 mo. 10 da. after the interest has been paid?

At "112" the bond will cost 5 × $112 = $560; and, as the interest accrued goes to the seller, I shall have to pay 2 mo. 10 da. interest on $500 at 7%, or $6.81 in addition.

On transactions in United-States bonds reckon the exact number of days as 365ths of a year. On State bonds reckon calendar months, and days as 30ths of a month.

6. When Erie consolidated 7s are quoted "111⅞, interest to seller," 3 mo. after payment of interest, what does a $100 bond cost?

7. When District-of-Columbia 3-65s are quoted "$86\frac{7}{8}$, interest to seller," with $2\frac{1}{2}$ mo. accrued interest, what cost a $1000 bond?

3-65s means bonds that bear $3\frac{65}{100}\%$ interest; i.e., a $100 bond bears interest at 1¢ per day, all years being reckoned as having 365 da.

8. When United-States 4s are quoted at $102\frac{3}{8}$, with 1 mo. 18 da. accrued interest, what shall I pay for a $1000 bond? What is the actual cost exclusive of accrued interest?

9. When United-States $4\frac{1}{2}$s are quoted at 107, with 2 mo. 22 da. accrued interest, what is the actual cost of a $500 bond?

Interest and brokerage are always reckoned on the *face* of a bond or stock certificate.

10. I buy through a broker, who charges me $\frac{1}{8}\%$ for buying, $2500 United-States 4s, quoted $102\frac{1}{4}$. What amount do I pay? What is the net cost of the bonds above accrued interest, if the purchase is made May 25? (See **386**, *f*.)

11. If I buy Bay-City 8s, quoted at 111, 6 mo. after their date, interest to seller, and pay my broker $\frac{1}{5}\%$, what do $5000 in bonds cost me?

As there is 6 mo. accrued interest, which I have to pay to the seller, I pay for $100 in bonds $111 + the interest for 6 mo. at 8% + 20¢ for brokerage.

12. What do I pay for $1000 United-States 5s of 81, April 15, the quotation being $103\frac{5}{8}$, and brokerage $\frac{1}{10}\%$? What do the bonds stand me in, exclusive of accrued interest at time of purchase?

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13. If I buy a $500 $7\frac{1}{2}\%$ county bond 9 mo. after its date at 105, interest to seller, what do I pay for the bond? What per cent does my investment yield?

I pay $553.125 for the bond, and it yields me $35 interest per year. The question then is, What per cent of $553.125 is $35?

14. In "The New-York Independent" Fisk & Hatch say, under date of May 1, 1879, that an investment in 6s of 81, which mature Dec. 31, 1880, and are selling at 106½, will barely pay 3% on the investment. How do they make it out?

Suppose I buy a $100 bond. I pay for it $106.50. On this I get $3 interest July 1, 1879; $3 Jan. 1, 1880; $3 July 1, 1880; and $3 Jan. 1, 1881. At this time the interest ceases, and the bond is worth exactly $100. I then have $112[1] for my investment of $106.50 for 1 yr. 8 mo.; i.e., from May 1, 1879, to Jan. 1, 1881. The interest on the $106.50 is, therefore, $112.00 — $106.50 = $5.50. Now, at 1%, $106.50 yields in 1 yr. 8 mo. $1.775. Hence $5.50 is 5.50 ÷ 1.775, or 3.1 times 1%, or 3.1% nearly.

15. What per cent on the investment do 6s at 95, interest payable annually, yield, if bought 3 mo. after the payment of the interest, interest to seller?

Compute on a $100 bond in all such questions. In this case such a bond costs $96.50. The question then is, What per cent on $96.50 is $6? In such computations the bond is supposed to run indefinitely, so that no note is taken of the fact that for the first year the investment yields its first interest only 9 mo. after it is made.

16. Considered as a permanent investment, what per cent on my money do 4% bonds, bought at 103¼, yield me?

17. Which is the better investment, — United-States 4½s bought at 104½ with 2 mo. accrued interest, or State 7s with 5 mo. accrued interest, bought at 107, interest to seller, considering each as a permanent investment, and taking no notice of the semi-annual payment of interest?

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[1] An exact analysis requires that we take account of the interest on the three payments made before Jan. 1, 1881. Compounding this each six months at 6%, and also *compounding* the interest on $106.50 in the same way, we get a little *less* than 3%.

SECTION VI.

EXCHANGE.

391. A merchant in Detroit wishes to pay a debt of $2500 in New York. He may send the money by a friend, by mail, or by express; but the most common and most convenient way is to step into a *bank* in Detroit, and, paying in his $2500 with a small percentage for their trouble, get the Detroit bank's order on a New-York bank for the $2500. This *order*, called a *Draft*, the Detroit merchant can send to his creditor in New York, who, by stepping into the New-York bank to which the order is addressed, will get his $2500.

A similar order given by a bank in this country upon a *foreign bank*, as one in London, Eng., is called a *Bill of Exchange.*

392. Exchange is a method of making payments in distant places by the use of *Drafts*, or *Bills of Exchange*, without the direct transmission of money.

When the exchange is between places in the same country, it is called *Inland* or *Domestic Exchange;* and when between places in foreign countries, it is called *Foreign Exchange.* Hence a *Draft* is a *Domestic Bill of Exchange.*

393. A **Draft,** or **Bill of Exchange,** is a written order for money, drawn in one place, and payable in another.

394. A **Bank** is a company authorized by law to issue paper money, receive deposits, deal in exchange, loan money, or buy and sell coin, bonds, stocks, etc.

Some banks make it their chief business to loan money, others to deal in exchange, others to receive deposits: while comparatively few are banks of issue; that is, issue paper currency.

DOMESTIC EXCHANGE.

395. [*Forms of Drafts.*]

FIRST NATIONAL BANK OF DETROIT,
$2500. DETROIT, MICH., FEB. 29, 1876.

At sight, pay to the order of Newcomb, Endicott, & Co.
TWENTY-FIVE HUNDRED DOLLARS.

SCHUYLER GRANT, *Cashier*.

To the NINTH NATIONAL BANK,
New York, N.Y.

See introductory note to the subject of Exchange. Newcomb, Endicott, & Co. are the merchants in Detroit who wish to pay $2500 to their creditor, John Smith, in New York. Having obtained this *draft*, they write on the back of it, "Pay to the order of John Smith," signing this order, "Newcomb, Endicott, & Co." When Mr. Smith in New York receives the draft, he takes it to the Ninth National Bank, and, writing his own name on the back, receives the money.

$3500. DETROIT, MICH., Feb. 29, 1876.

At ten days' sight, pay to the order of Sheldon & Co. THIRTY-FIVE HUNDRED DOLLARS, value received, and charge the same to the account of E. B. SMITH & Co.

To the TWELFTH NATIONAL BANK,
New York.

The first form of draft is of a draft drawn by one bank upon another: the second is a draft drawn by a bookseller's firm upon a bank. Each supposes that the party in Detroit has an account with the bank in New York, and has money in the bank.

Observe that the first above is a *Sight Draft*; i.e., it is to be paid as soon as presented to the bank in New York. The second is a *Time Draft*, and is not payable till *ten* days after presentation. It should be presented as soon as received, when the cashier writes on it "Accepted," giving the date of acceptance, and signing his name as cashier. This makes the bank liable for it, and is an agreement to pay it after ten days. If no time is specified when a draft is to be paid, it is payable at sight. Drafts are also made payable a certain *time* after date.

Ex. 1. — Suppose New-York exchange is at a premium of $\frac{1}{10}\%$ in Detroit, how much will Newcomb, Endicott, & Co. have to pay for the above draft?

Such exchange is generally at a slight premium in Detroit, since Detroit merchants and other business-men want much more New-York exchange than New-York business-men want of Detroit. This requires that Detroit banks should actually send money to the banks in New York, whereas there will be no need of New-York banks sending money to Detroit banks. Thus, if the trade between two places is equal, exchange between them should be about at par; but, when there is much inequality, exchange on the other place will be at a premium at the place which buys more than it sells, and at a discount at the place which sells more than it buys. Thus, there being comparatively little demand in New York for drafts on Detroit, such drafts will be at a slight discount.

In consequence of the above facts, if a New-York house wishes to pay a debt in a small Western place, they send to their creditor their check (order) upon a bank in New York, or a certificate of deposit. This will be at par in the Western place, and perhaps a little above.

2. What would A. T. Stewart's sight draft on the Twentieth National Bank, N.Y., for $1500, be worth in Niles, Mich., at $\frac{1}{12}\%$ premium?

3. What will a sight draft for $1825 on New Orleans sell for in New York at a discount of $\frac{1}{20}\%$?

4. What will it cost a New-York merchant to pay a New-Orleans debt of $2500, New-York exchange being at $\frac{1}{10}\%$ premium in New Orleans?

Every $1 face of draft will be worth $1.001 in New Orleans.

5. A New-York merchant sends his creditor in New Orleans a sight draft for $2497.50. New-York exchange being at $\frac{1}{10}\%$ premium in New Orleans, what will the New-Orleans man receive for the draft?

6. What will it cost me in Philadelphia to pay $1500 in St. Louis, Philadelphia exchange being at $\frac{1}{8}\%$ premium in St. Louis? In what way would I make the remittance? See 2d paragraph under Ex. 1.

7. Being in Omaha, I receive a draft on a New-York bank for $3000. New-York exchange being at $\frac{1}{8}$% premium in Omaha, what shall I receive for it?

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8. What is a $1200 St. Louis draft at $^{30}/_{33}$ da. on New York worth, New-York exchange being at 101, and the time discount being at 3%?

The nature of this transaction is, that a man in St. Louis buys at a bank there a draft on New York for $1200. Since New-York exchange is at 101 (i.e., at 1% premium), if his draft were a *sight* draft it would cost him $1200 × 1.01 = $1212. But, inasmuch as the bank in New York will not have to pay the draft till $^{30}/_{33}$ da. after its date, they will not charge the St. Louis bank with it till they pay it. Hence the St. Louis bank will have the use of the money $^{30}/_{33}$ da. before it will be charged to them in New York; i.e., before they have to pay it. Therefore they allow 3% discount on *the face of the draft* for the use of the money. 3% of $1200 for $^{30}/_{33}$ da. is $3.25. Hence the purchaser of the draft pays $1212 — $3.25, or $1208.75, or $1208.77 including stamp.

9. A merchant in Boston wishes to pay $980 in Milwaukee. Required the cost of a draft, payable in 60 days, exchange being at $1\frac{3}{4}$% discount, the bank or time discount being 4%.

The nature of this transaction is as follows: —

The Boston merchant steps into a bank, and buys a draft on a Milwaukee bank. Milwaukee exchange being at discount in Boston, a sight draft for $980 could be bought for $980 less $1\frac{3}{4}$% of $980, or for $962.85. But as the Boston bank will have the money $^{60}/_{63}$ da. before it will be charged to them by the Milwaukee bank, and as the Boston merchant will have paid the money $^{60}/_{63}$ da. before his creditor in Milwaukee receives it, it is but right that the Boston bank should allow the merchant for the use of the money. Hence they allow him 4% on $980 for the $^{60}/_{63}$ da.; i.e., $6.766. This, deducted from $962.85, leaves $956.084; and, adding the 2¢ for the stamp, the cost of the draft is $956.10+.

The justice of such an arrangement will appear very clearly if we suppose that the debt in Milwaukee is drawing interest. Now, no interest will be stopped until $^{60}/_{63}$ da. after the debtor has paid his money: hence the party which has the money these 63 days should pay interest on it.

10. Exchange at 2% premium, and time discount at 5%, what is the cost of \$1 exchange on draft for $^{60}/_{63}$ days? Then what of a draft for \$750? For a draft for \$85.50?

11. Exchange at $1\frac{1}{2}$% discount, and time discount at 4%, what is the cost of \$1 exchange on draft for $^{90}/_{93}$ da.? Then what of a draft for \$500? For \$184.25?

12. Exchange being at $101\frac{3}{4}$ (that is, $1\frac{3}{4}$% premium), and time discount at 5%, what % of its face is the cost of a $^{30}/_{33}$ da. draft? Of a $^{60}/_{63}$ da.? Of a $^{90}/_{93}$ da.? Of a 10 da.?

13. Exchange being at $98\frac{1}{2}$ ($1\frac{1}{2}$% discount), and time discount at $4\frac{1}{2}$%, what per cent of its face is the cost of a $^{30}/_{33}$ da. draft? Of a $^{60}/_{63}$ da.? Of a $^{90}/_{93}$ da.? Of a 10 da.?

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396. FOREIGN EXCHANGE.

1. James Howell, a young man from Chicago, is travelling in England, and his father, Thomas Howell, wishes to send to him in London \$1000. How will he effect it? and what amount in English currency will the son receive, sterling exchange being quoted at 4.89 sight? (See **202.**)

397. Mr. Howell will receive from the bank *three* bills of exchange (orders on the London bank) of the following form:—

£204 10s. CHICAGO, ILL., March 7, 1876.

At sight of this FIRST of Exchange (Second and Third of same date and tenor unpaid), pay to the order of James Howell *Two hundred and four pounds and ten shillings sterling*, value received, and charge the same to

BROWN, GALE, & CO.

TO SUNDERLAND & HATCH, London.

The other two bills will be exactly like this, except that in the second the word SECOND will be used where FIRST is in this, and the parenthesis will read, "First and Third of same date," etc. The third will read, "THIRD of exchange (First and Second of same date)," etc.

The object of this arrangement is that the three bills may be sent by different mails; and thus, if one is lost, the remittance will not fail. Of course, when one has been received and paid, the others are void.

2. Sterling exchange is quoted to-day (May 23, 1879), 60 days, 4.87½; sight, 4.89 (see **202**). What will it cost me to pay in London, Aug. 1, a bill of £325? What to pay it June 1? What will be the manner of doing the business?

3. I wish to send to a friend in Paris $500. Exchange being quoted 5.15⅞ (see **202**), for what will the bill of exchange be drawn?

4. I wish to send my correspondent in Berlin 1000 marks. The quotation being 95¾ (see **202**), what will the bill of exchange cost me?

SECTION VII.

GENERAL PROBLEMS IN SIMPLE INTEREST.

[Or *Percentage* involving the Element of *Time.*]

398. As in Simple Percentage (**308**), so here, there are two general methods of treating the subject; viz., by analyzing each problem on its own merits, or by the use of special rules or formulas.

399. SOLUTION BY ANALYSIS. — Aside from the ordinary rules for finding *interest*, or percentage, and *amount*, all that is needed is, when any other element than interest or amount is required, *Find the effect produced under the given conditions by 1 of the thing required, and compare this with the given effect.*

400. Solution by Formulas. — Letting P stand for *principal*, r for *rate*, t for *time*, I for *interest*, or percentage, and A for amount, we have, by (**347**), the two

Fundamental Formulas.

(1.) $P \times r \times t = I$;
(2.) $P (1 + r \times t) = A$.

The second is obtained by remembering that $P + I = A$, and since $I = P \times r \times t$, $P + P \times r \times t = A$, or $P (1 + r \times t) = A$.

[These formulas are given, in order that teachers having the desire may explain them fully, and use them. We propose to solve the problems by analysis.]

401. *Examples.*

1. Principal $576, rate per cent 6, time 2 yr. 3 mo. What is the interest?

This is the ordinary problem in simple interest, and may be solved in any of the ways already given.

2. Principal $576, interest $77.76, time 2 yr. 3 mo. What is the rate per cent?

The thing required is *rate per cent.* Hence we find what interest would be yielded at 1% in the given time, and compare this with the given interest.

Analysis. — At 1% for 2 yr. 3 mo. the interest on $576 is $12.96. But $77.76 is 6 times $12.96, which we learn by dividing $77.66 by $12.96. Hence, as the principal yields 6 times the interest it would at 1%, the rate per cent is 6.

3. Principal $576, interest $77.76, rate per cent 6. What is the time?

The thing required is *time.* Hence we find what interest would be yielded in 1 yr. at the given rate, and compare this with the given interest.

Analysis. — In 1 yr., at 6%, $576 yields $34.56 interest. Hence it requires as many years to yield $77.76 as $34.56 is contained times in $77.76; i.e., $2\frac{1}{4}$ years, or 2 yr. 3 mo.

4. Interest $77.76, rate per cent 6, time 2 yr. 3 mo. What is the *principal?*

The thing required is *principal*, etc.

ANALYSIS. $1 principal yields $0.135 interest in 2 yr. 3 mo. at 6%. Now, to yield $77.76 requires as many dollars principal as $0.135 is contained times in $77.76, which is 576. Hence the principal is $576.

5. Amount $102.81 at 10% for 3 yr. 9 mo. 18 da. What is the principal?

ANALYSIS. — $1 principal at 10% for $3\frac{4}{5}$ years yields $1.38 amount. Now, to yield $102.81 requires as many dollars principal as $1.38 is contained times in $102.81. $102.81 \div 1.38 = 74.50$. Hence the principal is $74.50. [This is the ordinary problem in common discount. See **383**.]

6. Amount $102.81, on $74.50 at 10%. What is the time?

The interest is $102.81 — $74.50 = $28.31, the *given effect*. The thing required is *time*. The interest on the principal ($74.50) for 1 yr. at 10% is the effect produced by 1 of this kind.

7. Amount $102.81, on $74.50 for 3 yr. 9 mo. 18 da. What is the rate per cent? Same as Ex. 9.

8. At what per cent will $75 yield $28.125 in 6 yr. 3 mo.? At what % will it yield $15.30 in 2 yr.?

9. How long does it take $750 to amount to $942 at 6%? How long at 5%? At 3%?

10. What principal yields $150 at 4% in 7 yr. 2 mo. 15 da.? In 3 yr.? In $5\frac{1}{2}$ yr.?

11. In what time will $120 yield $16.56 interest at 6%?

12. If $584 in 2 yr. 8 mo. 7 da. yield $$94.121\frac{1}{3}$, what is the %?

13. I wish to obtain $150 at bank for $^{60}/_{63}$ at 10%. For what amount must I make my note?

The *thing inquired about* is the *Face of my Note.* Now, as the interest of $1 for $^{60}/_{63}$ da. at 10% is $.01726, $1 face of note will yield $1 — $.01726, or $.98274. The *effect to be produced* is $150. Hence $150 ÷ .98274 = $152.63, the face of the note.

14. For what must I make my note in order to get at bank

$350 for $^{30}/_{33}$ da. when they are discounting at 8%? What to obtain the same sum at the same rate for $^{60}/_{63}$? For $^{90}/_{93}$?

15. I receive $495.50 for my note at $^{30}/_{33}$ da. at bank when they are discounting at 10%. What is the face of the note?

16. I receive at bank $122.90 for my note of $125 at $^{30}/_{33}$ da. What is the rate per cent of discount?

17. My annual income is $1500 from stocks, which yield 7%. What amount of stocks do I hold?

18. I have 4% bonds, which yield me $500 quarterly. What amount of bonds do I hold?

19. What amount of United-States 5s of 81 must a man have to yield an income of $400 per quarter?

20. What amount of United-States 4s must I have to yield me $600 quarterly?

RATIO AND PROPORTION.

CHAPTER VI.

RATIO.

402. **Ratio** is the quotient of one number divided by another.

Thus the ratio of 12 to 4 is $12 \div 4$, or 3. The ratio of 5 to 7 is $5 \div 7$, or $\frac{5}{7}$.

NOTE.—If the numbers are concrete, they must be of the same kind, since we cannot divide one concrete number by another of a different kind. Thus the ratio of $10 to $5 is 2; but to ask, "What is the ratio of $10 to 5 miles?" is absurd.

403. The first number named is called the **Antecedent**, and the second the **Consequent**. The two together constitute the *Terms* of the ratio, or a *Couplet*.

404. The ratio between two numbers is indicated by writing the antecedent before the consequent, and the colon (:) between them; or by writing the *antecedent* as the *numerator* of a fraction, and the *consequent* as the *denominator*.

Thus 8 : 4 is read, "The ratio of 8 to 4;" so also $\frac{8}{4}$ may be read, "The ratio of 8 to 4,"—both forms meaning exactly the same thing.

405. The term *Ratio* is also applied to such forms as 6 : 2, $\frac{7}{3}$, etc.; that is, to the indicated operation of division, *the sign* : being an equivalent of $\div$.

Thus we speak of the ratio 6 : 2 (not the ratio *of* 6 : 2), the ratio $\frac{4}{5}$; reading, "The ratio 6 to 2," "The ratio 4 to 5." The ratio *of* 6 to 2 is 3, or the *value* of the ratio 6 : 2 is 3. So the ratio *of* 4 to 5 is $\frac{4}{5}$ (4-fifths), or the *value* of the ratio $\frac{4}{5}$ (read "4 to 5") is 4-fifths.[1]

1. What is the ratio of 15 to 3? 8 : 2? 9 : 3? 10 : 2? 5 : 7? 4 : 8? 1 : 3? 3 : 1? 7 : 11? 11 : 7?

2. Which is greater, 12 : 3 or 8 : 4? 6 : 3 or 9 : 3? 5 : 6 or 7 : 8? 2 : 3 or 5 : 4? 10 : 5 or 6 : 3? 5 : 8 or 15 : 24? $\frac{4}{8}$ or $\frac{3}{6}$? $\frac{7}{11}$ or $\frac{3}{5}$?

3. Mention several ratios which are each equal to 15 : 3. Several which are equal to $\frac{5}{8}$. To $\frac{2}{3}$. To 3 : 5.

Principle.

406. *A ratio has all the properties of a common fraction with the antecedent for its numerator, and the consequent for its denominator.*

4. What effect does it produce on a ratio to *multiply* the antecedent by 2? by 3? by any number? Try it on 2 : 4. What effect to *divide* the antecedent by any number? Try it.

5. What effect does it produce on a ratio to multiply its consequent? To divide its consequent? Try it.

6. How do you compare two common fractions to ascertain which is the greater? (p. 132.) How then do you compare two ratios?

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7. If 24 is the antecedent and 4 the ratio, what is the consequent?

Having the antecedent and ratio given, how do you find the consequent?

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[1] This double use of the word "ratio" has given no little trouble to students. That the word is habitually used by mathematicians in both of these ways no one at all conversant with mathematical writing can doubt. Thus when we ask, "What is the ratio of 12 to 4?" all answer "3;" and all with equal unanimity speak of the ratio a : b, — "The ratio *a* to *b*."

8. If 3 is the consequent and 7 the ratio, what is the antecedent? If 45 is the consequent and the ratio $\frac{1}{9}$?

9. If 7 is the antecedent, what is the consequent when the ratio is $\frac{7}{5}$? When it is $\frac{2}{3}$? When it is 6?

10. If 28 is the antecedent, what is the consequent when the ratio is 7? When it is 4? When it is 14? When it is $\frac{2}{3}$? When it is $\frac{4}{5}$?

11. Antecedent 10, ratio 2, what is the consequent? Antecedent 27, ratio 9? 3? $\frac{1}{2}$?

PROPORTION.

407. Proportion is an equality of ratios, the terms of the ratios being expressed. The equality is indicated by the ordinary sign of equality (=), or by the double colon (::).

Thus $8:4=12:6$, or $8:4::12:6$, is a proportion. It is read, "8 is to 4 as 12 is to 6." The expression $\frac{8}{4}=\frac{12}{6}$ may be read in the same way, and means the same thing.

408. Two ratios at least are required for proportion: hence we have two antecedents and two consequents. Of four terms which constitute a proportion, the 1st and 4th are called **Extremes**, and the 2d and 3d **Means**.

1. Is 15 : 3 :: 10 : 2 a true proportion? What is the ratio of 15 to 3? What of 10 to 2? Are they equal?

2. Show which of the following are true proportions: —

1. 20 : 5 :: 8 : 2	7. 10 : 7 :: 20 : 14
2. 2 : 12 :: 5 : 30	8. 3 : 7 :: 12 : 26
3. 5 : 35 :: 8 : 64	9. 13 : 27 :: 117 : 243
4. 2 : 3 :: 14 : 21	10. $2\frac{1}{2}$: 5 :: $3\frac{1}{3}$: $6\frac{2}{3}$
5. 7 : 11 :: 35 : 55	11. 1.05 : 8.4 :: 1 : 8
6. 8 : 3 :: 16 : 9	12. .05 : 7 :: .3 : 42

3. If the first 3 terms of a proportion are 18 : 6 :: 21 what is the 4th term?

The ratio of 18 to 6 is 3: hence the 4th term must be $\frac{1}{3}$ of 21, so that the ratios may be equal. Is $18 : 6 :: 21 : 7$ a true proportion? Why?

4. Find the 4th term of $7 : 3 :: 5 : —$.

What is the ratio of $7 : 3$? Then, if 5 is the antecedent and $\frac{7}{3}$ the ratio, what is the consequent? Is $7 : 3 :: 5 : 2\frac{1}{7}$ a true proportion? Why?

5. Find the lacking term of $12 : — :: 8 : 6$.

Which ratio is given? What is the ratio of $8 : 6$? If 12 is the antecedent and $\frac{4}{3}$ the ratio, what is the consequent?

6. Find the lacking term of $13 : 7 :: — : 11$.

We have the ratio of $13 : 7$, $\frac{13}{7}$. Hence the lacking term is $11 \times \frac{13}{7}$, or $\frac{143}{7} = 20\frac{3}{7}$.

7. Find the lacking term of $— : 43 :: 5 : 17$.

The given ratio is $\frac{5}{17}$. Hence we have $43 \times \frac{5}{17} = \frac{215}{17} = 12\frac{11}{17}$. Is $12\frac{11}{17} : 43 :: 5 : 17$ true?

Principle.

409. *The product of the means of a proportion is equal to the product of the extremes.*

This is evident, since the 1st *mean* is the 1st *extreme divided* by the ratio, and the 2d *mean* is the 2d *extreme multiplied* by the ratio. Hence the product of the means is $\frac{\text{1st } \textit{Extreme}}{\textit{Ratio}} \times \text{2d } \textit{Extreme} \times \textit{Ratio}.$ In this the ratio cancels, and leaves the product of the extremes.

8. Find by means of this principle the lacking term in $13 : 5 :: 12 : —$.

The two means being given, we know their product, 60; but this is also the product of the extremes. Now, if 60 is the product of the extremes and 13 is one of the extremes, the other is $\frac{60}{13}$, or $4\frac{8}{13}$.

9. In like manner find the lacking term in each of the following, giving the explanation: —

1. 2 : 7 :: 5 : —
2. — : 4 :: 11 : 6
3. 5 : 12 :: — : 8
4. 34 : — :: 17 : 16
5. 131 : 47 :: 1.5 : —
6. $12\frac{1}{2}$: $6\frac{2}{3}$ :: 8 : —
7. 23.05 : 4.5 :: 7.1 : —
8. 1.05 : 342 :: 100 : —
9. 42 : 6 :: — : 30
10. $\frac{2}{3}$: $\frac{4}{5}$ :: $\frac{5}{8}$: —
11. 112 : 16 :: 49 : —
12. $11\frac{3}{5}$: $2\frac{1}{7}$ :: 4 : —

RULE OF THREE.

410. The **Rule of Three** is an old term applied to the method of solving problems in which *Three Terms* of a proportion are given and the fourth is to be found.

1. If 8 yards of a certain kind of cloth cost \$35, how much will 42 yards of the same cloth cost?

It is evident that the same ratio exists between the cost of the two quantities as between the quantities, since the price per yard is the same. Hence the ratio of 8 yd. to 42 yd. is the same as the ratio of the cost of 8 yd., \$35, is to the cost of 42 yd. Stated as a proportion, this is 8 yd. : 42 yd. : : \$35 : the cost of 42 yd.

We have therefore to find the 4th term of the proportion 8 : 42 : : 35 : —. This is $\frac{35 \times 42}{8} = \$183.75$. Hence 42 yd. will cost \$183.75.

2. If it require 12 bbl. of flour per year for a family of 10, how many barrels will it require for a family of 6?

The proportion is 10 : 6 : : 12 : —. Hence we have $\frac{6 \times 12}{10} = 7\frac{1}{5}$.

Let the pupil give the reasons: 1st, for the statement of the proportion; 2d, for the method of finding the 4th term.

From the above analyses we deduce the following: —

411. Rule. — I. *Make that number the third term which is of the same kind as the answer sought.*

II. *If the nature of the problem requires the answer to be greater than the third term, make the* greater *of the two remaining terms the* second term; *otherwise make the* less *of those terms the second term: the term still remaining will be the* first *term.*

III. *Divide the product of the means by the given extreme, first indicating the operations in the form of a fraction, and then cancelling as much as practicable.*

NOTE. — In such questions there are always 3 quantities given, and *two of these are of the same kind.* Now it is necessary that we determine, from the nature of the case, whether the same ratio (relation) exists between the other two, one of which is *not* given, as exists between the two which are given. Not every problem in which three terms are given and a 4th required can be solved by proportion.

3. If 14 cords of wood cost \$98, what will 32 cords cost?

4. If 12 acres yield 384 bu. of wheat, what will 36 acres yield at the same rate?

What are the two quantities of the same kind? Does the same ratio exist between the quantities of wheat produced as between the quantities of land?

5. If the interest on \$350 at a certain rate and for a certain time is \$65, what is the interest on \$700 for the same time and rate?

Will *twice* as great a principal give *twice* as much interest at the same rate and for the same time? What are the two like terms? Are they not all alike? They are all *dollars.*

6. If \$100 principal amounts to \$122.50 for a certain rate and time, what principal does it require to amount to \$422.62½ for the same rate and time?

Amount.		*Amount.*		*Principal.*		*Principal.*	
\$122.50	:	\$422.62½	::	\$100	:	$\frac{42262.5}{122.5}$	(= \$345).

NOTE. — This is the question in true discount.

7. What is the present worth of a note \$422.62½, due 2 yr. 3 mo. hence, without interest, money being worth 10%?

Same as Ex. 6. $100 *now* amounts to $122.50 at the given rate and time. Hence the question is, How much *now* will it take to amount to $422.62½ in the given time at the given rate?

8. What is the present worth of a note which amounts to $350, 1 yr. 6 mo. 15 da. hence, money being worth 7%?

Find the amount of *any sum* for the given time and rate ($100 is convenient). Then state and work the proportion.

9. What is the present worth of $725.50, due in 6 yr. at 8%?

10. What is the present worth of $2000, due in 3 yr. 6 mo., interest at 7%?

The examples in (**383**) can be solved by proportion, and may be reviewed and solved in this way.

11. A bankrupt paid 43 cents on every dollar of his debts. How much did he pay on a debt of $569.31?

12. A difference of 15° in longitude makes an hour's difference in time. What is the difference in time between Boston, which is 71° 4′ 20″ W. long., and Washington, which is 77° 1′ 30″ W. long.?

13. If we measure a distance and find it to be 500 yd., measuring by a yard-measure which is afterward found to be one-eighth of an inch too short, what is the distance?

14. If a person by travelling 10 hours a day perform a journey in 31 days, in how many days will he perform the same journey if he travel 13 hours a day?

15. If a person perform a certain journey in 13½ days by travelling 10 hours a day, in what time will he perform the journey if he travel 11¼ hours per day?

16. Moll and Van Beek, in 1823, found that sound travels 332.05 meters in a second. What is the velocity per second in feet?

17. How far off is a battery when the flash precedes the report 15 sec., no allowance being made for the progressive motion of light?

18. It is found that the eclipses of Jupiter's moons occur 16 min. 26.6 sec. sooner when the earth is on the side of her orbit nearest Jupiter than when she is on the opposite side. The diameter of the earth's orbit being 183,000,000 miles, what is the velocity of light per second?

19. How many times would light go round the earth in a second, the earth's circumference being called 24,000 miles?

20. The nearest of the fixed stars are probably 100,000,-000,000,000 miles from the earth. How long would one of them continue to be seen on the earth after it was annihilated, were such a thing possible?

21. If a staff 5 ft. long casts a shadow 3 ft., how high is a steeple whose shadow at the same time is 90 ft.?

PARTITIVE PROPORTION.

412. Partitive Proportion is a term applied to the division of a number into parts which shall be in the ratio of given numbers.

Ex. 1. — Divide 150 into three parts, which shall be to each other as 2, 3, and 5.

150 is the *sum* of three parts, which are to bear the ratio to each other of 2, 3, and 5, of which the *sum* is 10. Hence the proportions are, —

10 : 150 :: 2 : *the first part.*
10 : 150 :: 3 : *the second part.*
10 : 150 :: 5 : *the third part.*

2. Divide 35 into 2 parts, which shall be to each other as 3 to 4. As 2 to 5. As 1 to 6.

3. Divide 1 into 3 parts, which shall be to each other as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{1}{5}$.

4. Divide 7 into 4 parts, which shall be to each other as 3, 5, 8, and 2.

5. Divide \$5000 among 3 persons so that the first shall have twice as much as the second, and the second twice as much as the third.

6. Divide \$100 into 4 parts, which shall be to each other as \$800, \$700, \$1000, and \$500.

7. Of \$1900 A is to have a certain sum, B twice as much, C twice as much as B, D as much as A and C, and E as much as B and D. How much is each to have?

8. A and B found a purse containing \$85, and agreed to share it in the ratio of $\frac{2}{3}$ to $\frac{3}{4}$. What did each receive?

9. A, B, and C entered into partnership. A put in \$340, B \$460, and C \$500. They gained \$390. What was the gain of each, the gain being divided in the ratio of the shares in the capital?

10. A, B, and C purchased a farm for \$3500, of which A furnished \$1500, B \$1500, and C \$500. They received \$280 rent for the farm. How much of this rent is due each?

11. A, B, C, and D hired a pasture for \$120. A put in 120 sheep, B 160, C 180, and D 140. How much ought each to pay?

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12. Divide \$70 between A, B, and C, in such a manner that A's share shall be to B's as 2 to 3, and B's to C's as 4 to 5.

As often as A has \$2 B is to have \$3, and C is to have $\frac{5}{4}$ as much as B, or $\frac{\$15}{4}$. Hence we are to divide \$70 into three parts, which shall be to each other as 2, 3, and $\frac{15}{4}$, or as 8, 12, and 15.

13. Three persons in a joint speculation gain \$1000, which is to be divided so that the first share shall be to the second as 3 to 2, and the second to the third as 5 to 6. Required *the shares.*

COMPOUND PROPORTION.

413. A Compound Ratio is the product of two or more simple ratios, as a *Compound Fraction* is the product of two or more simple fractions.

Thus the ratio of 3 to 5 is $\frac{3}{5}$, and the ratio of 2 to 7 is $\frac{2}{7}$. Hence the compound ratio of 3 to 5 and 2 to 7 is $\frac{3}{5} \times \frac{2}{7}$, or $\frac{6}{35}$.

414. A Compound Proportion is an equality between a Compound Ratio and a Simple Ratio, the terms of each of the simple ratios being expressed.

If 240 is to some number which we wish to find, in the compound ratio of 3 : 4, 5 : 7, and 1 : 11, we write

$$\left.\begin{array}{r} 3 : 4 \\ 5 : 7 \\ 1 : 11 \end{array}\right\} :: 240 : \text{the number sought.}$$

Ex. 1. — If 3 men, working 10 hours per day, can cut 51 cords of wood in 6 days, how much can 5 men cut in 7 days, working 8 hours per day?

The question is about the number of cords of wood.

First consider the number of men. 5 men will cut $\frac{5}{3}$ as much as 3 men, if they work 6 da. of 10 hr. each, or $\frac{5}{3}$ of 51 cd.

Second, in 7 da. the 5 men will cut $\frac{7}{6}$ as much as in 6 da., if they work 10 hr. per day, or $\frac{7}{6}$ of $\frac{5}{3}$ of 51 cd.

Third, but working 8 hr. per day, the 5 men will cut only $\frac{8}{10}$ as much as though they worked 10 hr.; i.e., —

$$\frac{8}{10} \text{ of } \frac{7}{6} \text{ of } \frac{5}{3} \text{ of 51 cd.} = \frac{\overset{\overset{\cancel{2}}{\cancel{4}}}{\cancel{8}} \times 7 \times \cancel{5} \times \overset{17}{\cancel{51}}}{\underset{\cancel{2}}{\cancel{10}} \times \underset{3}{\cancel{6}} \times \cancel{3}} = \frac{119}{3} = 39\tfrac{2}{3} \text{ cd.}$$

In the form of a proportion this may be stated thus: —

$$\left.\begin{array}{r} 5 : 3 \\ 7 : 6 \\ 8 : 10 \end{array}\right\} :: 51 \text{ cd.} : \text{the number of cords sought.}$$

From this analysis we deduce the following: —

415. Rule. — I. *Make that number the third term which is of the same kind as the answer sought.*

II. *Arrange the couplets of the compound ratio as in simple proportion, considering one condition at a time.*

III. *Divide the product of the means by the product of the given extremes, first indicating the operations in the form of a fraction, and then cancelling as much as practicable.*

2. If a footman can travel 150 miles in 5 days, when the days are 12 hours long, in how many days may he travel 275 miles when the days are 10 hours long?

3. If 5 oxen require an acre of grass for 9 days, how many acres will 20 oxen require for 30½ days?

4. If 4 men eat 64 pounds of bread in 2 weeks, how many pounds will 16 men eat in 7 weeks?

5. If a man travel 100 miles in 3 days of 13 hours length, how far might he travel in 33 days of 14¼ hours length?

6. If 2 yards of cloth 1½ yd. wide cost $10.25, what cost 13 yards of like quality, which is 1¾ yd. wide?

7. If a family of 10 persons, in 2 weeks, spend $200, how long ought a family of 13 persons to be in expending $500?

8. If 6000 lb. of bread will supply a garrison of 100 men 2 mo., how long will 12000 lb. last three such garrisons?

9. If 7 men can mow 84⅘ acres in 12½ days, working 8 hours per day, how many days of 10 hours each will 20 men require to mow 254⅖ acres?

10. If 40 men in 8 days of 9 hours each build a wall 120 rd. long, 9 ft. high, and 3 ft. thick, how many men will be required to build a wall 162 rd. in length, 12 ft. high, and 9 ft. thick, in 18 days, by working 12 hr. each day?

11. If 100 men, by working 6 hr. each day, can in 27 da. dig 18 cellars, each 40 feet long, 36 ft. wide, and 12 ft. deep, how many cellars, each 24 ft. long, 27 ft. wide, and 18 ft. deep, can 240 men dig in 81 da. of 8 hr. each?

12. If 24 men, by working 8 hr. a day, can in 18 da. dig a ditch 95 rd. long, 12 ft. wide, and 9 ft. deep, how many men, in 24 da. of 12 hr. each, will be required to dig a ditch 380 rd. long, 9 ft. wide, and 6 ft. deep?

Powers and Roots.

CHAPTER VII.

INVOLUTION AND EVOLUTION.

INVOLUTION.

416. A **Power** is the product arising from multiplying a number by itself a certain number of times.

Thus $3 \times 3 \times 3 = 27$; and 27 is called the 3d power of 3. $5 \times 5 = 25$; and 25 is the 2d power of 5. So $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$; and $\frac{8}{27}$ is the 3d power of $\frac{2}{3}$. The 2d power of .4 is .16. Of 1.2 is 1.44.

417. **Involution** is the process of raising a number to any required power. The number to be involved is called the *First Power*, or the *Root*.

418. The **Square** of a number is its 2d *power*, and the **Cube** of a number is its 3d *power*.

1. What is the square of 4? Of 7? Of $\frac{1}{2}$? Of $2\frac{1}{3}$? Of .35? The cube of 2? Of 10? Of 6? Of $1\frac{2}{3}$?

2. Write the squares and cubes of the 9 digits, and commit them to memory thoroughly.

419. A *Figure*, written at the right and a little above a number, indicates the power of that number. It is one form of what is called an *Exponent*.

Thus $4^2 = 4 \times 4$, or 16. $2^3 = 2 \times 2 \times 2$, or 8. $3^4 = 3 \times 3 \times 3 \times 3$, or 81. $.3^5 = .00243$. $1.1^3 = 1.331$.

3. What is the square of 23? Of 341? Of 3580? Of $2\frac{3}{4}$? Of 4.3? Of $\frac{6}{7}$? Of .35?

4. What is the cube of 6? Of 10? Of 1.5 Of $\frac{1}{2}$? Of .2? Of .04? Of $\frac{3}{5}$? Of $2\frac{2}{3}$? Of 1.07?

5. What is the value of these expressions: 2.1^2? 12^3? 7^4? $(\frac{2}{3})^2$? $.051^2$? $.01^3$? 1.01^2? 246^2? 10^5? 100^4? $(342)^2$? $(1834)^3$?

EVOLUTION.

420. A **Root** is one of the equal factors into which a number is conceived to be resolved. The *Square Root* of a number is one of *two* equal factors into which the number is conceived to be resolved. The *Cube Root* is one of three equal factors.

421. The *Radical* or *Root Sign* is $\sqrt{\ }$. When written thus, $\sqrt{25}$, it indicates that the square root of 25 is to be taken; that is, that 25 is to be resolved into 2 equal factors, and one of them taken. To indicate the cube root, 3 is written in the sign. Thus $\sqrt[3]{125}$ means the cube root of 125. It is 5.

$\sqrt{9}$ is 3, because 3 is one of the 2 equal factors which compose 9.

$\sqrt[3]{343}$ is 7, because $7 \times 7 \times 7 = 343$.

1. What is the square root of 16? Of 36? Of 144? Of 81? Of 49? Of 1? Of 4? Of 9? Of 25? Of 121? Of 100? Tell why in each case.

2. What is $\sqrt[3]{8}$? $\sqrt[3]{27}$? $\sqrt[3]{1}$? $\sqrt[3]{1728}$? $\sqrt[3]{64}$? $\sqrt[3]{125}$? $\sqrt[3]{343}$? $\sqrt[3]{729}$? $\sqrt[3]{1000}$? $\sqrt[3]{216}$?

422. Finding the root of a number is called *Extracting the Root.*

423. Evolution is the process of extracting roots.

424. A number is said to be a *Perfect Power* when it can be produced by multiplying some number by itself.

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425. ***Extraction of Roots of Perfect Powers.***

As Evolution is the process of finding one of a certain number of equal factors which compose a number, it is but a process of factoring, — resolving a number into equal factors.

Ex. 1. — Show what $\sqrt{16} =$. $\sqrt[5]{32} =$. $\sqrt[3]{1728} =$.

2. What is the square root of 1764?

$$\begin{array}{r|l} 2 & 1764 \\ \hline 2 & 882 \\ \hline 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

Resolving 1764 into its prime factors, we find them to be 2, 2, 3, 3, 7, 7. Hence

$$2 \cdot 3 \cdot 7 \times 2 \cdot 3 \cdot 7, \text{ or } 42 \times 42 = 1764,$$

and 42 is the square root of 1764, being one of the two equal factors which compose it.

3. As above, find the square root of each of the following: 11025, 32400, 245025, 145600, 48841.

4. What is the cube root of 74088?

As above, resolving 74088 into its prime factors, we find $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 7$; i.e., $2 \cdot 3 \cdot 7 \times 2 \cdot 3 \cdot 7 \times 2 \cdot 3 \cdot 7$, or $42 \times 42 \times 42$. Hence $\sqrt[3]{74088} = 42$.

5. As above, find the cube root of each of the following: 46656, 621875, 18399744, 4741632.

General Method of Extracting the Square Root.

426. This method is based on the two following Principles: —

427. PRINCIPLE I. — *In squaring a number, the square of any order above units falls twice as many places to the left of units as the order itself.*

Thus the square of tens falls two places to the left, as $(50)^2 = 2500$; the square of hundreds falls four places to the left, as $(300)^2 = 90000$, or $(900)^2 = 810000$, etc.

428. PRINCIPLE II. -- *The square of any number made up of tens and units is the square of the tens, + twice the product of the tens by the units, + the square of the units.*

Let us show this by squaring 68. Multiplying in the ordinary way, only writing each product by itself, we see at once that the square of 68 is (6 tens)^2 + 2 (6 tens × 8) + 8^2 = 4624.

$$\begin{array}{rr} & 68 \\ & \underline{68} \\ 8^2 = & 64 \\ 8 \text{ tens} \times 8 = & 48 \\ 8 \times 6 \text{ tens} = & 48 \\ (6 \text{ tens})^2 = & \underline{36} \\ & 4624 \end{array}$$

But it is necessary to prove this truth in a more general way, as it is the foundation of the very important *Rule for the Square Root.*

For this purpose, instead of using 8 for the units and 6 for the tens of the number we wish to square,

let u stand for *any number* of units,

and t for *any number* of tens.

Then, as 68 is 6 tens + 3 units, our number will now be represented thus, $t + u$.

We will now square $t + u$, observing, first, that tu means the same as $t \times u$, and that $2tu$ means twice tu. We multiply first by u, saying "u times u is u^2, and u times t is tu." Hence $t + u$ multiplied by u is $tu + u^2$. So t times $t + u$ is $t^2 + tu$. Adding these partial products (as in common multiplication), we have $t^2 + 2tu + u^2$. Which corresponds with the principle.

$$\begin{array}{r} t + u \\ \underline{t + u} \\ tu + u^2 \\ \underline{t^2 + tu} \\ t^2 + 2tu + u^2 \end{array}$$

Ex. — Find the square of 56 according to Principle II. Of 87. Of 243. Of 3469. Of 43. Of 426. Of 1325.

87^2 = (8 tens)^2 + 2 (8 tens × 7) + (7^2) = 6400 + 1120 + 49 = 7569. We may regard 243 as 24 tens and 3 units, and thus have 243^2 = (24 tens)^2 + 2 (24 tens × 3) + 3^2. Pupil, complete the work.

3469 may be regarded as 346 tens and 9 units, etc.

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429. Rule. — I. *Separate the number into periods by placing a point over units and over each alternate figure therefrom.*

II. *Write as the highest order in the root the square root of the greatest square in the left-hand period, subtract its*

square from that period, and to the remainder annex the next period, thus forming a new dividend.

III. *Double the root already found, regarding it as tens, as a Trial Divisor. By this divide the new dividend, annex the quotient to the root, and add it to the trial divisor, thus forming the True Divisor. Multiply the true divisor by the last root figure, subtract the product from the dividend, and to the remainder annex the next period.*

IV. *Repeat the process described in the last paragraph till the work is complete.*

When any trial divisor is not contained in the dividend, place a zero in the root, and also at the right of the divisor, and bring down the next period.

If any figure obtained for the root proves too large, diminish it by 1, and repeat the work.

Approximate roots may be obtained by annexing decimal periods of two zeros each. Decimal periods must always be full, since the square of any decimal has an even number of figures. Why?

Ex. 1.—Extract the square root of 4624.

OPERATION.

$$(t+u)^2 = t^2 + 2tu + u^2 = 4\dot{6}2\dot{4}\ (68 = t+u.$$

$$\begin{array}{rr|l} & t^2 = 36 & \\ 2t = 120 & & \overline{1024} = 2tu + u^2 = (2t+u)\,u. \\ u = \underline{\ \ 8} & & \\ 2t + u = 128 & & \underline{1024} = (2t+u)\,u. \end{array}$$

EXPLANATION.—By placing a point over units figure, and over each alternate figure to the left, we see that the highest order in the root is *Tens*, according to Principle I., and that the square of the tens is in 46. Now the greatest square in 46 is 36. Hence 6 is the *tens* of the root.

Now letting $t + u$ represent the root, we have

$(t+u)^2 = t^2 + 2tu + u^2 = 4624$, by Principle II.

But having found t to be 6 (tens), we subtract its square from 4624, and have $2tu + u^2 = (2t+u)\,u = 1024$ remaining.

Now, as u is small with reference to $2t$, we may, *for a trial*, put $(2t) \times u$, or (12 tens) $\times\ u = 1024$. Hence $1024 \div 120$ will give

the units figure of the root approximately at least. (In this case it gives it exactly.) Now, completing the divisor $2t + u$ by adding the units figure, we find the *True Divisor* to be 128. This multiplied by 8 gives 1024. Hence 68 is the exact square root of 4624.

QUERIES. — Why do we point off as we do the number whose square root is to be extracted?

Ans. — In order to ascertain what the highest order in the root will be, and what its square is, according to Principle I.

Why do we double the root already found, regarding it as tens, for a trial divisor?

Ans. — Because the remainder, which we use as a dividend, is approximately equal to twice this figure regarded as tens, multiplied by the next figure of the root.

Why do we add the root figure to the divisor?

Ans. — Because the *True Divisor* is twice the root previously found regarded as tens + the figure last found.

Why do we bring down only one period at a time?

Ans. — Because the square of the root as far as found in that step can form no part of the remaining periods. Thus, when we are finding the hundreds figure of the root, we do not need the two right-hand periods, since the square of hundreds falls beyond four places from units (Prin. I.).

2 and 3. Extract the square root of 74529, and 2125764.

OPERATION. — When there are more than two figures in the root.

```
      7̇45̇29̇ ( 273                2̇12̇57̇64̇ ( 1458
      4                          1
  47 ) 345                  24 ) 112
       329                        96
 543 ) 1629                285 ) 1657
       1629                       1425
                          2908 ) 23264
                                 23264
```

NOTE. — The same explanation which has been given when the root consists of two figures can be readily extended to any number. Thus, in extracting the square root of 74529, the square of the first two figures 27 (tens) falls in orders from hundreds upwards. We may therefore proceed to find these two figures exactly as if we were extracting the root of 745. Having found these, we may take them as tens, and consider the root as made up of 27 (tens) and some number of units, etc.

4–12. Extract the square root of 2209, 361, 2601, 4900, 120409, 412164, 123201, 6718464, 966289.

13–15. Extract the square root of 87.512. Of 2. Of .4.

OPERATIONS.

```
        87.5120  ( 9.354+          2  (1.4142+           .40  (.632+
        81                         1                     36
  183 ) 651                  24 ) 100              123 ) 400
        549                       96                     369
 1865 ) 10220               281 ) 400             1262 ) 3100
         9325                     281                    2524
18704 ) 89500              2824 ) 11900                   576
        74816                     11296
         4684             28282 ) 60400
```

All that is peculiar in extracting the roots of numbers wholly or in part decimals is to make the number of decimals even by annexing a 0 where there is an odd number of significant figures. This arises from the fact that the square of any decimal must have an even number of figures, since there is as many decimals in the quotient as in both factors. We point off, as directed in the rule, by placing a point over each alternate figure from units.

16–25. Extract the square root of .0256, 5.32, 28.6, 34.3, 7.5, 4.9, .0049, .049, 582.431, 3.671, extending the root to three places of decimals when it is not exact.

26–36. Extract the square root of $\frac{49}{121}$, $\frac{144}{519}$, $\frac{2}{7}$, $\frac{3}{4}$, $\frac{5}{11}$, $\frac{128}{347}$, $\frac{43}{87}$, $1\frac{2}{3}$, $2\frac{1}{4}$, $1\frac{1}{25}$, $3\frac{2}{5}$.

Since a fraction is involved by raising numerator and denominator separately to the required power, its root may be extracted by extracting the root of each term separately.

If neither of the terms is a perfect power, it is best to reduce the fraction to a decimal, extending the operation to such degree of accuracy as may be desired.

When the denominator is a perfect power, and the numerator is not, extract the root of the denominator, and also of the numerator, extending the work as far as desired, and then divide the latter by the former.

General Method of Extracting the Cube Root.

430. The general rule for extracting the cube root is based on the two following principles: —

431. PRINCIPLE I. — *In cubing any number, the cube of any order above units falls three times as many places to the left as the order itself.*

Thus the cube of 3 tens, or $(30)^3 = 27{,}000$. The cube of 5 hundreds $(500)^3 = 125{,}000{,}000$.

432. PRINCIPLE II. — *The cube of any number made up of tens and units is the cube of the tens, + 3 times the square of the tens multiplied by the units, + 3 times the tens multiplied by the square of the units, + the cube of the units.*

DEMONSTRATION. — Let the number we propose to cube be represented by $t + u$, as in (**492**). Now the square of $t + u$ is by (**492**) . . . $t^2 + 2tu + u^2$

Multiplying this by $t + u$, we have the cube.

$$\begin{array}{r} t^2 + 2tu + u^2 \\ t + u \\ \hline t^2u + 2tu^2 + u^3 \\ t^3 + 2t^2u + tu^2 \quad\;\; \\ \hline t^3 + 3t^2u + 3tu^2 + u^3 \end{array}$$

The multiplication is explained thus: multiplying by u, we have u^2 multiplied by u, which makes u^3, just as 2 squared (2^2) multiplied by 2 makes 2 cubed (2^3). $2tu$ multiplied by u makes $2tu^2$; for $2tu$ is $2 \times t \times u$, and, putting in another factor of u, we have $2 \times t \times u \times u$, or $2tu^2$. In like manner the other terms are multiplied. In adding the partial products we notice that 2 times tu^2 and 1 time tu^2 make 3 times tu^2, or $3tu^2$. So 2 times t^2u and 1 time t^2u make $3t^2u$. Finally, we observe that this result, $t^3 + 3t^2u + 3tu^2 + u^3$, agrees with our statement of the principle.

433. Rule. — I. *Separate the number into periods by placing a point over units, and over each third figure therefrom.*

II. *Write as the highest order in the root the cube root of the greatest cube in the left-hand period, subtract its cube from that period, and to the remainder annex the next period, thus forming a new dividend.*

III. *Take 3 times the square of the root already found, regarded as tens, for a Trial Divisor. By this divide the new dividend, which will give approximately the next figure in the root.*[1] *Then form the True Divisor by adding to the Trial Divisor 3 times the product of the preceding part of the root (regarded as tens) by the last figure, and the square of this last figure. Multiply this True Divisor by the last root figure, subtract the product from the last dividend, and to the remainder annex the next period.*

IV. *Proceed as described in the last paragraph till the work is complete.*

[See notes under rule for Square Root.]

1. Extract the cube root of 262144.

OPERATION.

$$(t+u)^3 = t^3 + 3t^2u + 3tu^2 + u^3 = 26\dot{2}14\dot{4}\ (64 = t + u.$$

$$\begin{array}{rr|l} & t^3 = 216 & \\ 3t^2 = & 10800 & 46144 = 3t^2u + 3tu^2 + u^3, \text{ or} \\ 3tu = & 720 & (3t^2 + 3tu + u^2)\,u.\text{[2]} \\ u^2 = & 16 & \\ 3t^2 + 3tu + u^2 = & 11536 & 46144 = (3t^2 + 3tu + u^2)\,u, \text{ or} \\ & & 3t^2u + 3tu^2 + u^3. \end{array}$$

EXPLANATION. — Pointing off into periods, by placing a point over units and over each third figure therefrom, we see that the highest order in the root is tens (Principle I.), and that its cube is contained in 262. Now the greatest cube in 262 is 216, the cube root of which is 6. Hence 6 is the tens figure of the root.

Now, letting $t + u$ represent the root, we have $(t+u)^3 = t^3 + 3t^2u + 3tu^2 + u^3 = 262144$.

But, having found t to be 6 (tens), we subtract its cube from 262144, and have $3t^2u + 3tu^2 + u^3 = (3t^2 + 3tu + u^2)\,u = 46144$ remaining.

Now, as u is small with reference to $3t^2$, we may, *for a trial*, put

---

[1] The root figure thus found may be too large; but, if so, the fact will appear when the True Divisor is multiplied by it.

[2] This means $3t^2 + 3tu + u^2$ multiplied by u, which makes $3t^2u + 3tu^2 + u^3$.

$(3t^2) \times u = 46144$, and, as we know t, this becomes $10800 \times u = 46144$. Hence u is, approximately, $46144 \div 10800$.

We thus find that the units figure is probably 4. Now, completing the divisor by adding to it $3tu$, — i.e., 3 times the product of the root already found (remembering that it is tens) by the units, — and the square of the units, we find that the *True Divisor*, $3t^2 + 3tu + u^2$, is 11536. This multiplied by 4 gives 46144. Hence 64 is the exact cube root of 262144.

[Similar queries to those following the explanation of the method of extracting the square root may be raised here, and can be answered in a similar manner from the above explanation.]

2–5. Extract the cube root of 54872. Of 41063625. Of 354894912. Of 3416.53.

```
        54872 ( 38
        27
 2700 | 27872
  720 |
   64 |
 ---- |
 3484 | 27872
```

```
          41063625 ( 345
          27
   2700 | 14063
    360 |
     16 |
   3076 | 12304
 346800 | 1759625
   5100 |
     25 |
 351925 | 1759625
```

```
           354894912 ( 708
           343
   14700 | 11894912
 1470000 |
   16800 |
      64 |
 1486864 | 11894912
```

```
           3416.530 ( 15.06
           1
     300 | 2416
     150 |
      25 |
     475 | 2375
 6750000 | 41530000
   27000 |
     216 |
 6777216 | 40663296
           866704
```

6–13. Extract the cube root of the following: 157464; 74088; 571787; 15625; 2744; 1124864; 2571353; 651714-363.

[1] As this is not contained in 11894, we write 0 in the root, and bring down the *next period*.

14–26. Extract the cube root of 34285.7; 34.3472; 5; 48; 2; .4932; .8; .343; .27; $\frac{125}{256}$; $\frac{343}{729}$; $\frac{28}{125}$; $\frac{14}{33}$.

[See note under Ex. 26–36 in Square Root.]

Applications.

434. PROBLEM 1. — *Given the area of a square, to find its side.*

RULE. — Extract the square root of the area.

The reason for the rule is, that the area of a square is the square of its side, as a square is a rectangle whose sides are equal to each other. (215.)

NOTE. — If the area is given in a denomination which has no corresponding linear unit (as in *acres*), it should be reduced to a denomination which has such a linear unit (as to *rods*), before the root is extracted.

1. What is the side of a square which contains 74529 square feet?

2. How many rods on a side is a square acre? How many feet?

3. How many rods on a side is a square 40-acre lot? A square 80-A.?

4. What is the side of a square containing 10 Ars? What of one containing 40^{a}? 60^{a}? 100^{a}? 25^{Ha}? 70^{Ha}?

5. A man has a farm containing 520 acres, in the form of a rectangle, twice as long as wide. What are the dimensions?

6. What is the side of a square which contains twice as much as one 1 ft. on a side? 3 times as much? 4 times? 10 times? 9 times?

---

435. When *three* numbers are so related that the 1st is to the 2d as the 2d is to the 3d, the 2d is called a **Mean Proportional** between the other two. The third term is called a *Third Proportional to the other two.*

Thus, in the proportion 2 : 4 :: 4 : 8, 4 is a *Mean Proportional* between 2 and 8, and 8 is a *Third Proportional* to 2 and 4.

436. PRINCIPLE. — *A Mean Proportional between two quantities is the square root of their product.*

Thus let a stand for the *first* number, b for the *third*, and m for the *mean*; then $a : m :: m : b$, and $m^2 = ab$, or $m = \sqrt{ab}$.

Ex. 1. — Find the mean proportional between 4 and 9. 27 and 3. 5 and 11. $\frac{2}{3}$ and $\frac{1}{2}$.

2. Find a third proportional to 7 and 4. To 8 and 12. To 12 and 8. To 4 and 6. To 6 and 4. To $\frac{2}{3}$ and $\frac{4}{5}$. To $\frac{3}{5}$ and $\frac{2}{3}$.

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437. PROBLEM 2. — *Given the contents, to find the edge of a cube.*

RULE. — Extract the cube root of the contents.

[Reason and note similar to the above.]

1. What is the edge of a cube containing 45,499,293 cu. ft.? One containing 487.42 cu. ft.?

2. What must be the edge of a cubical box which shall contain a bushel? (See **229.**) What of one containing 100 bushels?

3. How many feet on an edge is a cubical mass containing 389,017?

4. How many feet on an edge is a cube whose content is a cord?

5. Mr. A. received $97.20 for digging a cellar at 20¢ per cubic yard. The cellar was 3 times as wide as it was deep, and twice as long as it was wide. What were its dimensions in feet?

6. The edge of a cubical vessel is 1 ft. What is the edge of one which contains 2 times as much? 3 times? 4 times? 8 times? 27 times? 40 times?

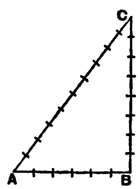
CHAPTER VIII.

PROBLEMS IN MENSURATION.

[For definitions of point, line, angle, rectangle, square, solid, parallelopiped, and cube, see *Measures of Extension*, p. 171, *et seq.*]

438. PROBLEM 1.— *To find one side of a right-angled triangle when the other two are given.*

439. A **Triangle** is a plane (flat) figure with only 3 sides. When one of its angles is a right angle, it is called a *Right-Angled Triangle.* If you make a right-angled triangle whose *Base*, **AB**, is 6, and whose *Perpendicular*, **CB**, is 8, and then measure the *Hypothenuse*, **AC**, you will find the latter to be just 10. Now, notice that $6^2 + 8^2 = 10^2$, and in Geometry this is found true of all right-angled triangles. *The square of the hypothenuse equals the sum of the squares of the other sides.*



440. Since $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$, $AC = \sqrt{(\overline{AB}^2 + \overline{BC}^2)}$;

Also, $\overline{AB}^2 = \overline{AC}^2 - \overline{BC}^2$, $AB = \sqrt{(\overline{AC}^2 - \overline{BC}^2)}$;

And $\overline{BC}^2 = \overline{AC}^2 - \overline{AB}^2$, $BC = \sqrt{(\overline{AC}^2 - \overline{AB}^2)}$.

[It is important that the pupil become familiar with this form of statement, and be able to put it into common language.]

Ex. 1.—What is the length of a brace in a building which goes into the post 3 ft. from the angle, and into the beam 2 ft. from the angle?

2. What is the length of a brace which reaches from corner to corner of a gate which is 10 ft. long and $4\frac{1}{2}$ ft. high?

3. What length of rafter is required for a building 30 ft. wide, the ridge of the roof being 10 ft. higher than the plates, making no allowance for the projection beyond the plates?

4. What is the distance from one corner on the floor of a room to the diagonally opposite corner in the ceiling, the room being 18 ft. by 20 ft., and 12 ft. high?

5. Wishing to find the distance between two trees between which a pond lay, I measured from one of the trees a line perpendicular to the line joining the trees, a distance of 100 rods, and from the end of this line measured a right line to the other tree, 160 rods. What was the distance between the trees?

6. Having a pole which I knew to be 30 ft. long, and wishing to find the height of a wall, I put one end of the pole against the top of the wall, and found that the other end rested on the ground 12 ft. from the wall. What was the height of the wall?

7. What must be the length of the rafter for a *quarter-pitch* roof on a house 32 ft. wide? What for a third-pitch roof? Half-pitch? Whole-pitch?

Quarter-pitch means that the height of the ridge above the plates is $\frac{1}{4}$ the span, etc.

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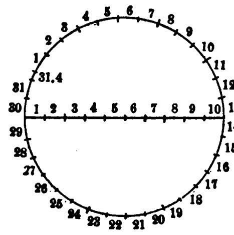
441. A **Circle** *is a plane (flat) figure bounded by a curved line, every point of which is equally distant from a point within called the* **Centre.** *The curved line is called the* **Circumference.** *A line drawn from the centre to the circumference is called a* **Radius.** *A line drawn through the centre and terminated by the circumference is a* **Diameter.**

442. PROBLEM 2. — *To find the circumference of a circle when the diameter is given.*

By drawing a circle very carefully, say 1 inch in diameter, as in the margin, and dividing the diameter into 10ths inches, a nice pair of dividers can be opened one 10th inch, and made to step around the circumference. If it is all done with nicety, it will be found to be a little over 31 steps around when it is 10 across. Hence we can learn that the circumference is a little more than 3.14 times the diameter. *In Geometry it is shown that the circumference is very nearly* 3.1416 *times the diameter.* Nobody knows the exact ratio: it is represented in Geometry by the Greek letter π (pi).



Ex. 1.—How long a bar of iron will it take to make a tire for a wheel which is 50 in. in diameter?

2. What is the diameter of a tree which measures $33\frac{1}{2}$ ft. around?

3. As the earth revolves on its axis once in 24 hours, at what rate per hour do we move in consequence of this rotation, the radius of the earth being 3962 miles?

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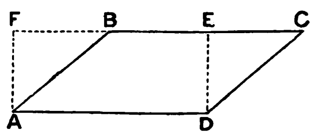
443. Parallel lines *are lines which run in the same direction. A plane figure bounded by four sides is a* **Quadrilateral.** *When each side is parallel to the side opposite, the figure is a* **Parallelogram**; *and, when the angles are all right angles, the figure is a* **Rectangle.**

444. Problem 3.—*To find the area of a parallelogram.*

Let ABC be any parallelogram. Draw DE perpendicular to AD, and consider the triangle EDC as cut off, and placed on the other end in the position AFB. Then the rectangle AFED has the same area as the parallelogram ABCD. But the area of the rectangle is the product of the base and altitude (**215**).



Hence the area of any parallelogram is the product of its base and altitude.

Ex. 1. — A piece of velvet is cut "on the bias" at both ends; i.e., diagonally across. If the piece is $\frac{1}{2}$ yd. wide, how much velvet is there in a piece which measures 8 yd. along the selvage, the entire figure being a parallelogram?

2. How many square yards in the plastering of a ceiling of a room 20 ft. by 18?

3. There are two parallel roads $\frac{1}{2}$ a mile apart. Mr. A.'s farm runs obliquely across from one to the other in the form of a parallelogram. How many acres in his farm if it measures 1 mi. along the road?

4. How many bricks (a common brick is $2 \times 4 \times 8$ in.) laid on the side will it take to pave a cellar 15×18 ft. on the bottom? How many if laid on the edge?

5. How many tiles 6 in. square will lay a floor 20×36 ft.?

6. What is the cost of plastering a room 20×24 ft. and $10\frac{1}{2}$ ft. high, including the ceiling, at 25¢ per square yard, there being 3 doors $7\frac{1}{2}$ ft. high and 3 ft. 9 in. wide, and 4 windows 6 ft. high and $3\frac{1}{2}$ ft. wide?

7. How many yards of cloth 27 in. wide will it take to line 12 yd. $1\frac{1}{4}$ yd. wide?

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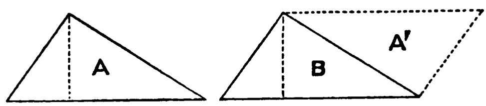
445. PROBLEM 4. — *To find the area of a triangle.*

Suppose A is the triangle whose area we seek. Make another just like it, as A′, and then put the two together, as B and A′. Then the two make



a parallelogram, whose area is the product of the base and altitude. But the triangle is half the parallelogram. *Hence the area of a triangle is half the product of the base and altitude,* or half of either multiplied into the other.

446. Note.—If the three sides only are given, *from half the sum of the three sides subtract each side separately, multiply the half sum and these remainders together, and extract the square root of the product.*

The pupil cannot see *why* this is so without considerable knowledge of Geometry.

1. What is the area in acres of a triangle whose base is 40 rods and altitude 24 rods?

2. What is the area in acres of a triangular field whose base is 85 rods and altitude 75 rods?

3. How many square inches in a triangular board whose sides are respectively 12, 18, and 24 inches?

4. How many acres in a triangular field whose sides are respectively 20, 30, and 40 rods?

5. What is the area in acres of a triangular piece of ground whose base is 7.52 ch., and altitude 5.32 ch.?

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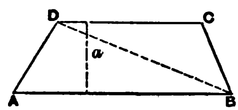
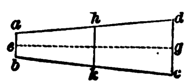
447. Problem 5. — *To find the area of a trapezoid.*

A **Quadrilateral** is a plane figure with four sides. If two of these are parallel, and the other two not, the figure is a **Trapezoid.** ABCD is a trapezoid, and a is its altitude. It is evident that the area is the area of two triangles having the same altitude as the trapezoid, and, for their bases, the upper base and the lower base. *Hence the area of a trapezoid is ½ the product of the altitude into the sum of the bases,* or half the sum of the bases into the altitude.



Let *abcd* be a tapering board. Such a board is a trapezoid, and *ab* and *cd* are the bases, and *eg* the altitude. Now, the width across the middle, as *hk*, is half the sum of the two bases, or ends. *Hence, to find the area of such a board, multiply its width, measured in the middle, by its length.*



1. How many acres in a trapezoid whose parallel sides are 45 and 60 rods, and the distance between them 50 rods?

2. How many square feet in the surface of a board 14 ft. long, and 21 in. wide in the middle?

448. *In measuring boards the* SURFACE MEASURE *alone is considered, unless the board is more than an inch thick.*

When the lumber is $1\frac{1}{4}$ in. thick, $\frac{1}{4}$ is to be added to the superficial measure; when $1\frac{1}{2}$ in. thick, $\frac{1}{2}$ is to be added; when 2 in. thick, the superficial measure is to be doubled. Why?

QUERY. — Why is it that in measuring boards we may multiply the length IN FEET by the width in INCHES, and divide the product by 12 to get the square feet?

3. How many square feet in a load of boards in which there are 10 boards 12 ft. long and 10 in. wide, 8 boards 16 ft. long and 8 in. wide, 20 boards 14 ft. long and 10 in. wide, the widths all being measured in the middle? How much would this load amount to at $14 per M?

4. How many feet of boards in a load consisting of 30 pieces of $1\frac{1}{4}$ in. stuff, 12 ft. long 6 in. wide; 40 pieces of $1\frac{1}{2}$ in. stuff, 10 ft. long and 9 in. wide; and 20 bolts of "siding" ($\frac{1}{2}$ in. stuff), 12 ft. long and 5 in. wide, with 6 pieces in each bolt?

5. How much flooring $1\frac{1}{4}$ in. thick will it take for the floors of 10 rooms, 2 being 18 × 15 ft. (18 ft. by 15 ft.) each, 2 15 × 12 ft. each, 1 12 × 16 ft., 1 12 × 12 ft., 2 10 × 12 ft. each, and 2 10 × 10 ft.? How much will it cost at $18 per M?

6. My house-lot is a corner-lot 4 rods by 8. What will the lumber cost for a 2 in. plank-walk, 6 ft. wide, at $18 per M, with 3 by 4 in. scantling for stringers, the stringers to be laid crosswise, and so that each 12 ft. plank shall rest on 4 stringers, the scantling costing $12 per M?

7. Carpenters often charge by the *square* (10 by 10 ft.) for laying floors, ceiling, roofing, etc. What will the floor-

ing of four rooms with $1\frac{1}{4}$ in. plank cost at $20 per M for the plank, 75¢ per square for laying, and 20 lb. of nails at 3¢ per lb., the rooms measuring respectively 18 ft. square, 15 by 18 ft., 12 by 15 ft., and 16 ft. square?

8. Mr. A. has a farm, two of whose sides are north and south lines $\frac{1}{2}$ a mile apart. One of these sides is 60 rods, and the other 80. How many acres in his farm?

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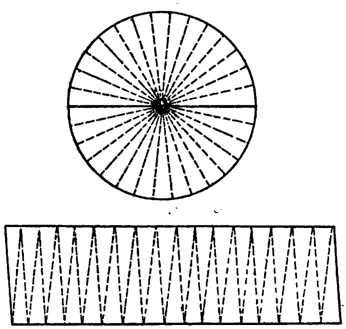
449. PROBLEM 6. — *To find the area of a circle.*

If you cut out of a tough piece of cardboard a circle, then cut it in two on a diameter, and cut from the centre through *almost* to the circumference, as represented by the dotted lines, you can straighten the semicircumferences out, and make two comb-like pieces, which you can slip together and thus make a very perfect parallelogram. From this you will see that *the area of a circle is the product of its radius into its semicircumference.*



Now, if the radius is r, we have learned that the circumference is $3.1416 \times 2r$, and the semicircumference $3.1416 \times r$. If we multiply this by r, we have $3.1416 \times r^2$. Thus we see that *the area of a circle is the square of the radius multiplied by* 3.1416.

1. What is the area in square feet of a circle whose diameter is 28 inches? What is the circumference?

2. Two 5 in. stove-pipes run into a 7 in. pipe. Is the 7 in. pipe large enough to carry the smoke of the two?

3. How many square feet in a circle 5 ft. 6 in. in diameter?

4. What is the area in acres of a circular piece of ground whose circumference is 1 mile?

5. A horse is tied by a rope to a stake in a meadow. The rope being attached to his head, how long must it be so that he can graze over an acre?

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450. PROBLEM 7. — *To find the amount of carpeting required for a room whose dimensions are given.*

In order to do this, we must know which way the breadths are to run, the width of a breadth, and how much will be wasted on a breadth in cutting so as to match the figures. Often it is necessary to know whether it is practicable to split a breadth. When these facts are known, *we find the number of breadths which will be required, and the length of a breadth including waste, and multiply the two together.*

1. How much carpeting, $\frac{3}{4}$ yd. wide, will it take to carpet a room 18 ft. by 15 ft. if the breadths run lengthwise, and it wastes 6 in. on a breadth in matching? How much, if the breadths run crosswise, and it wastes 2 in. in matching?

2. What will it cost to carpet a room 14 ft. by 17 ft. with yard-wide carpeting at 89¢ per yard, which wastes 4 in. on a breadth in matching, if the breadths run crosswise? What, if the breadths run lengthwise, and it wastes 6 in. in matching?

3. Wall-paper is usually 18 inches wide, and 8 yards make a roll. How many rolls must we buy (we can buy only *whole* rolls) to paper a room 16 × 18 ft. whose walls are 10 ft. high, no allowance being made for doors and windows? What will it cost at 40¢ per roll if an allowance of $\frac{1}{4}$ be made for doors and windows?

Find how many strips of 10 ft. each it takes.

4. How many rolls of wall-paper must we buy to paper 2 rooms, one 12 × 16 ft. and the other 15 × 18, each 10 ft. high, there being 7 doors and windows in one room, and 6 in the other, each of which is $3\frac{1}{2}$ ft. wide; it being understood *that* the strips are not to be pieced, but that the 4 ft. which

each roll will overrun will piece out above the doors and windows?

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451. **Similar Figures** are such as have the same shape.

452. *The areas of similar plane figures are in the same ratio as the squares of their like sides, or lines.*

1. There are two fields of the same shape (similar): one is 10 rods on a certain side, and the other is 15 rods on the corresponding side. What is the ratio of their areas?

2. Two circles have their diameters respectively 6 and 8. What is the ratio of their areas?

453. *The volumes of similar solids are in the same ratio as the cubes of their corresponding edges, or lines.*

1. How many times as large as a 2 in. ball is a 3 in. ball?

2. A cubical bin 5 ft. on an edge contains (practically) 100 bu. How much does one 6 ft. on an edge contain? 10 ft.? 4 ft.?

3. Which is the stouter (bulkier, "fleshier"), a man 5 ft. 10 in. in height who weighs 175 lb., or one 6 ft. who weighs 180 lb.?

4. In the last example, what must be the weight of the 6 foot man in order that he may have the same proportions as the other?

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454. PROBLEM 8. — *To find the volume of a right parallelopiped.*

(See **216.**)

1. How many cubic feet in a pile of 4 foot wood (that is, sticks of wood 4 feet long) piled in the usual way, the pile being 32 feet long and 5 feet high? How many cords?

2. How many cords of wood in a pile of 4 foot wood 130 ft. long and 6 ft. high?

SUGGESTION. — A convenient way to solve such an example is this: —

$$\frac{\overset{65}{\cancel{130}} \times \cancel{4} \times \overset{3}{\cancel{6}}}{\cancel{4} \times \cancel{4} \times 8} = \frac{195}{8} = 24\tfrac{3}{8}.$$

3. How many cords in a pile of 6 ft. wood 148 feet long and 8 feet high?

4. How many cords in a pile of wood 36 ft. long, 4 ft. wide, and 4 ft. high? When wood is piled in this way, how many feet in length of the pile does it take to make a cord?

5. Why does $\frac{250}{8}$ give the number of cords in a pile of 4 ft. wood 4 ft. high and 250 ft. long?

6. How many cords in three piles of 4 ft. wood, the first 36 ft. long and 4 ft. high, the second 42 ft. long and 5 ft. high, and the third 20 ft. long and 6 ft. high?

7. How many cords in a pile of 4 ft. wood 200 ft. long and 7 ft. high? How many, if the wood be 3 ft. long? If 2 ft. long? How many in each case, if the pile be 6 ft. high? 5 ft. high? 8 ft. high?

8. How much wood in a load consisting of 3 lengths of 4 foot wood, the load being 3 ft. 2 in. wide and 2 ft. 6 in. high?

9. How much wood in a load consisting of 2 lengths of 4 ft. wood, the average width of the load being 2 ft. 9 in. and the height 3 ft.?

10. How many cubic yards in a ditch $\frac{1}{2}$ mi. long, and which averages 3 ft. deep and 4 ft. wide?

11. At 20¢ per cubic yard what does it cost to excavate a cellar 20 ft. by 30 ft., and $6\frac{1}{2}$ ft. deep?

12. At 30¢ per cubic foot what cost a stick of timber *$15\frac{1}{2}$ in.* square and 20 ft. long?

13. What is the cost of digging a trench 650 ft. long, $2\frac{1}{2}$ ft. wide at top, and $1\frac{1}{2}$ ft. wide at bottom, and averaging $3\frac{1}{4}$ ft. deep, at 25¢ per cu. yd.?

The average width is 2 ft.

14. The end (cross-section) of a certain railroad tunnel is 525 sq. ft., and the length of the tunnel $\frac{3}{8}$ of a mile. How many cubic yards were removed in the excavation?

15. In a room 125 ft. by 90 ft., with ceiling 30 ft. high, and seating 2500 persons, how soon will the air have been all breathed, allowing 10 cu. ft. per minute for each person?

16. Milwaukee bricks are $8\frac{1}{2} \times 4\frac{1}{2} \times 2\frac{3}{8}$ in. If it takes 40000 common bricks ($8 \times 4 \times 2$ in.) for a particular structure, how many Milwaukee bricks will it take?

17. Philadelphia and Baltimore bricks are $8\frac{1}{4} \times 4\frac{1}{8} \times 2\frac{3}{8}$ in. How many such bricks would be required for the structure in Ex. 16?

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455. Doyle's Rule for calculating the amount of square-edged inch boards which can be sawed from a round log is this: *From the diameter in inches subtract 4; the square of the remainder will be the number of square feet of inch boards yielded by a log 16 ft. in length.*[1]

The yield of logs of the same diameter is in the ratio of their lengths. The log is scaled; i.e., its diameter is measured, at the top end.

1. What is the board measure of a log 16 ft. long and 18 in. in diameter? One 20 in. in diameter? 30 in.?

2. What is the board measure of a log 12 ft. long and 2 ft. in diameter? 31 in.? 40 in.? 37 in.?

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[1] This rule, so admirable in its simplicity, is the foundation of the table in *Scribner's* popular *Lumber and Log Book*, which is said to have a larger sale than all other books of the kind together, and is a generally recognized standard among lumbermen. Nevertheless, in a scientific point of view, the rule is but a rude approximation. See the Author's SCIENCE OF ARITHMETIC, p. 277.

3. What is the board measure of a log 18 ft. long and 36 in. in diameter? Of one 15 ft. long? 10 ft.? 20 ft.?

456. Scribner's, or the Lumberman's, Rule for computing the amount of square timber yielded by a given log. — *Call ½ the sum of the extreme diameters the average diameter, and ⅔ of this diameter the side of the log when hewn square.*

1. How many cubic feet of square timber can be cut from a log 30 in. in diameter at top, and 36 at butt, and 48 ft. long?

$\frac{30+36}{2}=33$, the av. diameter; ⅔ of $33=22$, side of square stick.

$\frac{22\times 22\times \cancel{48}^{\,4}}{\cancel{12}\times \cancel{12}_{\,3}}=\frac{484}{3}=161.4$, nearly. This is the result given in *Scribner*. He usually "saves" the fractions in this table by reckoning the next unit.

2. Compute the following by Scribner's Rule: Diameters 24 and 29 in., length 35 ft. Diameters 20 and 32 in., length 50 ft. Diameters 36 and 48 in., length 42 ft.

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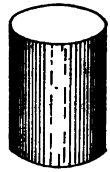
457. PROBLEM 9. — *To find the area of the surface and the volume of a right cylinder with a circular base.*



A *Right Cylinder* with a circular base (which is usually called simply a cylinder) is a solid which may be conceived as generated by revolving a rectangle about one of its sides. The bases (or ends) of the cylinder are circles. The *Altitude* is the distance between the bases. *The* AREA *of the surface* (i.e., *the convex surface*), *besides the bases, is the circumference of the base multiplied by the altitude. The* VOLUME *is the area of the base multiplied by the altitude.*

The reasons for these rules are evident. The convex surface may be thought of as a thin film, which, taken off, could be spread out into a rectangle whose base would be the circumference of the cylinder, and whose altitude, the altitude of the cylinder. As to the volume: as the area of the base is 3.1416 times the radius square, it may be thought of as a square of a certain size (say in inches), and then 1 in. in depth would contain as many cubic inches as there were square inches in the base, etc.

1. How many barrels of water does a cylindrical cistern contain which is 8 ft. in diameter and 8 ft. deep? 6 ft. in diameter and 7 ft. deep?

2. How many gallons of water does a pipe 3 in. in diameter discharge in an hour, if the current flows 200 ft. a minute?

3. The specific gravity of green oak being .85, and a cubic inch of water weighing $252\frac{1}{2}$ grains, what is the weight of a green-oak log 2 ft. in diameter and 12 ft. long?

4. How many cubic feet does a cylindrical vat contain, which is 20 ft. in diameter and 15 ft. deep?

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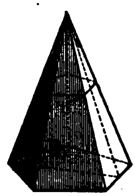
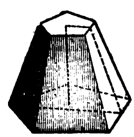
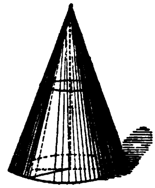
458. A **Right Pyramid** is a solid which has equal triangles for its faces, and its base an equilateral polygon. *The Vertex* is the point in which the triangles meet. *The Altitude* is the perpendicular distance from the vertex to the base. *The Slant Height* is the distance from the vertex to the middle of one side of the base.



459. The **Frustum** of a pyramid is a portion of a pyramid included between the base and any plane cutting the pyramid parallel to the base. The *Altitude* and *Slant Height* are the portions of these lines of the pyramid pertaining to the frustum.

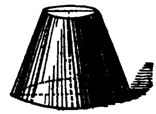




460. When the base of a Right Pyramid, or of a frustum, passes into a circle, the solid becomes a Right Cone, or frustum thereof.

461. PROBLEM 10.—*To find the area of the convex surface of a right pyramid or cone.*

Multiply the perimeter of the base by $\frac{1}{2}$ the slant height.



462. PROBLEM 11. — *To find the area of a frustum of a right pyramid or cone.*

Multiply $\frac{1}{2}$ the sum of the perimeters of the bases by the slant height.

463. PROBLEM 12. — *To find the volume of a right pyramid or cone.*

Take $\frac{1}{3}$ the product of the base and altitude.

464. PROBLEM 13. — *To find the volume of a frustum of a right pyramid or cone.*

Multiply the sum of the two bases and a mean proportional between them by $\frac{1}{3}$ the altitude.

1. What is the convex surface of a right pyramid whose base is a polygon of 5 sides, each of which is 10 ft., and whose slant height is 14 ft.?

2. What is the volume of a pyramid whose base is a polygon of 4 sides, each of which is 200 ft., and whose altitude is 150 ft.?

3. How many barrels will a cistern in the form of an inverted frustum of a right cone contain, the diameter of the bottom being 8 ft., of the top 10 ft., and depth 7 ft.?

465. PROBLEM 14. — *To find the area of the surface and the volume of a sphere whose radius is given.*

The area of the surface is 4 times the area of a circle with the same radius.

The volume is the product of $\frac{1}{3}$ the radius into the area of the surface.

1. Find the area of the surface and the volume of the earth, its diameter being 7958 miles.

2. What is the area of the surface, and what the volume, of a sphere 2 ft. in diameter?

STRENGTH OF BEAMS.

466. *The* STRENGTH *(power to support weight) of rectangular* BEAMS, *supported at both ends, is in the ratio of their cross-sections (ends), multiplied by their depths.*

1. How much stronger is a 3 by 4 in. beam when set on edge than when lying flat? A 2 by 8? A $2\frac{1}{2}$ by 10?

2. Which is the stronger beam, one 6 in. square, or one $2\frac{1}{2}$ by 10 in., set on edge? What is the ratio of their strengths?

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467. THERMOMETER SCALES.

1. The scale of the common thermometer (*Fahrenheit*) is divided into 180° between the freezing point (32°) and the boiling point (212°). The *Centigrade* scale is divided into 100°, from freezing (0°) to boiling (100°). What is the relative length of the degrees?

2. When the temperature is 68° F., what is it Centigrade? When 85° F., what is it C.? When 20° below 0, or —20° F., what is it Centigrade?

3. 29° C. is what F.? 30° C.? —18° C.? —15° C.?

Review

CHAPTER IX.

468. ONE HUNDRED TEST EXERCISES IN ARITHMETICAL OPERATIONS.

1. $3\frac{1}{2} - \frac{\frac{2}{3}}{4} + \frac{4}{\frac{1}{5}}$.

2. $\frac{3}{5}$ of $4\frac{2}{5} \times .25$.

3. $1.33\frac{1}{3} \times 4\frac{1}{5} \div \frac{2}{\frac{3}{5}}$.

4. $\frac{11\frac{2}{3}}{12\frac{3}{5}} + \frac{5\frac{7}{8}}{\frac{3}{16}} - \frac{\frac{2}{7}}{4}$.

5. $(5\frac{1}{2} + 3) \times (2 - \frac{3}{7})$.

6. $\frac{4 - \frac{2}{3}}{3} - \frac{3\frac{1}{2} + 2}{8}$.

7. $\frac{5\frac{1}{3} + 2\frac{3}{4}}{4.5 - 1\frac{1}{3}} \div 6.25$.

8. $\frac{1\frac{1}{4} \times \frac{4}{3}}{5\frac{1}{2}} \div \frac{3 - \frac{2}{3}}{\frac{1}{2} \text{ of } 7}$.

9. $\frac{4\frac{1}{3} + 2\frac{1}{5} - 3}{\frac{3}{5} \text{ of } 1.25} \times (\frac{2}{3} + \frac{1}{2})$.

10. $\frac{5.8 \div .002}{1.6} + \frac{.45}{5}$.

11. $\frac{1 \div .0001}{.5 \div 50} \times 400$.

12. $\frac{42.68 \div .002}{\frac{1}{5} \text{ of } 13} \div .8$.

13. $\frac{500 \div (\frac{2}{3} \text{ of } .066)}{(\frac{1}{2} \text{ of } .7) \div (4 - \frac{.2}{20})}$.

14. $\frac{.3 + .03 + .003}{3.5 \div .07}$.

15. $\frac{56 \div .007}{\frac{1}{2} \text{ of } .04} \div \frac{.02}{20}$.

16. $\frac{(\frac{2}{3} + 4.2) \div (.125 \times \frac{5}{2})}{.375 \times (\frac{3}{5} - .16\frac{2}{3})}$.

17. $\frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} \times \frac{1 + \frac{1}{3}}{1 + \frac{1}{3}}$.

18. $\frac{.05 \times 1.02}{\frac{3}{4}} \div \left(\frac{1}{6\frac{1}{2}} - \frac{.005}{50}\right)$.

19. $\frac{3.7 + 1.05 + .508}{.43 - .005} \times (\frac{2}{3} - .5)$.

20. $(.0008 \div .008) \times 10000$.

21. $\frac{.468}{200} \times \frac{\frac{5}{6}}{2.12\frac{1}{2}} \div 5\frac{3}{8}$.

22. $\dfrac{1 - .0001}{.5 \div 5} + 3\frac{1}{2}$.

23. $\dfrac{4 - .002}{3 \div .03} + \dfrac{.04 \div .0002}{.01\frac{1}{2}}$.

24. $\dfrac{\sqrt{.4} + \sqrt{.9}}{\sqrt{.36} + \sqrt{.16}} + \dfrac{\sqrt{.4 + .9}}{\sqrt{.36 - .16}}$.

25. $\dfrac{\sqrt{\frac{4}{9}} \div \sqrt{\frac{16}{25}}}{\sqrt{\frac{1}{5}} \times \sqrt{5}}$.

26. $\sqrt{\frac{2}{3}} : \sqrt{\frac{3}{2}} :: \sqrt{7} : ?$

27. $\dfrac{\sqrt{4.9} + \sqrt{3.6}}{\sqrt{.25} - \sqrt{.01}}$.

28. $\dfrac{\sqrt{.0036} + \sqrt{490}}{\sqrt{.025} - \sqrt{.0025}}$.

29. $\sqrt{16} \times \sqrt{4}$, and $\sqrt{16} + \sqrt{4}$.

30. Is $\sqrt{16} \times \sqrt{4} = \sqrt{16 \times 4}$?

31. Is $\sqrt{16} + \sqrt{4} = \sqrt{16 + 4}$?

32. $\sqrt{\frac{3}{7}} \times \sqrt{\frac{7}{3}} \times \sqrt{\frac{2}{5}} \times \sqrt{\frac{5}{2}}$.

33. $\sqrt{\frac{3}{7}} + \sqrt{\frac{7}{3}} + \sqrt{\frac{2}{5}} + \sqrt{\frac{5}{2}}$.

34. $5\frac{1}{2} : 3\frac{1}{4} :: \frac{2}{3} : ?$

35. $\frac{2}{5} : 1\frac{1}{5} :: 8 : ?$

36. Why is $\frac{2}{3} : \frac{4}{5} :: \frac{15}{7} : ?$ the same as $\frac{3}{2} \times \frac{4}{5} \times \frac{15}{7} = ?$

37. $\sqrt{\frac{3}{4}} : \sqrt{\frac{3}{9}} :: 10 : ?$

38. $.05 : 3 :: 1.02 : ?$

39. $.4 : ? :: 6\frac{1}{2} : .03$.

40. $? : .001 :: .02 : .3$.

41. $4.0\frac{3}{4} : 1.00\frac{1}{2} :: .0\frac{2}{5}$?

42. $\sqrt{2\frac{1}{2}} + \sqrt{4\frac{1}{5}}$

43. $\sqrt{2\frac{1}{2}} \times \sqrt{4\frac{1}{5}}$.

44. $\sqrt{16} \div \sqrt{4}$, and $\sqrt{16} - \sqrt{4}$.

45. Is $\sqrt{16} \div \sqrt{4} = \sqrt{16 \div 4}$?

46. Is $\sqrt{16} - \sqrt{4} = \sqrt{16 - 4}$?

47. $\sqrt{\frac{2}{5}} \div \sqrt{\frac{2}{5}}$.

48. $\sqrt{\frac{2}{5}} - \sqrt{\frac{2}{5}}$.

49. $\sqrt{1\frac{1}{2}} : \sqrt{2\frac{2}{3}} :: 6 : ?$

50. $1\frac{1}{2} : 2\frac{2}{3} :: 36 : ?$

What is the relation between the answers to the last two? Why?

51. $5 : \sqrt{.6} :: \sqrt{.15} : ?$

52. $\sqrt{.8} : 3 :: \sqrt{1.1} : ?$

53. $\sqrt{\frac{2}{3}} : \sqrt{\frac{9}{16}} :: \sqrt{\frac{4}{5}} : ?$

54. $\sqrt{\frac{2}{3} \times \frac{9}{16} \times \frac{4}{5}} = ?$

Are the answers to the last two alike? Why?

55. $\dfrac{3\frac{2}{5} + 5\frac{3}{4}}{5\frac{1}{2} - .025}$.

56. $\sqrt{.002} : \sqrt{.004} :: \sqrt{7} : ?$

57. $\sqrt{10} : \sqrt{5} :: \sqrt{\frac{3}{5}} : ?$

58. $\sqrt{\frac{3}{4}} \times \sqrt{\frac{4}{3}}$.

59. $\sqrt{\frac{3}{4}} + \sqrt{\frac{4}{3}}$.

60. $\sqrt{\dfrac{.005 \div .01}{3 - 2.01}}$.

61. $\dfrac{(4\frac{1}{2} - 3\frac{1}{5})\ (.2 + 3)}{.3\ (5.6 \div .07)}$.

62. $\dfrac{(3 - \frac{1}{3}) \div (1 - \frac{2}{5})}{.04\ (1 - .05)}$.

63. $\dfrac{(4 \div .04)\ (4 - .04)}{(3 \times .03)\ (3 + .03)}$.

64. $\dfrac{2.08 \div (1 - \frac{1}{5})}{(2 + \frac{1}{5}) - (\frac{1}{3} \text{ of } \frac{3}{5})}$.

65. $\dfrac{1}{\frac{2}{3}} : ? :: \dfrac{\frac{3}{5}}{2\frac{1}{2}} : 4$.

66. $2 \times 3\frac{1}{5} : (5 - 1\frac{1}{6}) :: ? : \frac{1}{2} \text{ of } \frac{3}{7}$.

67. $(2 - \frac{3}{5}) \div \dfrac{4\frac{1}{2}}{\frac{3}{7} \text{ of } \frac{5}{9}}$.

68. $(1\frac{1}{2} - \frac{5}{6}) \div (.02 - .002)$.

69. $\dfrac{5\frac{1}{2} \times 2\frac{2}{3}}{2 \div .02} \div .125$.

70. $(\frac{2}{3})^2 : (1\frac{3}{5})^3 :: 8 : ?$

71. $(5\frac{1}{2})^2 \div (\sqrt{2.5} \times \sqrt{.016})$.

72. $(4\frac{2}{5})^3 \div (.12)^2 \times \sqrt{.4}$.

73. $(2.01)^2 + (5.3)^3 - (2\frac{1}{8})^2$.

74. $\left(5 \div \dfrac{2\frac{1}{4}}{\frac{1}{2} \text{ of } \frac{3}{8}}\right) \times \dfrac{13\frac{1}{12}}{4.2}$.

75. $\dfrac{\sqrt{3.6} - \sqrt{.16}}{(\frac{2}{3})^2 - \sqrt{\frac{1}{8}}} \div \sqrt{\dfrac{1\frac{1}{2}}{\frac{2}{3} \text{ of } \frac{5}{6}}}$.

76. $\frac{2}{3}$ of \$5 is what part of £4?

77. $\frac{1}{5}$ of $\frac{2}{3}$ of $1\frac{1}{4}$ yd. is what part of $2\frac{1}{5}^{m}$?

78. $\frac{1}{5}$ of $7\frac{1}{2}$ ft. is $\frac{2}{3}$ of how many meters?

79. $5\frac{1}{2}^{Kg}$ is what part of 10 lb.?

80. $7\frac{1}{2}$ dg is what part of 10 gr.?

81. 4s. 6d. is what part of 2^{Nap}?

82. $5\frac{2}{3}^{m}$ is what part of \$$2\frac{1}{2}$?

83. $14\frac{2}{7}$ less $\dfrac{\frac{1}{2} \text{ of } 8\frac{2}{5}}{14\frac{7}{10}}$ is $\frac{2}{3}$ of $\frac{7}{9}$ of what number?

84. Divide $7\frac{1}{2}$ into 3 parts, which shall be to each other as $1\frac{1}{3}$, $1\frac{1}{2}$, and 2.

85. Divide 12 into 3 parts, which shall be to each other as $\frac{2}{3}$, $\frac{3}{5}$, and 2.1.

86. Divide 9 into 4 parts, which shall be to each other as 2.5, 1.1, $\frac{1}{2}$, and $3\frac{1}{5}$.

87. Divide \$1250 into 3 parts, which shall be to each other as $\frac{2}{3}$ of 6, $\frac{1}{5}$ of 20, and $\frac{1}{7}$ of 14.

88. Divide \$3800 into 4 parts, which shall be to each other as 2×60, 3×50, 4×35, and 7×16.

89. Does it increase a proper fraction, or diminish it, to *extract its root*? Why?

90. Does it increase a proper fraction, or diminish it, to involve it to a power? Why?

91. Answer the last two questions with reference to an improper fraction?

92. $\frac{2}{3}$ of $3\frac{3}{4}$ liquid gallons is what part of .5 bu.?

93. 2.6^{l} is what part of $\frac{2}{3}$ of 5 gal.?

94. 4 mm is what part of .01 in.?

95. 25^{Dl} is what part of 10 bu.?

96. 3.2^{m} is what part of 1 rod?

97. A is 1.6^{m} in height, and B 5 ft. $10\frac{1}{2}$ in. What is the ratio of their statures?

98. What is the ratio of ℥ ij to 4^{g}?

99. What is the difference between $\frac{3}{7}$ of $4\frac{1}{2} \times \frac{9\frac{3}{4}}{\frac{13}{20}} \times \frac{1}{15}$ of $\frac{7}{9}$ of £43 18s. $11\frac{1}{2}$d., and $3\frac{8}{9} \times \frac{1}{17\frac{1}{2}}$ of .56 of $1.75 \times 6\frac{1}{2}$ times \$97.18?

100. Reduce $\frac{2}{3}$ of $4\frac{1}{2} \times 7\frac{4}{5} \times \frac{9}{19\frac{1}{2}}$ of $\frac{5}{9}$ of 3 oz. 4 dr. 2 scr. 5 gr. to the decimal of $\frac{6}{11}$ of .63 of $2\frac{37}{42} \times \frac{3}{13}$ of $6\frac{1}{2}$ times 7 lb. 3 oz. Av.

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469. ONE HUNDRED TEST EXERCISES IN APPLIED ARITHMETIC.

1. What per cent is 5 of 25? Of 10? Of 43?

2. Of what number is 11 6%? Of what 10%? Of what 7%?

3. I send an 8% note for \$1000, dated July 1, 1876, to Buffalo for collection. The maker of the note, having gone into bankruptcy, pays only 75¢ on \$1. The note is collected by my broker, April 30, 1877. What does he remit, charging $\frac{1}{2}$% for collecting?

4. The bank is discounting at 8%. I wish \$200 for 60 da.

For what must the note be made? What for $350 for 30 da. at 10%? For $250 for 90 da. at 5%?

5. I send a 6% note for $500, dated May 10, 1875, with an indorsement July 1, 1876, of $150, to a broker in Chicago for collection. He collects it March 10, 1877, and charges me ¾% for collecting. What does he remit to me?

6. A real-estate broker received $2593.75 for the purchase of land. Reserving 3¾% commission on the purchase, what number of acres of land could be bought at $125 per acre?

7. If I buy bonds for 85 cents on a dollar which pay 3% semi-annual interest on their face, what per cent per annum does this give me on my investment, money being worth 10%?

8. What will be the duty in our currency on a case of silk mantillas, invoiced in Paris at 13950 francs, the rate of duty 60% *ad valorem?*

9. I invest $2000 in certain goods, which I sell at 50% advance, but at a cost of 3% on the sales for selling. Allowing 5% loss by selling on credit, what per cent do I make by the transaction?

10. How long does it take a principal to double at 6%? 7%? 8%? 10%?

11. How long does it take a principal to double itself at compound interest at 5%? At 10%? At 4%?

12. $5600. PHILADELPHIA, Jan. 11, 1871.

For value received, on demand, I promise to pay James Jones, or order, five thousand six hundred dollars, with interest, without defalcation. JOHN SMITH.

Indorsements: May 19, 1871, $500; Sept. 5, 1871, $200; Jan. 1, 1872, $300; April 17, 1872, $150.

What is due Jan. 11, 1873, by the United-States-Court Rule? What by the Merchant's Rule?

13. At what per cent will \$240.80 amount to \$325.08 in 5 yr. 10 mo.?

14. At what per cent will a given principal double in 12 yr.? In 15 yr.?

15. What principal will amount to \$1617 in 3 yr. 6 mo. 15 da. at 8%?

16. Suppose an annual premium of \$68.25 is paid for insuring a house worth \$2275, what per cent is paid?

17. At a rate of $1\frac{1}{2}$% a year a warehouse is insured for $\frac{3}{4}$ of its value, paying thereon a premium of \$202.50. What is the whole value of the warehouse?

18. A tax of \$50,000 net is to be raised in a certain city on a valuation of \$2,000,000. Supposing 3% to be uncollectible, and allowing 5% for collecting, what tax must be levied? What will be a man's tax who is assessed on \$3500?

19. If the property in the city of Cleveland be assessed at \$70,000,000, what must be the tax on each dollar to raise \$294,000 for school purposes? At two-tenths of a mill on each dollar how much will be raised for the support of the library?

20. What per cent annual interest do I make by investing in railroad stock at 75, which pays 3% semi-annually, allowing 6% on the mid-year payment to the close of the year?

21. I bought 25000 feet of boards at \$12.25 per thousand, and sold $\frac{1}{2}$ of them for what $\frac{2}{3}$ of the whole cost. What per cent did I gain on the part sold?

22. A merchant bought goods at 25% below their nominal price, and sold them at 20% above, thereby making \$1920. How much did he invest?

23. I mark down 10% from the retail price goods which I was selling at 25% advance on cost. At what per cent advance on cost do I now propose to sell them?

24. In consequence of a rise of a certain article in the

market, I mark up 5% on my former retail price goods which I was selling at 20% advance on cost. At what per cent profit do I now propose to sell them?

25. A New-York bank in which I hold stock declares a 4% dividend. I draw a draft for the amount due me, and sell it at 1% premium in Omaha, receiving $707. How many shares do I own?

26. A bought 230 bales of cotton, each bale containing 450 lb., at $10\frac{3}{4}$ cents a pound, on a credit of 9 mo. He sold the cotton immediately for $12000 cash, and paid the present worth of the debt at 8%. What was his gain?

27. At what must cloth be bought to sell it at $9 per yd., and make $12\frac{1}{2}$% profit?

28. A jeweller has a watch which cost him $150. He wishes to mark it so that he can fall 5% on the asking price, and still make 20%. How must he mark it?

29. A bookseller sells a book for $1.20, and makes 25% thereby. What would he have made had he sold it at $1.28?

30. When $\frac{2}{3}$ the selling price equals the cost, what per cent is made? When $\frac{1}{2}$ the selling price equals the cost? When the selling price is $\frac{5}{6}$ of the cost, what per cent is lost?

31. How much water must be added to 1 gal. pure alcohol to make a mixture 75% alcohol? How much, to make one 50%? 40%?

32. A railroad has been constructed through a farm, making it necessary to build fences at a cost of $750, which must be renewed every 15 years. What should the owner receive to meet this expenditure, at 6% compound interest?

33. If a square orchard contains 2916 trees, how many are in a row on each side?

34. A man has a rectangular board 128 in. long and 32 in. wide, from which he makes a square table as large as possible. Required its length, no allowance being made for *sawing.*

35. What would it cost to enclose a square lot containing 160 acres, with a fence costing at the rate of \$4 per rod?

36. What cost 3 piles of 4 foot wood, one 58 ft. long and 5 ft. high, another 70 ft. long and $5\frac{1}{2}$ ft. high, and the other 65 ft. long and 6 ft. high, at \$5.50 per cord?

37. I have a cylindrical cistern 6 ft. deep and $6\frac{1}{2}$ ft. in diameter. How much shall I increase its capacity if I increase each of its dimensions 25%? 50%? 100%?

38. How much is a rectangular bin increased in capacity by increasing two of its dimensions 10%? By increasing all three of its dimensions 10%? If I double two of its dimensions? If I double all three dimensions?

39. Suppose a rectangular field to be three-eighths of a mile in length, and one-fourth of a mile in width. What distance will be saved by walking directly from any corner to the corner diagonally opposite, instead of going by the line of the fence from the one point to the other?

40. A ladder 30 ft. in length was found to reach just to the eaves of a building when its foot was 12 ft. from the foundation. What was the height of the building?

41. The eaves of a house are at the same height, and 30 feet apart. The ridge-pole is 12 feet higher than the eaves, and just midway between them. The house is 40 feet long. How many shingles will it take to cover the roof, if each shingle covers a space 6 inches long and 4 inches broad?

42. A third-pitch "square roof" is to be put on a rectangular house, 36 by 42 ft., with a flat deck at top 8 ft. above the plates. What will be the size of the deck, what the length of the side rafters, and what the 4 corner rafters? (See p. 335, Ex. 7.)

43. How many revolutions in a minute does a 6 foot drive-wheel of an engine make when the engine is running at the rate of 30 mi. an hour?

44. If a ball 1 foot in diameter weighs 100 lb., what is the weight of one 3 feet in diameter, made of the same material?

45. How large a square can be cut from a circle 36 in. in diameter?

46. How large a cube can be cut from a sphere 2 ft. in diameter?

47. If a solid globe of 4 in. diameter weigh 20 lb., what will one of the same material 6 in. in diameter weigh?

48. How many boxes of common double tin, 100 12×17 in. sheets in a box, will take to cover a hemispherical dome of 20 ft. diameter, allowing $\frac{1}{2}$ in. lap on end and side of each sheet?

49. What is the area of a trapezoid whose parallel sides are 30 and 50 rods, and whose altitude is 40 rods?

50. What is the diameter of a sphere whose surface is 100 sq. ft.?

51. What is the diameter of a sphere whose volume is 150 cu. ft.

52. The sun's diameter being 852573 mi., and the earth's radius 3979 mi., how many times as large as the earth is the sun?

53. The diameter of the moon being to that of the earth as 3 : 11, what is the relation between their volumes?

54. What is the diameter of a grindstone when it is $\frac{1}{3}$ worn away, its original diameter having been 2 ft.? What when $\frac{2}{3}$ worn? When $\frac{1}{2}$ worn? When ^{1}? $\frac{3}{4}$?

55. If a ball of thread is 4 inches in diameter, what will be the diameter in each of three conditions, — when $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of it are wound off? What part of the thread will be left when the diameter is reduced to 2 inches?

56. When Gen. Tom Thumb (Charles S. Stratton) was 5 years old, he measured 2 ft. in height, and weighed 16 lb. What would be the weight of a man of similar form who was 6 ft. tall?

57. What are the dimensions of a rectangular box containing 3000 cubic feet, the dimensions being to each other as 2, 3, and 4?

58. How many yards of carpeting $\frac{3}{4}$ yd. wide must I buy to carpet a room 20 by 25 feet, the strips running lengthwise of the room, and there being 4 inches waste on each strip in matching?

59. I sell 12 logs at $10 per M, board measure. Six logs are 12 ft. long, and six 14 ft. The first scale 28 in., 30 in., 40 in., and three 32 in.; the others, two 30 in., three 35 in., and one 20 in. What do the logs bring me, reckoning by *Scribner's* "Log Book"?

60. How many cubic feet of hewn timber in 3 logs measuring 15 ft. long, 21 and 25 in. in diameter; 20 ft. long, 26 and 30 in. in diameter; and 32 ft. long, 30 and 36 in. in diameter, measured by *Scribner's* rule?

61. How many minutes will there be in the month of February, 1880?

62. A lady bought 6 silver spoons, each weighing 3 oz. 3 pwt. 8 gr., at $2.25 an ounce, and a gold chain weighing 14 pwt., at $1.25 a pwt. What was the cost of both spoons and chain?

63. What is the difference between 32 liters and a bushel? What is the difference between a barrel and $1\frac{1}{4}$ hektoliters?

64. How many gallons does a tub 18 in. deep contain, whose top is 16 in. in diameter, and bottom 20 in.?

65. If telegraph-poles are 66 ft. apart, and a train passes one every 3 sec., what is the rate in miles per hour?

66. A physician having 1 ℔ ℥ iij ʒ iv ℈ ij gr. xij of a certain medicine, put it up in gr. xx packages. How many did it make?

67. The United-States "Trade Dollar" (silver) weighs 420 gr., and the common half-dollar 12.5^g. How much more is the trade dollar actually worth than two common half-dollars?

68. How many bushels in a bin 3.2^m long, 1^m wide, and 2^m deep. How many hektōls? How many kilōgs of wheat will it contain at 60 lb. to the bushel?

69. Cleveland, O., is in longitude 81° 47′ W., and Boston in 71° 4′ 9″ W. When it is 5 A.M. at Boston, what time is it at Cleveland?

70. What is the difference in longitude between two places whose difference in time is 52 min. 18 sec.?

71. In crossing the State of Michigan from Detroit to South Haven, which are nearly on the same parallel, I find my watch, which is set to Detroit time, 12½ min. fast at South Haven. Now 51 mi. make a degree of longitude on this parallel. What is the width of the State at this point?

72. The specific gravity of milk is 1.032. What does 1^{Dl} weigh?

73. A pail containing 1^{Dl} of cider is filled, and the cider found to weigh 10.18^{Kg}. What is the specific gravity of cider?

74. One millil of sulphuric acid is found to weigh 1.842^{g}. What is its specific gravity?

75. The specific gravity of linseed-oil is .94. How much would a cask of 2^{Hl} weigh?

76. I find that a liter of alcohol weighs 8^{Hg}. What is its specific gravity?

77. The specific gravity of cast iron being 7.25, what is the weight of a 12 in. cast-iron shell (hollow sphere), the shell being 1½ in. thick?

78. The specific gravity of iron being 7.25, what is the weight of a 10 in. cast-iron cannon-ball?

79. The specific gravity of common loose earth is about 1.5. A cubic yard makes a good-sized load for a span of horses. What does it weigh?

80. The specific gravity of common rocks is about 2.5. What is the weight of the earth if its material averages the same?

81. Required the time of day, provided the time past noon equals ⅔ of the time to midnight.

82. A quantity of flour lasts a man and wife 9 days, and the wife alone 27 days. How long would it last the man alone?

83. If a 2 in. pipe will fill a cistern in 6 hours, how long will it take a 3 in. pipe to fill it, the water flowing at the same velocity?

84. A agreed to labor for $2.50, on condition that he should forfeit 50¢ every day he was idle. At the end of 100 days he received $190. How many days was he idle?

85. A and B together can do a piece of work in 6 days; but B alone requires 10 days to do it. In what time can A do it alone?

86. Three men, A, B, and C, agree to do a certain piece of work. A and B can do the work in $6\frac{2}{3}$ days, B and C in 12 days, and A and C in 10 days. How long will it take each separately to do it?

87. In a mixture of gold and silver consisting of 100 oz. there are 6 oz. of silver. How much gold must be added that there may be $\frac{2}{5}$ oz. of silver to 10 oz. of gold?

88. A man bought a bar of gold at $192 per lb. and sold it for $16 per oz., weighing it in both cases by avoirdupois weight. How much did he gain, the true weight of the bar being 5 pounds? Did he gain, or lose, by selling by avoirdupois instead of troy?

89. French coin is of the same fineness as our own. A franc is 19.3 cents. Our gold dollar weighs 25.8 gr., and our silver dollar 420 gr. What is the weight of a silver franc in grams? What of a Napoleon?

90. When the temperature is 80° F., what is it C.? Blood-heat is 98° F.: what is it C.? —28° F. is what C.?

91. 57° C. is what F.? —18° C. is what F.?

92. What day of the week was the 4th of July, 1776? What day of the week will it be in 2000?

93. On what day of the week were you born?

94. On what day of the week will Christmas come this year? What the 4th of July?

95. If 20 yards of cloth, 1 yard wide, shrink 4% in length and 5% in breadth by sponging, what will be the loss in square yards?

96. At the breaking out of the late Franco-Prussian war the German government made a 5% war loan, which was taken at 88; and the French a 3%, which was taken at $65\frac{1}{2}$. Which paid the higher rate of interest, assuming that both ran indefinitely? Which, if the loans ran only 10 years?

97. Calling the diameter of the earth 7912 miles, and the height of the highest mountain 28000 feet, what elevation would represent the mountain on an artificial globe 2 feet in diameter?

98. Bought cranberries at $4.80 per bushel, and sold them at 15¢ per qt., measuring them out by liquid measure, whereas I bought them by dry measure. What per cent was my profit?

99. Compare the amount of a $500 10% note running 4 yr. at simple interest, annual deferred int. and compound int.

100. How much must I invest in United-States 4% bonds at $102\frac{3}{8}$ to give me a quarterly income of $500?

APPENDIX.

ERATOSTHENES' SIEVE.

1. To find the prime numbers between any given limits. — *Write down all the odd numbers, 1, 3, 5, 7, 9, etc. Over every third from 3 write 3; over every fifth from 5 write 5; over every seventh from 7 write 7; over every eleventh from 11 write 11; and so on. Then all the numbers which are thus marked are composite, and the others, together with 2, are prime.* (Why?)

Also, the figures thus placed over are the factors of the numbers over which they stand. (Why?)

Ex. — Find all the prime numbers less than 100.

				3			3.5	
1	3	5	7	9	11	13	15	17
	3.7		5	3			3.11	5.7
19	21	23	25	27	29	31	33	35
	3.13			3.5		7	3.17	
37	39	41	43	45	47	49	51	53
5.11	3.19			3.7	5.13		3.23	
55	57	59	61	63	65	67	69	71
	3.5	7.11		3		5.17	3.29	
73	75	77	79	81	83	85	87	89
7.13	3.31	5.19		3.11				
91	93	95	97	99				

Hence, rejecting all the numbers which have *superiors*, the primes less than 100 are 1, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, together with the number 2.

This process may be extended indefinitely, and is the method by which primes are found even by modern computers. It was invented by Eratosthenes, a learned librarian at Alexandria (born B.C. 275). He inscribed the series of odd numbers upon parchment, then, cutting out such numbers as he found to be composite, his parchment with its holes somewhat resembled a *sieve:* hence this method is called "*Eratosthenes' Sieve.*"

GENERAL METHOD OF FINDING THE GREATEST COMMON DIVISOR.

This rule is based on the two following principles:—

2. PROP. 1.—*A divisor of a number is a divisor of any number of times that number.*

This is self-evident, since, if a is contained in b c times, it is contained in $2b$ twice as many times, in $3b$ 3 times as many times, etc.

3. PROP. 2.—*A divisor of any two numbers is a divisor of their sum, and also of their difference.*

This is also self-evident. Thus, if a is contained in m 5 times, and in n 3 times, it is evident that it is contained in their sum 8 times, and in their difference 2 times.

4. Rule.—*To find the G. C. D. of two numbers, divide the greater by the less, and this divisor by the remainder, continuing to divide the last divisor by the last remainder until there is no remainder. The last divisor is the G. C. D. sought.*

DEMONSTRATION.—In order to demonstrate this rule, let us find the G. C. D. of 42 and 138. Performing the operation according to the rule, as in the margin, we are to prove that 6 is the G. C. D. of 42 and 138.

```
42 ) 138 ( 3
     126
     ---
      12 ) 42 ( 3
           36
           --
            6 ) 12 ( 2
                12
                --
```

As 42 is its own G. D., if it divides 138 it is the G. C. D. sought. Trying it, we find a remainder 12. Now any divisor of 42 is a divisor of 3 times 42, or 126 (PROP. 1), and any divisor of 126 and 138 is a divisor of their difference 12 (PROP. 2). Hence the G. C. D. sought cannot be greater than 12. Moreover, any number which divides 12 and 42 divides 138, which is the sum of 12 and 3 times 42 (PROPS. 1 and 2). Thus the question is reduced to finding the G. C. D. of 12 and 42.

In like manner we can reduce it to the question of finding the G. C. D. of 6 and 12. But this is 6. Hence 6 is the G. C. D. of 42 and 138.

Ex. 1.—Find the G. C. D. of 9131 and 13133.

An arrangement like that in the margin will be found convenient in performing the divisions. Placing the first divisor, the smaller number, on the right, divide and write the quotient at the right of both. Thus 9131 is contained in 13133, 1 time, with 4002. Now, using this remainder as divisor, we continue the work as in the margin.

OPERATION.

13133	9131	1
9131	8004	2
4002	1127	3
3381	621	1
621	506	1
506	460	4
115	46	2
92	46	2
23		

2. Find the G. C. D. of 4420, 3094, and 1326. Of 1445, 1190, and 204.

The common method of solving such problems is to find the G. C. D. of the two least numbers, and then of this G. C. D. and the next larger of the numbers, etc. But familiarity with the principles on which the operations are based will suggest better ways. Thus, for the above, we have the following: —

4420	3094	1326	2
2210	1547	663	2, 3
2210	1326	663	10
	221		
	2		
	442 = G. C. D.		

1445	1190	204	5, 8
1360	1020	170	2
85	170	34	2, 5, 2
68	170	34	
17 = G. C. D.			

In the *first* of these we reserve the evident common factor 2 as a factor in the G. C. D. sought. Hence we have to find the G. C. D. of 663, 1547, and 2210. Dividing 1547 by 663, we reduce the problem to finding the G. C. D. of 663, 221, and 2210. Then, as 221 is found to be the G. C. D. of these, $221 \times 2 = 442 =$ G. C. D. sought.

In the second, the problem is first reduced to finding the G. C. D. of 204, 170, 85. Then, as 85 is found to be a divisor of 170, it becomes a question between 85 and 204. The order of operation in this case is, $1190 \div 204$; $1445 \div 170$; $170 \div 85$; $204 \div 85$; $85 \div 34$; $34 \div 17$.

The student will have no difficulty in applying this method of finding the G. C. D. of several numbers, if he is careful to mark at each step what numbers are now to be examined, and always divides first by the least number, proceeding in order to the greatest under comparison.

LEAST COMMON MULTIPLE.

5. Rule. — I. *Write the numbers in a horizontal line, and divide by any prime number that will divide two or more of them without a remainder, placing the quotients and numbers undivided in a line below.*

II. *Divide this line as before, and thus proceed till no two numbers are divisible by any number greater than 1. The continued product of the divisors and numbers in the last line will be the L. C. M. of the numbers.*

Ex. 1. — Find the L. C. M. of 45, 81, 96, and 35.

5)	45	81	96	35
3)	9	81	96	7
3)	3	27	32	7
	1	9	32	7

5.3.3.9.32.7 = L. C. M.

Applying the rule to the solution of this example, we have the work in the margin.

The principle is the same as given in the text.

2. **Find the L. C. M. of 54479 and 35741.**

In this case it is not easy to discern a common factor, if the numbers have one. We may therefore apply the method for finding the G. C. D. Having found the G. C. D., we can divide the smaller number by it, and find the factor by which the larger number is to be multiplied, in order to give a product which will contain the smaller.

3. **Find the L. C. M. of 31861, 88409, and 63269.**

PROGRESSIONS.

6. An Arithmetical Progression is a series of numbers which increase or decrease by a common difference, as 3, 5, 7, 9, 11; or 28, 23, 18, 13, 8.

7. The Last Term of an *increasing* arithmetical progression is evidently equal to the first term + the common difference taken as many times as there are terms less 1. Thus the 5th term of the 1st series above is 3 + 4 times 2 = 11.

The last term of a *decreasing* arithmetical progression is equal to the first term — the common difference multiplied by the number of terms less 1. Thus the 5th term of the 2d series above is 28 — 4 times 5 = 8.

8. The Sum of an arithmetical progression is ½ the sum of the extremes multiplied by the number of terms.

This will be evident from an inspection of this operation. Hence the $Sum = \left(\frac{3+11}{2}\right) \times 5 = 35$, the Sum of the Series.

$$\begin{array}{r} 3 + 5 + 7 + 9 + 11 = \text{sum.} \\ 11 + 9 + 7 + 5 + 3 = \text{sum.} \\ \hline 14 + 14 + 14 + 14 + 14 = \textit{twice} \text{ the sum.} \end{array}$$

Ex. 1. — First term 7, common difference 4, series increasing, find the 10th term and the sum.

2. First term 134, common difference 7, series decreasing, find the 8th term and the sum.

3. A 10% note for $300, bearing annual interest, has been running 8 yr., and no interest has been paid. What is due, allowing simple interest on the deferred payments of annual interest?

The 8th year's interest is $30, the 7th $33, the 6th $36, etc. Hence the interest is an arithmetical progression of 8 terms, of which 30 is the first, and 3 the com. diff. The last term is therefore 51, and the sum $\left(\frac{51+30}{2}\right) \times 8 = 324$. Amount of note, $624.

9. A Geometrical Progression is a series of numbers which increase or decrease by a common multiplier, called the *rate*. If the rate is more than 1, the series is increasing; if less than 1, it is decreasing. Thus 3, 9, 27, 81, 243, is an increasing geometrical progression, rate 3. 6561, 729, 81, 9, is a decreasing geometrical progression, rate $\frac{1}{9}$.

10. The Last Term of a *geometrical progression* is the first multiplied by the rate raised to a power whose index is 1 less than the number of terms. This appears when we consider that the 2d term is the first multiplied by the rate, the 3d is the first multiplied 2 times in succession by the rate, etc.

11. The Sum of a *geometrical progression* is the difference between the last term multiplied by the rate and the first term, divided by the *rate* — 1 if the series is *increasing*, and by 1 — *the rate* if it is decreasing.

Thus, taking the series 3, 9, 27, 81, 243, of which the rate is 3, the sum is $\frac{3 \times 243 - 3}{3 - 1}$, or $\frac{729 - 3}{2} = 363$. An inspection of the following will indicate the reason for the rule: —

$$\begin{array}{rl} 729 + 243 + 81 + 27 + 9 \phantom{{}+3} & = 3 \text{ times the sum.} \\ 243 + 81 + 27 + 9 + 3 & = \text{the sum.} \\ \hline 729 \qquad\quad - \qquad\quad 3 & = (3 - 1) \text{ times the sum.} \end{array}$$

Again,

$$\begin{array}{rl} 6561 + 729 + 81 + 9 \phantom{{}+1} & = \text{the sum.} \\ 729 + 81 + 9 + 1 & = \tfrac{1}{9} \text{ the sum.} \\ \hline 6561 \qquad\quad - \qquad\quad 1 & = (1 - \tfrac{1}{9}) \text{ times the sum.} \end{array}$$

Ex. 1. — First term of a geometrical progression 7, rate 4. What is the 8th term? What the sum?

2. First term 6250, rate $\frac{1}{5}$. What is the 6th term? What the sum?

DIVISION OF UNITED-STATES PUBLIC LANDS.

12. When a new territory is to be surveyed, the first thing the surveyor does is to run one or more north and south lines through some convenient parts of it. These are run with great care, are carefully marked by posts, stones, marks upon trees, or other means, throughout their entire length, and are called PRINCIPAL MERIDIANS. In a similar way one or more east and west lines, called BASE LINES, are run and marked. After this the whole country is checked up into townships by running north and south lines parallel to the principal meridian, and six miles apart and east and west lines in a similar manner parallel to the base line. The north and south rows of these townships are called *Ranges*, and are numbered east and west from the *principal meridian*. The townships in each row are numbered north and south from the *base line*. Thus in Ohio the western boundary of Pennsylvania is the eastern principal meridian from which ranges are numbered up to Range XX. West, which reaches the western boundary of Huron County. Again: the western boundary of Ohio is another principal meridian from which ranges are numbered eastward to Range XVII. East. The base line in this State runs along the southern boundary of Paulding, Seneca, and Huron Counties. From this line townships are numbered, as Town 1 North, Town 2 North, etc.; Town 1 South, Town 2 South, etc. Townships are designated thus: T. 2 N., R. 4 E.; T. 8 S., R. 5 W., etc. The first of these is read, "Town 2 North, Range 4 East," and locates Brown Township in Paulding County, 6 miles north of the base line, and 18 miles east of the west meridian, or west line of the State.

The base line in Michigan runs along the north line of the second tier of counties; viz., Wayne, Washtenaw, etc. The principal meridian commences at the Ohio line, and runs north between Lenawee and Hillsdale Counties. The *names* which we apply to the townships form no part of a description of them as given in deeds: such description is by range and number. Sometimes (usually) the *political* divisions we call townships correspond to these surveyed and recorded towns: but often other divisions are made for political purposes; otherwise all the townships would be regular (except those along lake and river boundaries, etc.).

The following cuts illustrate the division of a township into sec-

tions and the subdivisions of sections. These sections are designated by number, beginning at the north-east corner, and numbering

A TOWNSHIP.

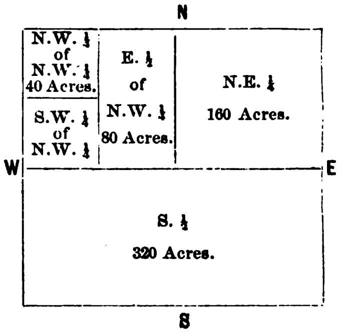
N

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

W — E

S

A SECTION.



across to the west, then back to the east, and thus back and forth. The division of the section is not made in the original survey; but the middle of the sides on the boundary-lines are marked so that all that is afterward necessary is to run the lines across in order to divide the sections into quarters. For purposes of sale these quarters are again divided into halves and quarters. The following designations will be readily understood: "North-west quarter of the north-west quarter of Section 10," written, "N.W. ¼ of N.W. ¼ of Sec. 10." See cut of section. So the east half of the north-west quarter is designated, "E. ½ of N.W. ¼ of Sec. 10."

Descriptions may be written on the board, and the pupils required to illustrate the location by diagrams on their slates.

N.E. ¼ of S.E. ¼, Sec. 20, T. 7 N., R. 9 E.
W. ½, S.W. ¼, Sec. 17, T. 10 S., R. 6 W.
S. ½, Sec. 28, T. 1 S., R. 15 E.
E. ½, N.E. ¼, Sec. 8, T. 6 N., R. 18 W.
W. ½, Sec. 16, T. 13 N., R. 11 E.
E. ½, S.E. ¼, Sec. 32, T. 5 S., R. 13 E.

THE METRIC SYSTEM.

13. The *Metric System*, originally devised and adopted by the French, makes *The Meter* the fundamental unit. It was designed that the meter should be $\frac{1}{10000000}$ part of a quadrant of a meridian of the earth. With this design an arc of the meridian, starting from the parallel of Dunkirk in the extreme north of France, and running the entire length of France, and terminating in the parallel of Barcelona in the north of Spain, was measured by Delambre and Méchain, as directed by the French Government. From this measurement the whole quadrant was computed, and the meter established as $\frac{1}{10000000}$ part of it. It is now known that there are irregularities in the form of the earth which would make such measurements give different results when taken in different places, and that the meter thus established is about $\frac{1}{5000}$ of an inch too short.

The meter being thus established, the *liter* is made a cubic *decim*, and this amount of pure water at the temperature of melting ice is made the kilōg.

The Metric System has now come to be adopted by most civilized nations, although generally only permissively, as a system legally recognized, but which may be used by the people, or not, as they see fit. Nevertheless, all nations, except the French (and they to a considerable extent), continue to use their various and older systems. It is, however, coming to be pretty generally accepted for scientific and philosophical purposes, and its cosmopolitan character makes it specially desirable that it should be understood by all who lay any claim to general intelligence. That the Linear Measures, Measures of Weight and of Capacity, will soon be in general use in this country, scarcely admits of a doubt; and hence the subject demands a place in our schools.

In attempting to teach the metric system, it is of the first importance that the pupils be made familiar with the measures themselves. *The Metric Bureau*, Boston, is organized for the purpose of furnishing apparatus for teaching, and information upon this subject.

14. Value of Foreign Coins in U. S. Money (Gold) as proclaimed by the Secretary of the Treasury, Jan. 1, 1878.

COUNTRY.	UNIT.	METAL.	U. S.
Argentine Republic [1]	Peso fuerte	G.	$1.00
Austria	Florin	S.	.45,3
Belgium	Franc	G. & S.	.19,3
Bolivia	Dollar	G. & S.	.96,5
Brazil	Milreis of 1000 reis	G.	.54,5
Bogota	Peso	G.	.91,2
Canada [1]	Dollar	G.	1.00
Central America	Dollar	S.	.91,8
Chili	Peso	G.	.91,2
Cuba [1]	Peso	G.	.92,5
Denmark	Crown	G.	.26,8
Ecuador	Dollar	S.	.91,8
Egypt	Pound of 100 piasters	G.	4.97,4
France	Franc	G. & S.	.19,3
Great Britain	Pound sterling	G.	4.86,6½
Greece	Drachma	G. & S.	.19,3
German Empire	Mark	G.	.23,8
Hayti	Dollar	S.	.95,2
India	Rupee of 16 annas	S.	.43,6
Italy	Lira	G. & S.	.19,3
Japan	Yen	G.	.99,7
Liberia	Dollar	G.	1.00
Mexico	Dollar	S.	.99,8
Netherlands	Florin	S.	.38,5
Norway	Crown	G.	.26,8
Paraguay [1]	Peso	G.	1.00
Peru	Dollar	S.	.91,8
Porto Rico	Peso	G.	.92,5
Portugal	Milreis of 1,000 reis	G.	1.08
Russia	Ruble of 100 copecks	S.	.73,4
Sandwich Islands	Dollar	G.	1.00
Spain	Peseta of 100 centimes	G. & S.	.19,3
Sweden	Crown	G.	.26,8
Switzerland	Franc	G. & S.	.19,3
Tripoli	Mahbub of 20 piasters	S.	.82,9
Tunis	Piaster of 16 caroubs	S.	.11,8
Turkey	Piaster	G.	.04,3
U. S. of Colombia	Peso	S.	.91,8
Uruguay [1]	Patacon	G.	.94,9

[1] Taken from the Treasury circular for 1875, as they are not mentioned in 1878.

PRESENT UNITED STATES COINAGE.

412½ gr.

SILVER.

192.9 gr.

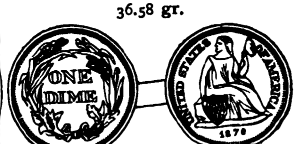
SILVER.

BRONZE.

96.45 gr.

SILVER.

36.58 gr.



SILVER.

5 grains

NICKEL.

NICKEL.

PRESENT GOLD COINAGE

OF THE UNITED STATES.

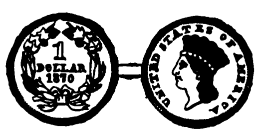
516 gr.

258 gr.

The Eagle—Ten Dollars. 1870.

Half-Eagle—Five Dollars. 1870.

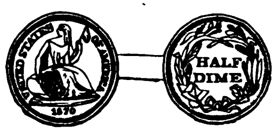
25.8 gr.



UNITED STATES COINS IN USE BUT NOT NOW COINED.

SILVER.

SILVER.



SILVER.

GERMAN COINS.

20 Mark
$5.70

GOLD.

28.5¢

SILVER.

ENGLISH COINS.

Sovereign—$4.8665

GOLD.

COPPER.

24.3¢

SILVER.

Farthing:

COPPER.

SILVER.

FRENCH COINS.

Napoleon—$3,86

GOLD.

5 Centimes—4.8¢.

BRONZE.

19.3¢.

SILVER.

ANSWERS.

This book will be supplied with or without the Answers, as may be desired.

HISTORIES OF THE UNITED STATES.

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3. The work is ***arranged in short sentences,*** so that the substance of each may be easily comprehended.
4. The ***most important events*** are indicated in the text by **heavy-faced letter.**
5. ***Full Questions*** are framed for every verse.
6. ***A Pronouncing Vocabulary*** is furnished in foot-notes wherever required.
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8. ***An Outline History*** of IMPORTANT EVENTS is given at the close of ***every chapter.***
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